Mappings with module conditions in metric spaces with measures

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Let us consider the following system of Borel measures which are associated with continua in metric spaces (X,d). A measure $m_\gamma^{(k)}$ is associated with continuum γ in (X,d) is defined for every Borel set B in (X,d) as Hausdorff measure H^k of $B\cap\gamma$ at fixed $k>0$. For every continuum $\gamma\in\Gamma$, set $m_\gamma := m_\gamma^{(1)}$. Now, let (X,d,μ) be a metric space with a Borel measure μ. μ-measurable functions $\rho : X\to[0,\infty]$, satisfying $\int\limits_X \rho\, dm_\gamma \geq 1$, for all curves $\gamma\in\Gamma$ are called **admissible functions** for Γ, abbr. $\rho\in adm\Gamma$. p**-modulus**, $p\in(0,\infty)$, of Γ is the quantity $M_p(\Gamma)=\inf\limits_{\rho\in adm\,\Gamma}\int\limits_X \rho^p(x)\,d\mu(x)$ where the infimum is taken over all $\rho\in adm\Gamma$. $M_p(\Gamma)=+\infty$ if $\Gamma=\emptyset$. Later $\Gamma(A,B;C)$ denotes the family of all continua $\gamma\in\Gamma$ connecting A and B in C, i.e., such that $\gamma\cap A\neq\emptyset$, $\gamma\cap B\neq\emptyset$ and $\gamma\setminus\{A\cup B\}\subseteq C$. A **generalized domain** in topological space T is an open set D whose each pair of points can be immersed in a continuum γ in D.

Let D and D' be generalized domains in (X,d,μ) and (X',d',μ'), respectively, $Q: X\to(0,\infty)$ be a μ-measurable function and $p\in(0,\infty)$. We say that homeomorphism $f: D\to D'$ is a **generalized ring Q-homeomorphism at a point** $x_0\in\overline{D}$ with respect to p-module, if $M_p(\Gamma(f(C_0),\, f(C_1);\, D'))\leq \int\limits_{A\cap D} Q(x)\cdot\eta^p(d(x,x_0))\, d\mu(x)$ for every ring $A=A(x_0,r_1,r_2)$ $0<r_1<r_2<\infty$, for any two continua $C_0\subset\overline{B(x_0,\, r_1)}\cap D$ and $C_1\subset D\backslash B(x_0,r_2)$ and for every Borel function $\eta:(r_1,r_2)\to[0,\infty]$, such that $\int\limits_{r_1}^{r_2}\eta(r)\,dr \geq 1$. A homeomorphism $f: D\to D'$ is called a generalized ring Q-homeomorphism if f is a generalized ring Q-homeomorphism at every point $x_0\in\overline{D}$. A generalized domain D in (X,d,μ) is called a **generalized quasiextremal distance (QED) domain** with respect to p-module, $p\in(0,\infty)$, if $M_p(\Gamma(E,F;X))\leq K\, M_p(\Gamma(E,F;D))$ for some $K\in[1,\infty)$ and for every continua E and F in D, cf. [1]. We also say that a space (X,d,μ) is **weakly flat at a point** $x_0\in X$ with respect to p-module, $p\in(0,\infty)$, if, for every neighborhood U of the point x_0 and every number $N>0$, there is neighborhood $V\subseteq U$ of x_0, such that $M_p(\Gamma(E,F;X))\geq N$ for any continua E and F in X intersecting ∂V and ∂U, cf. [2].

Теорема. *Let f be a generalized ring Q-homeomorphism with respect to p-module, $p\in(0,\infty)$, between QED generalized domains D and D' in weekly flat spaces X and X', respectively, and let $\overline{D'}$ and $\overline{D}$ be compact. If the function $Q: X\to[0,\infty]$ has finite mean oscillation at a point $x_0\in\partial D$, (see [2]), $\mu(B(x_0,2r))\leq c\cdot\log^{p-2}\frac{1}{r}\cdot\mu(B(x_0,r))$, $\forall\, r\in(0,r_0)$, and (X,d,μ) is upper p-regular with $p\geq 2$ at x_0, then f admits a continuous extension to the point x_0. If the last two conditions hold at every point of $x_0\in\partial D$, then f admits a homeomorphic extension to the boundary.*

1. Gehring F.W., Martio O. Quasiextremal distance domains and extension of quasiconformal mappings, J. Anal. Math., 1985, vol.24, P. 181–206.

2. Martio O., Ryazanov V., Srebro U., Yakubov E. Moduli in Modern Mapping Theory, Springer Monographs in Mathematics. - New York: Springer, 2009.