Partially invariant solutions in hydrodynamics describing fluid motions with planar trajectories

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We observe a family of partially invariant solution of ideal magnetohydrodynamics equations generated by groups of isometries of sphere and plane [1, 2, 3]. They can be treated as generalizations of the corresponding classical one-dimensional solutions with spherical and planar waves. In both cases the fluid motions possess the following properties.

1. 3D space is foliated by the level surfaces $r = \text{const}$, where r is either Cartesian or spherical coordinate.

2. The velocity and magnetic vectors are decomposed into orthogonal and transversal components to the level surfaces. Absolute values of both components and all thermodynamical functions are constants over each of the level surfaces. Angle ω of rotation of the velocity and magnetic vectors about the normal to the level surfaces changes across each level surface.

3. Particle trajectories and magnetic lines are planar curves.

These solutions are determined by a reduced systems of PDEs with two independent variables and a finite relation for the angle ω. In order to construct the whole 3D picture of motion one should calculate trajectory and magnetic line of any particle belonging to some initial level surface. Then it is necessary to set up and admissible field of directions over the chosen level surface according to the finite relation for function ω. The whole picture of motion is obtained by attaching the calculated trajectory and magnetic line to each point of the level surface according to the field of directions.

We also prove that the set of solutions satisfying properties 2 and 3 with the level surfaces given by $h(x, y, z) = \text{const}$ is exhausted by the constructed ones. It is found that such functions h should satisfy eikonal equation $|\nabla h| = 1$, and have Hesse matrix with algebraic invariants depending only on h. These two conditions select functions h only with planar, cylindrical and spherical level surfaces. Cylindrical case coincide with classical solutions with spherical waves.

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References

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