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METHODS QUASICONFORMAL MAPPINGS AND SPLITTINGS FOR SOLUTION OF A CLASS OF NONLINEAR BOUNDARY VALUE PROBLEMS

The efficient constructive approach to modeling of quasiideal processes in doubly-connected curvilinear domain bounded by equipotential lines and to solving the corresponding nonlinear elliptic boundary value problems was developed based on the synthesis of numerical methods complex analysis and numerical-analytical generalizations methods of summary representations (methods of splitting) .

We consider the stationary nonlinear quasiideal process described by the equation of motion $\vec{v} = \kappa \cdot grad\, \varphi$ and the continuity equation $div\, \vec{v} = 0$ [1] ($\vec{v} = \upsilon_x(x, y) + i\upsilon_y(x, y)$ – speed, κ – coefficient of conductivity, φ – potential of fields: $\varphi|_{L_*} = \varphi_*$, $\varphi|_{L^*} = \varphi^*$) in domain G_z ($z = x + iy$), bounded by closed equipotential lines L_* and L^*.

By entering, similar as [1], the function of flow $\psi = \psi(x, y)$, quasicomplex conjugate to φ, we arrive at the problem of quasiconformal mapping $\omega = \omega(z) = \varphi(x, y)+i\psi(x, y)$ domain $G_z^L = G_z/L$ (where L - imaginary incision along the selected line of flow) on the corresponding rectangular domain of complex quasipotential $G_\omega = \{\omega = \varphi + i\psi : \varphi_* < \varphi < \varphi^*,\ 0 < \psi < Q\}$ with an unknown parameter (total flow) $Q = \oint_{L_*} -\upsilon_y dx + \upsilon_x dy$:

$$\begin{cases} \frac{\partial\varphi}{\partial x} = \kappa(\varphi, \psi)\frac{\partial\psi}{\partial y}, \quad \kappa(\varphi, \psi)\frac{\partial\varphi}{\partial y} = -\frac{\partial\psi}{\partial x}, \quad (x, y) \in G_z^L; \\ \varphi\,|_{L_*} = \varphi_*, \quad \varphi\,|_{L^*} = \varphi^*, \quad \psi\,|_{L_0} = 0, \quad \psi\,|_{L^0} = \oint_{L_*} -\kappa\frac{\partial\varphi}{\partial y}dx + \kappa\frac{\partial\varphi}{\partial x}dy\ , \end{cases}$$

where $L_0,\ L^0$ - borders of the incision L.

In the case where $\kappa(\varphi, \psi) = \bar{\kappa}\,(\varphi)\,\underline{\kappa}\,(\psi)$, the corresponding inverse problem is reduced to quasiconformal mapping of $z(\omega) = x(\varphi, \psi)+iy(\varphi, \psi)$ domain G_ω on G_z^L:

$$\begin{cases} \mathrm{L}x \equiv \frac{1}{\bar{\kappa}(\varphi)}\frac{\partial}{\partial\varphi}\left(\frac{1}{\bar{\kappa}(\varphi)}\frac{\partial x}{\partial\varphi}\right) + \underline{\kappa}\,(\psi)\,\frac{\partial}{\partial\psi}\left(\underline{\kappa}\,(\psi)\,\frac{\partial x}{\partial\psi}\right) = 0, \\ \mathrm{L}y = 0, \quad (\varphi, \psi) \in G_\omega, \\ f_*\left(x\left(\varphi_*, \psi\right), y\left(\varphi_*, \psi\right)\right) = 0,\ f^*\left(x\left(\varphi^*, \psi\right), y\left(\varphi^*, \psi\right)\right) = 0,\ 0 \le \psi \le Q, \\ x\left(\varphi, 0\right) = x\left(\varphi, Q\right), \quad y\left(\varphi, 0\right) = y\left(\varphi, Q\right), \quad \varphi_* \le \varphi \le \varphi^*, \\ \left(\frac{\partial x}{\partial\varphi}\cdot\frac{\partial f_*}{\partial y} - \frac{\partial y}{\partial\varphi}\cdot\frac{\partial f_*}{\partial x}\right)\Big|_{\varphi=\varphi_*} = 0, \quad \left(\frac{\partial x}{\partial\varphi}\cdot\frac{\partial f^*}{\partial y} - \frac{\partial y}{\partial\varphi}\cdot\frac{\partial f^*}{\partial x}\right)\Big|_{\varphi=\varphi^*} = 0, \\ Q = \oint_{L_*}\frac{\bar{\kappa}(\varphi)\underline{\kappa}(\psi)}{J}\frac{\partial x}{\partial\psi}dx + \frac{\bar{\kappa}(\varphi)\underline{\kappa}(\psi)}{J}\frac{\partial y}{\partial\psi}dy, \quad J = \frac{\partial x}{\partial\varphi}\frac{\partial y}{\partial\psi} - \frac{\partial x}{\partial\psi}\frac{\partial y}{\partial\varphi}\ . \end{cases}$$

Numerical-analytical representations of the solutions this nonlinear boundary value problem based on the idea of gradual fixation of some parameters and a combination of the numerical (finite-difference) and analytical (splittings, integral representations, etc.) methods, which are generalizations of methods of of summary representations [2] that greatly accelerates achieve the desired of the conjugacy quasiconformal functions.

References

[1] Bomba A.Ya., Bulavatsky V.M., Skopetsky V.V. (2007) "Nonlinear mathematical models of geohydrodynamics processes", Naukova Dumka, Kyiv, Ukraine.

[2] Polozhii G.M. (1962) "The numerical solution of two-dimensional and three-dimensional boundary-value problems of mathematical physics and function of the discrete argument", Publishing KSU, Kyiv.