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Theorems on a fixed point for multivalued mappings

Let E^n be n-dimensional Euclidean space (real or complex), $\langle *, * \rangle$ be the inner product in E^n, A be some subset of E^n, *conv* A be a convex hull of A.

We consider the multivalued mappings (including single-valued and discontinuous mappings) of subsets of Euclidean space.

Let X and Y be some topological spaces. The mapping $F : X \to Y$ is called a multivalued mapping iff the set $F(x) \subset Y$ is the image of the point $x \in X$.

Let $F_1, F_2 : X \to Y$ be two multivalued mappings. The mapping F_2 is the restriction of F_1 iff $F_1(x) \supset F_2(x) \neq \varnothing$ for all points $x \in X$.

The mapping F satisfies "an acute angle condition" ("a strict acute angle condition") on the set A iff $X = Y$ and $\operatorname{Re} \langle x, y \rangle \geq 0$ ($\operatorname{Re} \langle x, y \rangle > 0$) for all pairs of points $x \in A$, $y \in F(x)$.

Definition. The restriction of function $f : A \to B$ to the subset $C \subset A$, i. e.

$$f \mid_C (x) = \begin{cases} f(x) \ , & x \in C, \\ \varnothing, & x \notin C \end{cases}$$

is called the limitation f to C.

Theorem 1. *Let D be a domain in Euclidean space E^n containing the origin 0. Let $K \subset \overline{D}$ be a subset of the closure of this domain and K has the following property (α): any ray, emanating from the origin, contains at least one point belonging to K. Suppose that the limitation $F \mid_K$ of multivalued mapping $F : \overline{D} \to E^n$ to K satisfies "the acute angle condition" and conv $F(K)$ is a compact set. If conv $F(K) \subset F(\overline{D})$ then $0 \in F(\overline{D})$.*

Corollary 1. *Let $K \subset \overline{D}$ be a subset of the domain $\overline{D}$ and K has the property (α). Suppose that the limitation $F \mid_K$ of multivalued mapping $F : \overline{D} \to E^n$ to K has the restriction $F_1 \neq \varnothing$ and conv $F_1(K)$ is a compact set. Let conv $F_1(K) \subset F(\overline{D})$. If $0 \notin F(\overline{D})$ then there exists a pair of points $x \in K$, $y \in F(x)$, such that* $\operatorname{Re} \langle x, y \rangle < 0$.

Theorem 2. *Let D be a domain in Euclidean space E^n containing the origin 0. Let $K \subset \overline{D}$ be a subset of the closure of this domain and K has the property (α). Suppose that the limitation $F \mid_K$ of multivalued mapping $F : \overline{D} \to E^n$ to K satisfies "the strict acute angle condition". If conv $F(K) \subset F(\overline{D})$ then $0 \in F(\overline{D})$.*

Corollary 2. *Let $K \subset \overline{D}$ be a subset of the domain $\overline{D}$ and K has the property (α). Suppose that the limitation $F \mid_K$ of multivalued mapping $F : \overline{D} \to E^n$ to K has the restriction $F_1 \neq \varnothing$ and conv $F_1(K) \subset F(\overline{D})$. If $0 \notin F(\overline{D})$ then there exists a pair of points $x \in K$, $y \in F(x)$, such that* $\operatorname{Re} \langle x, y \rangle \leq 0$.

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