Mizel‘s problem on a circle

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The problem is known in literature as **Mizel's problem**(A characterization of the circle): *A closed convex curve such that, if three vertices of any rectangle lie on it, so does the fourth, must be a circle.*

In 1961 A.S. Besicovitch [1] solved this problem. Later, a modified proof of this statement was presented by L.W. Danzer, W.H. Koenen, C. St. J. A. Nash-Williams, A.G.D. Watson [2, 3, 4, 5].

In 1989 T. Zamfirescu [6] proved a similar result for a Jordan curve (not convex a priory) and for a rectangle with the infinitesimal relation between its sides:

$$\left|\frac{a}{b}\right| \leq \varepsilon > 0,$$

where a and b are sidelengths of a rectangle.

In 2006 M. Tkachuk [7] obtained the most general result in this area for any arbitrary compact set $C \subset \mathbb{R}^2$, where the complement $\mathbb{R}^2 \backslash C$ is not connected.

T. Zamfirescu proved that *every analytic curve of constant width satisfying the infinitesimal rectangle property is a circle.* Our aim is to prove the theorem for a convex curve without the analyticity condition(Theorem 1 [6]) and to discuss new unsolved problems concerning Mizel's problem.

Theorem.[9] *Any convex curve of constant width satisfying the infinitesimal rectangular condition is a circle.*

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