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ON THE ZYGMUND-TYPE ESTIMATE FOR A QUATERNION SINGULAR CAUCHY INTEGRAL

Let $\mathbb{H}(\mathbb{C})$ be the algebra of complex quaternions $a = \sum_{k=0}^{3} a_k \boldsymbol{i_k}$, where $\{a_k\}_{k=0}^{3} \subset \mathbb{C}$, $\boldsymbol{i}_0 = 1$ be the unit, and $\boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3$ be imaginary quaternion units.

Let γ be a closed Jordan rectifiable curve in $\mathbb{R}^2 \ni \zeta := \zeta_1 \boldsymbol{i}_1 + \zeta_2 \boldsymbol{i}_2$, let $\alpha \in \mathbb{C}$. For a continuous function $f : \gamma \to \mathbb{H}(\mathbb{C})$ consider the quaternion singular Cauchy integral (see [1])

$$F_\alpha^{(2)}[f](t) := \int\limits_{\gamma} K_\alpha^{(2)}(\zeta - t)\,(d\zeta_2 \boldsymbol{i}_1 - d\zeta_1 \boldsymbol{i}_2)\,(f(\zeta) - f(t))\,, \qquad t \in \Gamma,$$

where $K_\alpha^{(2)}$ is a fundamental solution of the operator $D_\alpha^{(2)}[f] := \sum_{k=1}^{2} \boldsymbol{i_k} \frac{\partial f}{\partial \zeta_k} + \alpha f$.

We established in [2] sufficient conditions for existence of the integral $F_\alpha^{(2)}[f]$ and proved an upper Zygmund-type estimate for its modulus of continuity in terms of the modulus of continuity of the function f and a metric characteristic of the curve γ.

Let Γ be a closed Jordan rectifiable surface in $\mathbb{R}^3 \ni \zeta := \zeta_1 \boldsymbol{i}_1 + \zeta_2 \boldsymbol{i}_2 + \zeta_3 \boldsymbol{i}_3$. For a continuous function $f : \Gamma \to \mathbb{H}(\mathbb{C})$ consider the quaternion singular Cauchy integral

$$F_\alpha^{(3)}[f](t) := \int\limits_{\Gamma} K_\alpha^{(3)}(\zeta - t)\,\nu(\zeta)\,(f(\zeta) - f(t))\,ds_\zeta, \qquad t \in \Gamma,$$

where $\nu(\zeta) := \nu_1(\zeta)\boldsymbol{i}_1 + \nu_2(\zeta)\boldsymbol{i}_2 + \nu_3(\zeta)\boldsymbol{i}_3$ is the unit normal vector to the surface Γ, $K_\alpha^{(3)}$ is a fundamental solution of the operator $D_\alpha^{(3)}[f] := \sum_{k=1}^{3} \boldsymbol{i_k} \frac{\partial f}{\partial z_k} + \alpha f$, and ds_ζ is the surface square element.

Let $\theta_z(\delta)$ be the surface measure of the set $\Gamma_{z,\delta} := \{\zeta \in \Gamma : |\zeta - z| \leqslant \delta\}$.

Definition. A closed Jordan rectifiable surface Γ is called a regular surface, if there exists a positive constant K, such as for all $z \in \Gamma$ and for all $\delta > 0$ the next inequality holds true:

$$\theta_z(\delta) \leqslant K\delta^2.$$

We established in [3] sufficient conditions for existence of the integral $F_\alpha^{(3)}[f]$ on a regular surface and proved an upper Zygmund-type estimate for its modulus of continuity in terms of the modulus of continuity of the function f.

R E F E R E N C E S

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