New perspectives on Weyl algebras

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Among landmarks of the birth of quantum mechanics, the discovery of Weyl algebras occupies a pivotal place, profoundly impacting many branches of physics and mathematics. Their defining feature is capturing the noncommutativity of differential operators with polynomial coefficients, thus providing a natural framework for the algebraic description of the Heisenberg uncertainty principle and the canonical quantization of the classical phase space.

Since the seminal works of Dixmier [1, 2], the structure and representation theory of Weyl algebras and related objects (automorphism groups, endomorphism algebras, etc.) have been the subject of a great research interest, and they are related to well-known conjectures, such as, for example, the Jacobian conjecture.

In [3, 4], it is shown that Weyl algebras arise as algebras of generalized symmetries of the linear (1+1)-dimensional heat equation and the remarkable (1+2)-dimensional Fokker–Planck equation. This realization as symmetry algebras opens the way for a new perspective on the study of generators and relations of Lie algebras associated with the Weyl algebras, and on the gradings of these algebras.

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