Pukhtaievych R. P. (Kiev, Ukraine)

Monogenic functions in a finite-dimensional semi-simple algebra

Let $\mathbb{A}_n$ be a n-dimensional ($2 \le n < \infty$) commutative associative Banach algebra over the field of complex numbers $\mathbb{C}$. Let $I_1, I_2, \ldots, I_n$ be a basis of the algebra $\mathbb{A}_n$ with the multiplication table

$$I_k{}^2 = I_k, \quad I_k I_j = 0, \qquad k, j = 1, 2, \ldots, n, \;\; k \neq j.$$

The unit of $\mathbb{A}_n$ is represented as the sum of idempotents: $1 = I_1 + I_2 + \cdots + I_n$.

Let $E_m := \{\zeta = \sum_{j=1}^{m} x_j e_j : \;\; x_j \in \mathbb{R}\}$ be a linear span in $\mathbb{A}_n$ over the field of real numbers $\mathbb{R}$.

Let Ω be a domain in E_m. We say that a continuous function $\Phi : \Omega \to \mathbb{A}_n$ is *monogenic* in Ω if Φ is differentiable in the sense of Gateaux in every point of Ω, i.e. if for every $\zeta \in \Omega$ there exists an element $\Phi'(\zeta) \in \mathbb{A}_n$ such that

$$\lim_{\varepsilon \to 0+0} \left(\Phi(\zeta + \varepsilon h) - \Phi(\zeta)\right) \varepsilon^{-1} = h \Phi'(\zeta) \quad \forall h \in E_m.$$

$\Phi'(\zeta)$ is the Gateaux derivative of the function Φ in the point ζ.

We obtained a constructive description of monogenic functions $\Phi : \Omega \to \mathbb{A}_n$ by means of analytic functions of the complex variable. We proved that the mentioned monogenic functions have the Gateaux derivatives of all orders.