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Analogues of the Eisenstein integrals on the hyperbolic plane

The eigenfunctions of a perturbed Laplacian on the hyperbolic plane are studied. The integral representations of the kernel of the Poisson type for homogeneous eigenfunctions of this operator are obtained. Let $s \in \mathbb{C}$,

$$\mathcal{L} = 4(1-|z|^2)^2 \frac{\partial^2}{\partial z \partial \overline{z}} - 4s^2|z|^2 Id - 4s(1-|z|^2)\left(z\frac{\partial}{\partial z} - \overline{z}\frac{\partial}{\partial \overline{z}}\right),$$

where Id is the identity transformation. We set $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$.

Theorem i)Let $k \in Z$, $\lambda \in \mathbb{C}$, $\nu = \nu(\lambda) = \frac{1-i\lambda}{2}$.An arbitrary smooth function $f \in \mathbb{D} \to \mathbb{C}$, satisfying to the equation

$$(1) \qquad \mathcal{L}f = -(\lambda^2 + 4s^2 + 1)f$$

and the homogeneity condition

$$(2) \qquad f(e^{i\theta}z) = e^{ik\theta}f(z)$$

has the form $f(z) = (\mathfrak{D}_k f)(0) H^s_{\lambda,k}(|z|)(\frac{z}{|z|})^k$,where

$$(3) \qquad \mathfrak{D}_k = \begin{cases} \frac{1}{k!}\left(\frac{\partial}{\partial z}\right)^k, k \geq 0 \\ \frac{1}{(-k)!}\left(\frac{\partial}{\partial \overline{z}}\right)^{-k}, k < 0, \end{cases}$$

$H^s_{\lambda,k}(|z|) = |z|^{|k|}(1-|z|^2)^\nu F(\nu + s + \frac{|k|-k}{2}; \nu - s + \frac{|k|+k}{2}; |k|+1; |z|^2)$ F is the hypergeometric function.

(ii)Let $\eta \in \mathbb{S}^1$ and

$$(4) \qquad E^s_{\nu,\eta}(z) = \left(\frac{1-|z|^2}{|z-\eta|^2}\right)^\nu \left(\frac{1-z\overline{\eta}}{1-\eta\overline{z}}\right)^s, |z| < 1,$$

Then

$$(5) \qquad \frac{1}{2\pi}\int_0^{2\pi} E^s_{\nu,e^{i\theta}}(z)e^{ik\theta}d\theta = c_{\nu,k,s}H^s_{\lambda,k}(|z|)(z/|z|)^k,$$

Where

$$(6) \qquad c_{\nu,k,s} = \begin{cases} \frac{\Gamma(k+\nu-s)}{k!\Gamma(\nu-s)}, k \geq 0, \\ \frac{\Gamma(-k+\nu+s)}{(-k)!\Gamma(\nu+s)}, k > 0. \end{cases}$$

For other results in this direction, see [1-2].

1.V.V.Volchkov, Vit.V.Volchkov, Harmonic Analysis of Mean Periodic Function on Symmetric Spaces and the Heisenberg Group. 2009.

2. V.V.Volchkov, Vit.V.Volchkov, Offbeat Integral Geometry on Symmetric Speces. Basel, 2013,592 p.