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ALPHA-ACCESSIBLE DOMAINS IN $\mathbb{R}^n$, A NONSMOOTH CASE

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The lecture continues the study of α–accessible domains in $\mathbb{R}^n$ (see [1], [2]). They are starlike domains and satisfy cone condition which is important for applications. Conditions of α–accessibility of domain, defined by the inequality $F(x) < 0$, is obtained for a continuous function F in $\mathbb{R}^n$.

A domain $\Omega \subset \mathbb{R}^n$, $0 \in \Omega$, is α-accessible, $\alpha \in [0, 1)$, if for every point $p \in \partial\Omega$ there exists a number $r = r(p) > 0$ such that the cone

$$K_+(p, \alpha, r) = \left\{ x \in \overline{\mathbb{B}}^n(p, r) : \ \left(x - p, \frac{p}{\|p\|} \right) \geq \|x - p\| \cos \frac{\alpha\pi}{2} \right\}$$

is included in $\Omega' = \mathbb{R}^n \setminus \Omega$.

We assume that:

a) the function $F(x)$ is defined and continuous on $\mathbb{R}^n$;

b) the open set $D = \{x \in \mathbb{R}^n : F(x) < 0\} \ni 0$;

c) there exists derivatives $\dfrac{\partial F}{\partial l}(p)$ at the points of the set level $S = \{p \in \mathbb{R}^n : F(p) = 0\}$ in all directions $l \in (K_+(p, \alpha) - p) \setminus \{0\}$.

Theorem 1. *Let the assumptions a), b), c) be satisfied. If D is α–accessible domain for a certain $\alpha \in [0; 1)$ then derivatives $\dfrac{\partial F}{\partial l}(p) \geq 0$ for any direction $l \in (K_+(p, \alpha) - p) \setminus \{0\}$ and for any point $p \in S$.*

Theorem 2. *Let the assumptions a), b), c) be satisfied and D is bounded set. If for a certain $\alpha \in [0; 1)$ derivatives $\dfrac{\partial F}{\partial l}(p) > 0$ for any direction $l \in (K_+(p, \alpha) - p) \setminus \{0\}$ and for any point $p \in S$ then D is α–accessible domain.*

Theorem 3. *Let the assumptions a), b) be satisfied and D is bounded set. If for a certain $\alpha \in [0; 1)$ and an arbitrarily small $\delta > 0$ derivatives $\dfrac{\partial F}{\partial l}(p) \geq 0$ for any direction $l \in (K_+(p, \alpha) - p) \setminus \{0\}$ and for any point $p \in D^\delta = \{x \in D : \rho(x, S) < \delta\}$ then D is α–accessible domain.*

The conditions received in theorems give an opportunity to study starlikeness of the set in some cases, while the results from [3] are not applicable.

References

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