On partial preliminary group classification of some class of (1+3)-dimensional Monge-Ampère equations. Three-dimensional Abelian Lie algebras

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Monge—Ampère equations of various types and defined in spaces of different dimensions appear in solutions of numerous problems in geometry, theoretical physics, nonlinear elasticity, fluid dynamics, optimal mass transport, geometric optics, one-dimensional gas dynamics, and related fields.

Let us consider the following class of (1+3)-dimensional Monge-Ampère equations:

$$\det (u_{\mu\nu}) = F (x_0, x_1, x_2, x_3, u, u_0, u_1, u_2, u_3),$$

where $u=u(x), x=(x_0,x_1,x_2,x_3)\in M(1,3), u_{\mu\nu}\equiv \frac{\partial^2 u}{\partial x_\mu\partial x_\nu}, u_\alpha\equiv \frac{\partial u}{\partial x_\alpha}, \mu,\nu,\alpha=0,1,2,3$. Here, M(1,3) is a four-dimensional Minkowski space, F is an arbitrary real smooth function.

These classes have been investigated in numerous articles and books using variety of methods. We have used the classical Lie–Ovsiannikov approach to perform partial preliminary group classification of the above class of (1+3)-dimensional Monge–Ampère equations using three-dimensional Abelian nonconjugate subalgebras of the Lie algebra of the Poincaré group P(1,4).

In this report, we intend to present several results related to the partial preliminary group classification of this class.