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ANALOG OF MINIOWITZ THEOREM FOR SOME CLASS OF MAPPINGS WITH NON–BOUNDED CHARACTERISTICS

Let D be a domain in $\mathbb{R}^n$, $n \geq 2$, and $f : D \to \mathbb{R}^n$ be a continuous mapping. In what follows, m be the Lebesgue measure in $\mathbb{R}^n$, $\overline{\mathbb{R}^n} = \mathbb{R}^n \cup \{\infty\}$ be the one–point compactification of $\mathbb{R}^n$, and M be the conformal modulus of families of curves, $M(\Gamma) = \inf\limits_{\rho \in \mathrm{adm}\,\Gamma} \int\limits_{\mathbb{R}^n} \rho^n(x)\, dm(x)$, where inf is taken over all nonnegative Borel functions $\rho : D \to [0, \infty]$ with $\int\limits_{\gamma} \rho(x)\, |dx| \geq 1$ for each $\gamma \in \Gamma$ (that is can be written as $\rho \in \mathrm{adm}\,\Gamma$). A mapping $f : D \to \mathbb{R}^n$ is said to be a *discrete* if the preimage $f^{-1}(y)$ of every point $y \in \mathbb{R}^n$ consists of isolated points, and an *open* if the image of every open set $U \subset D$ is open in $\mathbb{R}^n$. Given a domain D and two sets E and F in $\overline{\mathbb{R}^n}$, $n \geq 2$, $\Gamma(E, F, D)$ denotes the family of all paths $\gamma : [a, b] \to \overline{\mathbb{R}^n}$ which join E and F in D, i.e., $\gamma(a) \in E$, $\gamma(b) \in F$ and $\gamma(t) \in D$ for $a < t < b$. Denote by $S(x_0, r_1)$ and $S(x_0, r_2)$ the corresponding boundaries of the spherical ring $A(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\}$ and let $S_i = S(x_0, r_i)$, $i = 1, 2$. Given a (Lebesgue) measurable function $Q : D \to [0, \infty]$, a mapping $f : D \to \mathbb{R}^n$ is called *ring Q–mapping at a point* $x_0 \in D$ if the conformal modulus satisfies the following inequality

$$M\left(f(\Gamma(S_1, S_2, A(x_0, r_1, r_2)))\right) \leq \int\limits_{A(x_0,r_1,r_2)} Q(x) \cdot \eta^n(|x - x_0|)\; dm(x) \qquad (1)$$

for any $A(x_0, r_1, r_2)$, $0 < r_1 < r_2 < r_0 = \mathrm{dist}(x_0, \partial D)$, and for every Lebesgue measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that $\int\limits_{r_1}^{r_2} \eta(r) dr \geq 1$. It is known that a conformal mapping f satisfies the (1) with $Q \equiv 1$, and quasiconfornal mapping satisfies the (1) with $Q \equiv K = const$. We say that a function $\varphi : D \to \mathbb{R}$ has *finite mean oscillation* at a point $x_0 \in D$ if

$$\limsup_{\varepsilon \to 0} \frac{1}{\Omega_n \cdot \varepsilon^n} \int\limits_{B(x_0,\,\varepsilon)} |\varphi(x) - \widetilde{\varphi_\varepsilon}| dm(x) < \infty$$

where $\widetilde{\varphi_\varepsilon} = \frac{1}{\Omega_n \cdot \varepsilon^n} \int\limits_{B(x_0,\,\varepsilon)} \varphi(x)\, dm(x)$. In the extended space $\overline{\mathbb{R}^n} = \mathbb{R}^n \bigcup \{\infty\}$, we use a *spherical (chordal)* distance $h(x, y) = |\pi(x) - \pi(y)|$, where π is a stereographical projection of $\overline{\mathbb{R}^n}$ onto the sphere $S^n(\frac{1}{2}e_{n+1}, \frac{1}{2})$ in $\mathbb{R}^{n+1}$:

$$h(x, \infty) = \frac{1}{\sqrt{1 + |x|^2}}, \quad h(x, y) = \frac{|x - y|}{\sqrt{1 + |x|^2}\sqrt{1 + |y|^2}}\,, \quad x \neq \infty \neq y\,.$$

The following result takes a place.

Theorem. *A family of all discrete open ring Q–mappings $f : D \to \overline{\mathbb{R}^n}$ at the point $x_0 \in D$ with $Q \in FMO(x_0)$ is equicontinuous at the point $x_0 \in D$ if and only if there exist $p = p(n, Q) > 0$, $C_n > 0$ and $\varepsilon_0(x_0) > 0$ such that*

$$h(f(x), f(x_0)) \leq C_n \left\{ \frac{1}{\log \frac{1}{|x - x_0|}} \right\}^p \qquad \forall\, x \in B(x_0, \varepsilon_0).$$