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AMBIGUOUS POINTS OF FUNCTIONS, DEFINED IN DOMAINS IN $\mathbb{R}^n$

Let g be any function defined in $E \subset \mathbb{R}^n$ with values in $\overline{\mathbb{C}}$, $A \subset E$, $\zeta \in \partial E \cap \overline{A}$. The cluster set $C(g, \zeta, A)$ of g along A consists of all $w \in \overline{\mathbb{C}}$, such that, for some sequence $z_N \in A$, $z_N \underset{N\to\infty}{\longrightarrow} \zeta$, we have $g(z_N) \underset{N\to\infty}{\longrightarrow} w$.

Let f be any function, defined in the unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$. Point $\zeta \in \partial\Delta$ is an ambiguous point of a function f if there exist two Jordan arcs Γ_1 and Γ_2, lying in the Δ, except the endpoint ζ, such that $C(f, \zeta, \Gamma_1) \cap C(f, \zeta, \Gamma_2) = \emptyset$. In 1955 F. Bagemihl [1] proved that an arbitrary function in Δ can have at most countable set of ambiguous points.

Examples have been given in [2], [3], [4] to show that this theorem fails for functions in the unit Euclidean ball $\mathbb{B}^n \subset \mathbb{R}^n$, $n \geq 3$. Moreover it shows that Bagemihl's theorem is not true even for continuous, harmonic or homeomorphic in $\mathbb{B}^n$ functions.

This problem was solved by P.J. Rippon in [5] with changing the definition of ambiguous point. The Jordan arcs Γ_1 and Γ_2 from Bagemihl's definition were replaced by

1) a subdomain D of $\mathbb{B}^n$ with $\partial D \cap \partial\mathbb{B}^n = \{\zeta\}$,

2) an Jordan arc lying in D with endpoint ζ at $\partial\mathbb{B}^n$, such that

$$C(f, \zeta, \partial D \setminus \{\zeta\}) \cap C(f, \zeta, \Gamma) = \emptyset.$$

This definition remains Bagemihl's theorem true for functions in $\mathbb{B}^n$: the set of a such points ζ is at most countable.

We have generalized the Rippon's theorem. In our case conditions on domain D are weaker, the intersection $\partial D \cap \partial\mathbb{B}^n$ can be an infinite set.

Theorem 1. *Let f be a function in $\mathbb{B}^n$, M is a fixed subset of $\partial\mathbb{B}^n$. Let $\zeta \in M$ such that there exist*

1) a domain $D \subset \mathbb{B}^n$, $\partial D \cap M = \{\zeta\}$,

2) a Jordan arc $\Gamma \subset D$ with endpoint ζ,

$$C(f, \zeta, \partial D \cap \mathbb{B}^n) \cap C(f, \zeta, \Gamma) = \emptyset.$$

Then the set of such points ζ is at most countable.

The Theorem 1 remains true if $\partial D \cap M$ is a finite set and it doesn't if $\partial D \cap M$ is an infinite set.

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References

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