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ON POWER SERIES APPROACH FOR ONE BOUNDARY VALUE PROBLEM FOR MONOGENIC FUNCTIONS OF BIHARMONIC VARIABLE IN A DISK

We consider the commutative algebra $\mathbb{B}$ over the field of complex numbers with the bases $\{e_1, e_2\}$ satisfying the conditions $(e_1^2+e_2^2)^2 = 0$, $e_1^2+e_2^2 \neq 0$. This algebra is unique and it is associated with the 2-D biharmonic equation. We consider monogenic functions (having the classic derivative in domains of the biharmonic plane $\mu := \{xe_1 + ye_2\}$, where x, y are real) with values in $\mathbb{B}$. For these functions, we consider a Schwarz-type boundary value Problem (associated with the main biharmonic problem) for a disk $D_R := \{\zeta \in \mu : ||\zeta|| \leq R\}$, $R > 0$: on finding a monogenic in D_R function

$$\Phi(\zeta) = U_1(x,y)\, e_1 + U_2(x,y)\, ie_1 + U_3(x,y)\, e_2 + U_4(x,y)\, ie_2, \tag{1}$$

$U_k\colon D \longrightarrow \mathbb{R}$, $k = \overline{1,4}$, when values of the components U_1 and U_3 are given on the circle ∂D_R, i.e. the following boundary conditions are satisfied:

$$U_1(x,y) = u_1(\zeta)\,, \quad U_3(x,y) = u_3(\zeta) \qquad \forall\, \zeta \in \partial D_R. \tag{2}$$

We call this problem as *(1-3) Problem.* Note, that any component of (2) is a biharmonic function.

Using polar coordinates $x = r\cos\theta$, $y = r\sin\theta$, rewriting boundary conditions (2) in the following form:

$$u_k(\theta) \equiv u_k(r\cos\theta + r\sin\theta\, e_2),\ k = 1,3,\ 0 \leq \theta \leq 2\pi. \tag{3}$$

We say that a function having the following Fourier's expansion:

$$f(\theta) = \frac{\alpha_0}{2} + \sum_{k=1}^{\infty} (\alpha_k \cos k\theta + \beta_k \sin k\theta)\,,\ 0 \leq \theta \leq 2\pi, \tag{4}$$

belongs to the class $\mathcal{F}_\alpha$, where $\alpha > 0$, if and only if, when members of convergent series $\sum_{k=1}^{\infty} |\alpha_k|$ and $\sum_{k=1}^{\infty} |\beta_k|$ satisfy the following condition: there exists a constant $M > 0$ such, that

$$|\alpha_k| \leq \frac{M}{n^{2+\alpha}}, \quad |\beta_k| \leq \frac{M}{n^{2+\alpha}}, k = 1, 2, \ldots. \tag{5}$$

Theorem 1. *If u_1 and u_3 belong to $\mathcal{F}_\alpha$, then solution of the* (1-3) *Problem for D_R is solvable if and only if the following condition is satisfied*:

$$\int\limits_{\partial D_R} u_1(\zeta)\, dx + u_3(\zeta)\, dy = 0. \tag{6}$$

Then any solution of the (1-3) *Problem is expressed in the form of the uniformly convergent in $\overline{D_R}$ power series*:

$$\Phi(\zeta) = \sum_{k=0}^{\infty} b_k \zeta^k, \tag{7}$$

where $b_k \in \mathbb{B}$, $k = 1, 2, \ldots$, are exactly expressions using the Fourier's coefficients of the expansions of u_1 and u_3.