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LOCALLY PIECEWISE LINEAR APPROXIMATION IN THE WEIGHTED SOBOLEV SPACES

Let G is an open bounded set in R^n , L_k and H_k are the k -dimensional Lebesgue and Hausdorff measures, A_p is class of locally integrable functions $w : R^n \rightarrow (0, \infty)$ satisfying the Muckenhoupt condition [1]

$$\sup_Q \frac{1}{|Q|} \int_Q w dx \left(\frac{1}{|Q|} \int_Q w^{1-q} dx \right)^{p-1} < \infty,$$

where the supremum is taken over all coordinate cubes $Q \subset R^n$, $|Q| = L_n(Q)$; $p, q \in (1, \infty)$ and $\frac{1}{p} + \frac{1}{q} = 1$.

For a weight function $w \in A_p$, by $L_{p,w}^1(G)$ denote the class of function $u : G \rightarrow (-\infty, +\infty)$, that are locally integrable in G , have generalized partial derivatives on G , and satisfy the condition $\int_G |\nabla u|^p w dx < \infty$.

Proof is based on the methods of the paper [2].

Theorem 1. For any $\varepsilon > 0$ and any function $u \in L_{p,w}^1(G)$, there is continuous locally piecewise linear function $L(x) \in L_{p,w}^1(G)$ such that

$$\int_G |\nabla u - \nabla L|^p \cdot w dx < \varepsilon.$$

Similar the statement were formulated for admissible function u in a problem of conformal capacity [3], [4], in that specific case, when $p = n$ and G – ring or shell in R^n .

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References

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