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THE POMPEIU PROPERTY WITH MOMENTS

Let $\mathbb{R}^n$ be a real Euclidean space of dimension $n \geq 2$ with Euclidean norm $|\cdot|$, and let

$$B_r = \{x \in \mathbb{R}^n : |x| < r\}.$$

Assume that $f \in L_{loc}(\mathbb{R}^n)$, let r be a fixed positive number and let

$$\int_{B_r} f(x+u)du = 0 \tag{1.1}$$

for all $x \in \mathbb{R}^n$. Does this imply that $f = 0$? The answer is in the negative (see, for instance, [1, Part 2]); however, under some additional assumptions f is indeed zero function. One such assumption is a sufficiently rapid decrease of f at infinity. For instance, it is known that if a function satisfying (1.1) belongs to the class $L^p(\mathbb{R}^n)$ for some $p \in [1, 2n/(n-1)]$, then $f = 0$, whereas for $p > 2n/(n-1)$ this does not hold any more (see [1], [2], where significantly more general and precise results in this direction were obtained). Another type of restrictions that ensure vanishing of f is related with increase of the number of possible values of r in condition (1.1). In particular, the property $f = 0$ is recovered by using two balls of appropriately chosen radii (see [1–4] and the extensive bibliography therein).

The authors consider condition (1.1) together with the vanishing first moments of f over balls for the case where f is defined in a ball B_R of radius $R > r$. In general we are asking about description and properties of the function space for f.

References

[1] *Volchkov V. V.* Integral Geometry and Convolution Equations. – Dordrecht: Kluwer Academic Publishers, 2003. – 454 p.

[2] *Volchkov V. V., Volchkov Vit. V.* Harmonic Analysis of Mean Periodic Functions on Symmetric Spaces and the Heisenberg Group. – London: Springer, 2009. – 671 p.

[3] *Zalcman L.* A bibliographic survey of the Pompeiu problem; in: B. Fuglede et al. (eds.), *Approximation by Solutions of Partial Differential Equations.* – Dordrecht: Kluwer Academic Publishers, 1992. – P. 185–194.

[4] *Zalcman L.* Supplementary bibliography to 'A bibliographic survey of the Pompeiu problem'; in *Radon Transforms and Tomography* // Contemp. Math. – 2001. – V. 278. – P. 69–74.