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LOCAL POLYNOMIAL APPROXIMATION ON ARCS IN THE COMPLEX PLANE

Besides the classical theory of polynomial approximation such as Jackson and Bernstein theorems defining global approximation, Nikolsky, Timan and Dzyadyk theorems determining local-global approximation, there exist purely local polynomial approximations. The first such theorem as well-known was obtained by P.M.Tamrazov and V.V.Berdzinskiy. They were considered a class of function on $[1, -1]$ interval defined in $x_0 \in [1, -1]$ point in which $|f(x) - f(x_0)| \leq const|x - x_0|$. They proved that there exists a such polynomial $P_n(x)$ for which $|f(x_0) - P_n(x_0)| \leq \frac{const}{n^\alpha}$.

On solving the problem of approximation on arcs we introduced a local class of functions $D_\alpha^\beta(z_0, \Gamma)$ ($z_0 \in \Gamma,\ \Gamma$ - arbitrary arc in a complex plane) ($0 < \alpha \leq 1,\ \beta \geq 0$), for which $\forall z_1, z_2\ \ |f(z_1) - f(z_2)| \leq C(\Gamma) max\{|z_0 - z_1|^\beta, |z_0 - z_2|^\beta\}, |z_1 - z_2|^\alpha$.

In addition, we considered also local class of functions introduced by A.A.Gonchar $H_\alpha^{\alpha+\beta}(z_0, \Gamma)$ consisting of functions $f(z) \in H^\alpha(\Gamma)$ (class of Holder α order), and for which $|f(z) - f(z_0)| \leq const|z - z_0|^{\alpha+\beta}$ ($0 < \alpha \leq 1,\ \beta \geq 0$). For these classes of functions, we obtained direct and inverse theorems of polynomial approximations, closer each other. That allows us to get the constructive characterization for classes of $D_\alpha^\beta(z_0, \Gamma)$ and $H_\alpha^{\alpha+\beta}(z_0, \Gamma)$.