Connection of PDEs by prolonged nonlocal transformations

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We consider an application of prolonged nonlocal transformations to the prolonged PDEs that do not admit the direct correspondence under appropriate nonlocal transformations in one step but admit such procedure in several steps using the auxiliary intermediate equations.

The systematical using of the nonlocal transformations has shown that wide family of known soluble equations admit an adequate interpretation within the framework of a given approach.

Nevertheless the invariance of such important equations as Korteweg-de Vries (KdV) equation, sine-Gordon (SG) equation and of others with respect to appropriate nonlocal transformations has appeared possible only by several steps, i.e., using appropriate intermediate equations, which are connected with each other by own nonlocal transformation. Therefore we are very interested in a solution of the corresponding problem of nonlocal invariance by one step. It has appeared that direct nonlocal invariance of such equations or, in some cases, of their differential consequences, becomes possible via appropriate generalization of the approach used.

The SG equation has long been known to admit a Bäcklund transformations (BT) from which many of its interesting properties were derived. Let choose the SG equation in the form

$$u_{xt} - \sin u = 0. \tag{1}$$

Assume we have only one equation from a system of well known ABT of this equation, and choose it in the form, solved with respect to u_x

$$u_x = -v_x + \frac{2}{b} \sin\left(\frac{u-v}{2}\right) = 0. \tag{2}$$

Here $v(x,t)$ is a solution of another example of the same equation

$$v_{xt} - \sin v = 0. \tag{3}$$

Let differentiate both the sides of equality (2) with respect to x and substitute the result into (1). Then we simplify an obtained result using an equality $v_{xt} = \sin v$ and its differential consequences. Solving the above result with respect to u_t we once again differentiate it with respect to x. So we have

$$u_{xt} - \sin u = H(u, u_x, v, v_x, v_t, v_{xt}) - \sin u. \tag{4}$$

Here H denotes for simplicity a quite determined function. Simplifying the right side of (4) with use of equations (3) and (2) we find that a result vanishes identically.

The KdV equation is considered from the such point of view.