Machine learning and prediction of trajectories in optimal control of Lie–Poisson systems

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Wide-spread adoption of unmanned vehicle technologies requires the ability to predict the motion of the combined vehicles from observations. This ability is especially important when observing adversary vehicles. While the general prediction of such motion for an arbitrary control mechanism is difficult, for a particular choice of control, based on symmetry reduction of an optimal control mechanism [2, 3], the dynamics is given by Lie–Poisson equations. Our goal is to learn the phase-space dynamics and predict motion solely from observations, without any knowledge of the control Hamiltonian or the adjacency matrix.

We propose the *control optimal Lie–Poisson neural networks (CO-LPNets)* for learning Lie–Poisson systems from data and predicting the dynamics of the system in the future. These methods learn the mapping of the phase space through the composition of Poisson maps, which are obtained as flows from Hamiltonians that could be integrated explicitly. CO-LPNets preserve the Poisson bracket and thus preserve Casimirs to machine precision. These methods are applicable to any Lie–Poisson control systems; to illustrate the prowess of the method, we apply these techniques towards systems of particles evolving on $\mathrm{SO}(3)^N$ group, N being the number of particles, which describe coupled satellite rotation about their center of mass. We also present the results for $\mathrm{SE}(3)^N$ dynamics, applicable to the movement of unmanned air and water vehicles. We also present some analytical results for the evolution on these groups extending the computations in [2, 3]. Numerical results demonstrate that CO-LPNets learn approximate trajectories, received from ground truth solutions, with good accuracy, over hundreds of time steps, with a very limited number of points necessary for learning the phase space dynamics.

This work is an extension of the earlier results in [1], which is devoted to the data-based computing of the Lie–Poisson system with the use of LPNets to the problems of optimal control of collective motion of particles. The novelty of our work, compared to previous results in this field, lies in learning and predicting the motion of coupled controlled Lie–Poisson systems using only data-based observations. No information about the type of control, the control directions, and the number of control dimensions is necessary for learning the whole phase space and predicting trajectories, as long as the optimal control procedure allows appropriate symmetry reduction. More precisely, our results learn the flow for *any* control Hamiltonian, arising from Lagrangian, which depends only on the controls u, so that the system admits the Lie–Poisson symmetry reduction as described in [2, 4]. Note that in our method, we never compute either the Hamiltonian H, its gradients, or the equations of motion. Instead, we compute only the composition of Poisson transformations, reproducing the dynamics in phase space of some Poisson system, coming from an unknown control Hamiltonian.

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