Generalized Convexity, Blaschke-type Condition in Unbounded Domains, and Application in Operator Theory

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It is known that zeros z_n of any bounded analytic function in the unit disc satisfy Blaschke condition $\sum(1-|z_n|) < \infty$. There are a lot of generalization of Blaschke condition to unbounded analytic functions (M.M.Djrbashian, W.Hayman and B.Korenblum, F.A.Shamoyan, and many others).

We consider the case of analytic functions f in the unit disc growing near a subset E of the boundary and obtain an analog of the above condition

$$\sum(1-|z_n|)\rho^{\kappa}(z,E) < \infty,$$

where $\rho(z,E)$ is the distance between z and E, $\kappa > 0$ depends only on growth of f and Minkowski dimension of the set E. Also, we show that our result is sharp.

Next, we introduce a notion of r-convexity for subsets of the complex plane. It is a pure geometric characteristic that generalizes the usual notion of convexity. Next, we investigate analytic and subharmonic functions that grow near the boundary in unbounded domains with r-convex compact complement. We obtain the Blaschke-type bounds for its Riesz measure and, in particular, for zeros of unbounded analytic functions in unbounded domains. These results are based on a certain estimates for Green functions on complements of some neighborhoods of r-convex compact set. Also, we apply our results in perturbation theory of linear operators in a Hilbert space. More precisely, we find quantitative estimates for the rate of condensation of the discrete spectrum of a perturbed operator near its the essential spectrum.