Olena Karlova (Chernivtsi, Ukraine)

On extension of Baire mappings and Lebesgue α-triples

A. Lebesgue [3] proved that each separately continuous function $f : \mathbb{R}^2 \to \mathbb{R}$ is of the first Baire class. In honour of this theorem we call a collection (X, Y, Z) of topological spaces *a Lebesgue α-triple*, or, briefly, *α-triple*, if any mapping $f : X \times Y \to Z$ which is continuous with respect to the first variable and of the α'th Baire class with respect to the second one belongs to the $(\alpha + 1)$'th Baire class, where $0 \leq \alpha < \omega_1$.

Remark that α-triples were studied by a couple of mathematicians (see [2] and the references given there). Among others we indicate the result of W. Rudin [5] who showed that (X, Y, Z) is an α-triple for every $\alpha \geq 0$ if X is a metrizable space, Y is a topological space and Z is a locally convex space. It is well-known that a union of several metrizable spaces is not necessary metrizable. That's why it is naturally to ask: is (X, Y, Z) an α-triple if $X = \bigcup\limits_{n=1}^{N} X_n$, where $2 \leq N \leq +\infty$ and (X_n, Y, Z) is an α-triple for each n?

Theorem 1. *Let $1 \leq \alpha < \omega_0$, X be a completely regular space, $E \subseteq X$ be a set of the functionally multiplicative class α and let Y be a Polish arcwise connected and locally arcwise connected space. If one of the following conditions hold*

(1) *X is perfectly normal,*
(2) *E is Lindelöf,*
(3) *X is normal and E is F_σ,*

then (X, E, Y) has the B_α-extension property.

Applying Theorem 1 we may prove the following classification theorem.

Theorem 2. *Let $1 \leq \alpha < \omega_0$, $\alpha \leq \beta < \omega_1$, $X = \bigcup\limits_{n=1}^{\infty} X_n$ a completely regular space, Y a topological space and let Z be a Polish contractible locally connected space such that for each n the following conditions hold:*

(1) *(X_n, Y, Z) is a β-triple;*
(2) *X_n is a set of the multiplicative class α in X_{n+1};*
(3) *$X_n \times Y$ is Lindelöf or $X \times Y$ is perfectly normal.*

Then (X, Y, Z) is β-triple.

References

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