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Cone condition and α-accessible domains

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The lecture will be based on a paper of P. Liczberski and V. V. Starkov [1].

The domains in $\mathbb{R}^n$ with some non-flates property are well-known in the analysis. In the Encyclopedia of Mathematics [2] such domains are defined as follows.

Let us denote by $K(p,e,\alpha,r)$ the closed circular cone with the vertex at a point $p \in \mathbb{R}^n$, an axis vector e, of the opening $\alpha\pi$, $\alpha \in (0,1)$ and the height $r \in (0,\infty]$. We say that a domain $\Omega \subset \mathbb{R}^n$ satisfies the "cone condition" if for every $p \in \Omega$ the cone $K(p,e(p),\alpha,r)$ is included in Ω for some fixed $\alpha \in (0,1)$ and $r \in (0,\infty]$.

The cone property and its generalizations are the main tool for solving very important various mathematical problems in the theory of integral representations of functions (see [3]), problems concerning the imbedding theorems (see, e.g., [2], (Chapt.1, Par.8), and problems occuring in the investigation of boundary behaviour of arbitrary functions (see e.g. [5]).

We say that a domain $\Omega \subset \mathbb{R}^n$, $0 \in \Omega$, is α-accessible, $\alpha \in [0,1)$, if for every point $p \in \partial\Omega$ there exists a number $r = r(p) > 0$ such that the cone $K_+(p,\alpha,r) \equiv K(p,p,\alpha,r)$ is included in $\Omega' = \mathbb{R}^n \setminus \Omega$.

We succeed in achieving a characterization of some geometric properties of α-accessible domains. In particular we showed that if $\Omega \subset \mathbb{R}^n$ is an α-accessible domain, $\alpha \in (0,1)$, then for every $p \in \partial\Omega$ and every $\eta \in (0,\alpha)$ there exists a number $r = r(\eta) > 0$, such that the bounded cone $K_-(p,\eta,r) \equiv K(p,-p,\eta,r)$ is included in Ω. We found an analytic test for α-accessibility of domains with smooth boundaries and an analytic condition for α-accessibility of domains with arbitrary boundaries. We obtained also another properties of such domains, some of them are new, even in the planar case.

As an application of the above we solved completely a problem on a characterization of all α-accessible domains in $\mathbb{C}^N$ which are biholomorphic to the Euclidean ball.

References

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