The properties of functions satisfying mean value equations

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The classes of functions on subsets of the compact plane that fulfill the conditions of the next type are studied in this work

$$\sum_{n=s}^{m-1} \frac{r^{2n+2}}{2(n-s)!(n+1)!} \left(\frac{\partial}{\partial z}\right)^{n-s} \left(\frac{\partial}{\partial \bar{z}}\right)^{n} f(z) = \frac{1}{2\pi} \int\int_{|\zeta - z| \leq r} f(\zeta)(\zeta - z)^s d\xi d\eta,$$

where $m \in \mathbb{N}$ and $s \in 0, ..., m-1$ are fixed. Also r is fixed or belongs to the set of two elements.

We point out that this equation holds for m -analytic functions. The set of functions from C^{2m-2-s} in some domain, that satisfies this equation with all possible z and r is of great interest.

Also a two-radii theorem is obtained. It turn out that this theorem characterizes the class of solutions for equation

$$\left(\frac{\partial}{\partial z}\right)^{m-s} \left(\frac{\partial}{\partial \bar{z}}\right)^{m} f = 0$$

in terms of the first equation.

Note that the case $s \geq m$ that corresponds to the zero integral mean value in the right hand side of the first equation, has been studied in the works by L.Zalcman and V.V.Volchkov.

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