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COMMUTATIVE ALGEBRAS OF TOEPLITZ OPERATORS ON THE BERGMAN SPACE

Let $\mathbb{B}^n$ be the unit ball in $\mathbb{C}^n$, with $n \geq 1$. Denote by $\mathcal{A}^2_\lambda(\mathbb{B}^n)$, $\lambda \in (-1,\infty)$, the standard weighted Bergman space, which is the closed subspace of $L^2_\lambda(\mathbb{B}^n)$ consisting of analytic functions. The Toeplitz operator T_a with symbol $a \in L_\infty(\mathbb{B}^n)$ acting on $\mathcal{A}^2_\lambda(\mathbb{B}^n)$ is defined as the compression of a multiplication operator on $L^2_\lambda(\mathbb{B}^n)$ onto the Bergman space, i.e., $T_a f = B_\lambda(af)$, where B_λ is the Bergman (orthogonal) projection of $L^2_\lambda(\mathbb{B}^n)$ onto $\mathcal{A}^2_\lambda(\mathbb{B}^n)$.

Note that for a generic subclass $S \subset L_\infty(\mathbb{B}^n)$ of symbols the algebra $\mathcal{T}(S)$ generated by Toeplitz operators T_a with $a \in S$ is non-commutative and practically nothing can be said on its structure. However, if $S \subset L_\infty(\mathbb{B}^n)$ has a more specific structure (e.g. induced by the geometry of $\mathbb{B}^n$, or with a certain smoothness properties) the study of operator algebras $\mathcal{T}(S)$ is quite important and has attracted lots of interest during the last decades.

At the turn of this century it was observed there exist many non-trivial algebras $\mathcal{T}(S)$ that are commutative on each standard weighted Bergman space. The talk aims to the description, classification, and the structural analysis of these commutative algebras.

The commutative C^*-algebras generated by Toeplitz operators are classified as follows: *given any maximal commutative subgroups of bihomorphisms of the unit ball $\mathbb{B}^n$, the C^*-algebra generated by Toeplitz operators whose symbols are constant on the orbits of this subgroup is commutative on each weighted Bergman space $\mathcal{A}^2_\lambda(\mathbb{B}^n)$.*

For each such commutative C^*-algebra there exists a unitary operator R, specific for each concrete algebra, such that any Toeplitz operator T_a from this algebra is *unitary equivalent to a multiplication operator*: $RT_aR^* = \gamma_a I$. This result gives us an easy access to the majority of the essential properties of Toeplitz operators, such as compactness, boundedness, spectral properties, invariant subspaces, etc.

As a leading example we analyze then the C^*-algebra generated by Toeplitz operators with radial symbols $a(z) = a(|z|)$, and show that, independently of a dimension n of the unit ball and a wight parameter $\lambda \in (-1,\infty)$, this algebra is isomorphic and isometric to the C^*-algebra of sequences $\mathrm{SO}(\mathbb{Z}_+)$ that *slow oscillate* in the sense of Schmidt (1924):

$$\mathrm{SO}(\mathbb{Z}_+) = \left\{ x \in \ell_\infty : \lim_{\frac{j+1}{k+1} \to 1} |x_j - x_k| = 0 \right\}.$$

As a byproduct we give a solution of the weighted extension of the classical Hausdorff moment problem.