Group Analysis of a System of Anisotropic Plane Plasticity

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The classical system of plane plasticity with a general yield criterion $f(\sigma, \tau) = 0$ has the following form [1]

$$\begin{aligned} \sigma_{x_1} - 2\tau\left(\theta_{x_1}\cos 2\theta + \theta_{x_2}\sin 2\theta\right) &= \tau_{x_1}\sin 2\theta - \tau_{x_2}\cos 2\theta, \\ \sigma_{x_2} - 2\tau\left(\theta_{x_1}\sin 2\theta - \theta_{x_2}\cos 2\theta\right) &= -\tau_{x_1}\cos 2\theta - \tau_{x_2}\sin 2\theta, \end{aligned} \tag{1}$$

where σ is a hydrostatic pressure, τ is a maximal shear stress, $\theta + \pi/4$ is the angle between the first principal direction of a stress tensor and the ox_1-axis, indices mean the derivation with respect to corresponding variables.

This system describes a plastic plane deformation of anisotropic materials, in particular, is applying in the static of soil medias [2]. In the case when $\tau = \tau(\theta)$ the system (1) has the form

$$\begin{aligned} \sigma_{x_1} - \theta_{x_1}\left[\tau \sin 2\theta\right]'_\theta + \theta_{x_2}\left[\tau\cos 2\theta\right]'_\theta &= 0, \\ \sigma_{x_2} + \theta_{x_1}\left[\tau\cos 2\theta\right]'_\theta + \theta_{x_2}\left[\tau\ \sin 2\theta\right]'_\theta &= 0, \end{aligned} \tag{2}$$

and is a hyperbolic one for any kind of the function τ, therefore this form of dependence is used widely in the theory of plasticity.

In the report we'll present a group classification of the system (2) using well-known methods [3]. There were found some specifications of τ, for which we have the extension of a basic group of symmetries. For the algebras of Lie, corresponding to this specifications the optimal systems of subalgebras were constructed and some invariant solutions were obtained.

This research was supported by PROMEP (Mexico).

References

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[3] L.V. Ovsyannikov, *Group Analysis of Differential Equations*, New York, Academic Press, 1982.