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BAIRE CATEGORIES AND WIMAN'S INEQUALITY FOR THE ANALYTIC FUNCTIONS

Let $f(z)=\sum_{n=0}^{+\infty}a_nz^n$ $(z\in\mathbb{C})$ be an analytic function in the unit disk and f_t be an analytic function of the form $f_t(z)=\sum_{n=0}^{+\infty}a_ne^{i\theta_nt}z^n$, where $t\in\mathbb{R}$, $\theta_n\in\mathbb{N}$, and h be a positive continuous on $(0,1)$ function increasing to $+\infty$ and such that $\int_0^1 h(r)dr=+\infty$. If the sequence $(\theta_n)_{n\geq 0}$ satisfies the inequality

$$\gamma(\theta)=\varlimsup_{n\to+\infty}\frac{1}{\ln n}\ln\frac{\theta_n}{\theta_{n+1}-\theta_n}\leq\delta\in[0,1/2), \tag{1}$$

then for every analytic functions f almost surely for t there exists a set $E=E(\delta,t)\subset(0,1)$ such that $\int_E h(r)dr<+\infty$ and

$$\varlimsup_{\substack{r\to1-0\\ r\notin E}}\Delta_h(r,f_t)=\varlimsup_{\substack{r\to1-0\\ r\notin E}}\frac{\ln M_f(r,t)-\ln\mu_f(r)}{2\ln h(r)+\ln\ln\{h(r)\mu_f(r)\}}\leq\frac{1+3\delta}{4+2\delta},$$

where $M_f(r,t)=\max\{|f_t(z)|\colon |z|=r\}$, $\mu_f(r)=\max\{|a_n|r^n\colon n\geq 0\}$ for $r\in[0,1)$.

Let $\theta=(\theta_n)_{n\geq0}$ be a fixed sequence satisfying condition (1), such that $\gamma(\theta)\leq\delta$. We define the following sets

$$F_{1h}(f,\theta,E)=\Big\{t\in\mathbb{R}\colon \varlimsup_{\substack{r\to1-0\\ r\notin E}}\Delta_h(r,f_t)\leq\frac{1+3\delta}{4+2\delta}\Big\},$$

$$F_{2h}(f,\theta)=\Big\{t\in\mathbb{R}\colon \varliminf_{r\to1-0}\Delta_h(r,f_t)\leq\frac{1+3\delta}{4+2\delta}\Big\}.$$

We conclude that for analytic functions in $\mathbb{D}$ there exists the set $E(f)$ of finite h-measure such that the set $F_{1h}(f,\theta)$ is "large" in the sense of Lebesque measure. Therefore, we obtain some information on set $F_{2h}(f,\theta)$.

The following question arises naturally: *does there exists a set $E=E(f)$ of the finite h-measure such that the set $F_{1h}(f,\theta,E)$ is residual in $\mathbb{R}$ for every analytic function f?*

We recall that a set $B\subset\mathbb{R}$ is called residual in $\mathbb{R}$, if its complement $\overline{B}=\mathbb{R}\backslash B$ is a set of the first Baire category in $\mathbb{R}$. It is clear, that if the answer to the question is affirmative, then the set $F_{2h}(f,\theta)$ are residual in $\mathbb{R}$. However for some analytic function the set $F_{1h}(f,\theta,E)$ is a set of the first Baire category. It follows from the following theorem.

Theorem 1. *Let a sequence $(\theta_n)_{n\geq0}$ such that for any $n\in\mathbb{N}$: $\theta_{n+1}/\theta_n\geq q>1$, $f(z)=\sum_{n=0}^{+\infty}e^{n^\varepsilon}z^n$, $\varepsilon\in(0,1)$, and $h(r)=(1-r)^{-1}$. Then there exists a constant $C=C(\theta,\varepsilon)>0$ such that for all sequences $(r_n)_{n\geq0}$ increasing to 1 the set*

$$F_3=\Big\{t\in\mathbb{R}\colon \varlimsup_{n\to+\infty}\frac{M_{f_t}(r_n)}{h(r_n)\mu_f(r_n)\ln^{1/2}\{h(r_n)\mu_f(r_n)\}}\leq C\Big\}$$

is a set of the first Baire category.

Theorem 2. *If sequence $(\theta_n)_{n\geq0}$ satisfies condition (1) and $h\in H$, then for every analytic function f the set $F_{2h}(f,\theta)$ is residual in $\mathbb{R}$.*