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ON PSEUDOREGULAR SOLUTIONS OF THE DIRICHLET PROBLEM FOR DEGENERATE BELTRAMI EQUATIONS

Here it is formulated criteria for the existence of solutions of the Dirichlet problem to degenerate Beltrami equations in the case of arbitrary finitely connected domains. Let D be a domain in the complex plane $\mathbb{C}$ and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a.e. (almost everywhere). The Beltrami equation is the equation of the form $f_{\overline{z}} = \mu(z) f_z$ where $f_{\overline{z}} = (f_x + i f_y)/2$, $f_z = (f_x - i f_y)/2$, $z = x + iy$, and f_x and f_y are the partial derivatives of f with respect to x and y, respectively. Set $K_\mu(z) = (1 + |\mu(z)|)/(1 - |\mu(z)|)$. The Beltrami equation is called degenerate if K_μ is essentially unbounded, i.e., $K_\mu \notin L^\infty(D)$.

A pseudoregular solution of the Dirichlet problem for the Beltrami equation in a bounded domain D in $\mathbb{C}$ is a mapping $f : D \to \overline{\mathbb{C}}$ which is continuous in $\overline{\mathbb{C}}$, discrete, open and $f \in W^{1,1}_{loc}$ outside of isolated poles in D with Jacobian $J_f(z) = |f_z|^2 - |f_{\overline{z}}|^2 \neq 0$ a.e. satisfying the Beltrami equations a.e. and the boundary condition $\lim\limits_{z \to p} Re f(z) = \varphi(p)$ $\forall p \in \Pi_D$ for a given continuous function $\varphi : \Pi_D \to \mathbb{R}$. Here Π_D is the set of prime ends of D, see, e.g., [1]. Recall that a mapping $f : D \to \overline{\mathbb{C}}$ is called *discrete* if the preimage $f^{-1}(z)$ of every point $z \in \overline{\mathbb{C}}$ consists of isolated points and *open* if the image of any open set $U \subseteq D$ is an open set in $\overline{\mathbb{C}}$.

The Dirichlet problem is well studied for uniformly elliptic systems (see, e.g., [2] and [3]). The Dirichlet problem for degenerate Beltrami equations in domains bounded by a finite number of mutually disjoint Jordan curves studied in [4].

Theorem 1. *Let D be a bounded domain in $\mathbb{C}$ whose boundary has $s \geq 2$ connected components and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a.e. such that*

$$\int\limits_D \Phi(K_\mu(z))dz < \infty$$

where $\Phi : [0, \infty] \to [0, \infty]$ is a nondecreasing convex function satisfying the condition

$$\int\limits_\delta^\infty \frac{d\tau}{\tau \Phi^{-1}(\tau)} = \infty$$

for some $\delta > \Phi(0)$. Then the Beltrami equation has a pseudoregular solution of the Dirichlet problem for any continuous function $\varphi : \Pi_D \to \mathbb{R}$, $\varphi(p) \not\equiv const$, with poles at $p \geq s - 1$ prescribed inner points in D.

1. E. F. Collingwood, A. J. Lohwater. The theory of cluster sets. - Cambridge, Cambridge University Press, 1966; transl. in Moscow: Mir, 1971.
2. B. V. Boyarskii. Generalized solutions of a system of first-order elliptic differential equations with discontinuous coefficients, Mat. Sb., 1957, 43(85), 451-503; transl. in Rep. Univ. Jyväskylä Dept. Math. Stat. 2009, 118, 1–64.
3. I. N. Vekua. Generalized Analytic Functions. - Moscow, Fizmatgiz, 1959 [in Russian]; transl. in London: Pergamon Press, 1962.
4. D. A. Kovtonyuk, I. V. Petkov, and V. I. Ryazanov. On the Dirichlet problem for the Beltrami equations in finitely connected domains, Ukr. Mat. Zh., 2012, 64, No. 7, 932-944 [in Russian]; transl. in Ukr. Math. J., 2012, 64, No. 7, 1064-1077.