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APPROXIMATION BY SHIFTS OF FINITE RADIAL FUNCTIONS

Let $\mathbb{R}^n$ be a real Euclidean space of dimension $n \geq 2$.

For $\lambda > 0$, let $N_\lambda = \{r > 0 : J_{n/2}(r\lambda) = 0\}$ where J_k is the kth-order Bessel function of the first kind.

In this note, we investigate the problem of approximation of functions in the space $L^p(H)$ where $2 \leq p < +\infty$ and

$$H = \{x = (x_1, \dots, x_n) \in \mathbb{R}^n : x_n > 0\}$$

by shifts of finite radial functions. We write $\widehat{g}$ for the Fourier transform of integrable function in $\mathbb{R}^n$. For $R > 0$, let B_R denotes the open ball in $\mathbb{R}^n$ with radius R centered at the origin. We write also $f * g$ for the convolution of functions f and g in the case where it there exists.

The following result is an analogue of well-known Wiener's theorem on approximation by shifts in the space $L(\mathbb{R}^n)$.

Theorem 1. *Let $p \in [2, +\infty)$, $f \in L^p(H)$, and assume that the support of function f is contained in*

$$H_t = \{x = (x_1, \dots, x_n) \in \mathbb{R}^n : x_n > t\}$$

for some $t > 0$. Suppose also that $g \in L(\mathbb{R}^n)$ is supported in the ball B_t and that the zero set of $\widehat{g}$ contains some sphere with the center at the origin. Then the function $f * g$ is the limit in $L^p(H)$ of a sequence of linear combinations of indicator functions of balls in H with radii $r \in N_\lambda$.

The condition on zero set of $\widehat{g}$ in previous theorem possesses for a broad class of functions g. The theorem is longer valid if instead of indicators of balls one takes certain finite radial functions.