Vladimir V. Kisil (Leeds, UK)

The Real and Complex Techniques in Harmonic Analysis from the Covariant Transform

There are two main approaches in harmonic analysis on the real line. The real variables technique uses various maximal functions, dyadic cubes and, occasionally, the Poisson integral. The complex variable technique is based on the Cauchy integral and fine properties of analytic functions.

Both methods seem to have clear advantages. The real variable technique:

- (1) does not require an introduction of the imaginary unit for a study of realvalued harmonic functions of a real variable (Occam's Razor);
- (2) allows a straightforward generalisation to several dimensions.

By contrast, access to the beauty and power of analytic functions (e.g., Möbius transformations, factorisation of zeroes, etc.) is the main reason to use the complex variable technique.

We demonstrated that both—real and complex—techniques in harmonic analysis have the same group-theoretical origin. Moreover, they are complemented by the wavelet construction. Therefore, there is no any confrontation between these approaches. In other words, the binary opposition of the real and complex methods resolves into Kant's triad thesis-antithesis-synthesis: complex-real-covariant.

References

- 1. V.V. Kisil, The Real and Complex Techniques in Harmonic Analysis from the Covariant Transform, 2012. arXiv:1209.5072.
- V.V. Kisil, Calculus of Operators: Covariant Transform and Relative Convolutions, 2013. arXiv:1304.2792.