

Lift of noninvariant solutions of heavenly equations from three to four dimensions

Abstract

The problem of constructing 4-dimensional metrics, which satisfy (anti)-self-dual complex vacuum Einstein equations, is still an unsolved actual problem in physics and geometry. In this paper we obtain new non-invariant solutions of the four-dimensional hyperbolic complex Monge-Ampère equation

$$u_{1\bar{1}}u_{2\bar{2}} - u_{1\bar{2}}u_{2\bar{1}} = -1 \tag{1}$$

This is a single equation to which complex vacuum Einstein equations with ultra-hyperbolic signature are reduced when the condition of (anti)-self-duality is imposed. We obtain its non-invariant solutions using non-invariant solutions of the 3-dimensional reduction of this equation. In particular, the Boyer-Finley equation

$$\psi_{z\bar{z}} = e^{\psi_x} \psi_{xx} \tag{2}$$

describes rotationally invariant solutions of (1). We use non-invariant solutions of (2) that was constructed earlier by the method of group foliation and lift these solutions to 4-dimensional solutions of Monge-Ampère equation (1).

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