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Uniform distance to the space of continuous functions

In [1, p.23] was obtained one result of a uniform distance from an arbitrary bounded function $g : X \to \mathbb{R}$ defined on paracompact space X to the space of bounded continuous functions $f : X \to \mathbb{R}$. The proof was based on the famous theorem of Hahn - Dieudonne - Tong - Katetov [2, p.105] which the authors have shown to paracompact space with a selection Michael's theorem. Meanwhile, the theorem is true for normal spaces. In addition, we have no one had found a uniform distance from some of the functions $g : \mathbb{R} \to \mathbb{R}$ to the space $C(\mathbb{R})$ of all continuous functions $f : \mathbb{R} \to \mathbb{R}$, in particular the unbounded function $g(x) = [x]$.Thus arose the natural question of extension of said result of [1].

We put

$$\|g\|_\infty = sup\{|g(x)| : x \in X\} \quad i \quad d(g, E) = inf\{\|g - f\|_\infty : f \in E\}.$$

for a function $g : X \to \mathbb{R}$ and non-empty set $E \subseteq \mathbb{R}^X$.

Let X is a topological space and $\mathcal{U}_x$ is the system of all neighborhoods of point x in X. We consider oscillation $\omega_g(x)$ of a function $g : X \to \mathbb{R}$ that determined by formula

$$\omega_g(x) = inf_{U \in \mathcal{U}_x} sup_{x', x'' \in U} |g(x' - g(x'')|.$$

Theorem 1. *Let X is a normal space and $g : X \to \mathbb{R}$ is a function. Than $d(g, C(X)) = \frac{1}{2}\|\omega_g\|_\infty$.*

For example, if $g(x) = [x]$ than $d(g, C(\mathbb{R})) = \frac{1}{2}$ and this distance realized on continuous function $f(x) = x - \frac{1}{2}$.

Moreover, were found uniform distances from some functions $g : \mathbb{R} \to \mathbb{R}$ to sets of quasi-continuous or somewhat continuous in zero functions [3].

References

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