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A NUMERICAL ALGORITHM OF SOLVING A NONLINEAR WAVE EQUATION

Let us consider the nonlinear differential equation

$$w_{tt}(x,t) = \varphi\left(\int_0^\pi w_x^2(x,t)\,dx\right) w_{xx}(x,t), \quad 0 < x < \pi, \quad 0 < t \le T, \tag{1}$$

with the initial boundary conditions

$$\begin{gathered} w(x,0) = w^0(x), \quad w_t(x,0) = w^1(x), \\ w(0,t) = w(\pi,t). \end{gathered} \tag{2}$$

Here $\varphi(z)$, $w^0(x)$ and $w^1(x)$ are the known functions, where

$$\varphi(z) \ge \alpha > 0, \quad 0 \le z < \infty,$$

and T, α are some constants.

The physical meaning of equation (1) consists in the description of a dynamic string in the conditions of nonlinear stress-strain dependence, while a particular form of this equation when $\varphi(z) = \alpha_0 + \alpha_1 z$ is applicable in the case of the fulfillment of Hooke's linear law and was obtained by G. Kirchhoff [2].

Let $\varphi(z) \in C^p[0,\infty)$, where p can be equal both to 1 and to 2. Assume $w^0(x)$ and $w^1(x)$ to be functions of the form $w^l(x) = \sum\limits_{i=1}^{\infty} a_i^{(l)} \sin ix$ and $|a_i^{(l)}| \le \frac{\omega}{i^{p+s+l-2,5}}$, $i = 1, 2, \ldots, l = 0, 1$, where ω and s are some positive constants. These requirements guarantee the existence of a local solution of problem (1), (2) [1]. To find it, we used an approximate algorithm consisting of the projection method and the difference scheme for obtaining an approximate solution with respect to spatial and time variables, while the resulting discrete system is solved by the iteration method. The algorithm error is estimated.

The question of constructing a numerical algorithm for obtaining a global solution of problem (1), (2) in general and particular cases and of estimating the algorithm accuracy is considered in [3] and [4].

References

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