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Cosmological Symmetry Breaking and Generation of Electromagnetic Field*

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Abstract. Cosmological phase transitions accompanied by some kind of symmetry breaking would cause the creation of topological defects and the resulting production of primordial magnetic field. Moreover, such a procedure inevitably affects the cosmic background radiation and it may be observed today. Motivated by the existence of stabilized embedded defects in the standard model of elementary interactions, we discuss their application to the cosmological electromagnetic field generation.

Key words: cosmology; defect; pion string; magnetic field

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1 Introduction

Particle cosmology predicts that our universe has experienced some phase transitions associated with the unification theory of the interactions. One of the important applications of such a transition to the cosmic evolution would be the generation of topological defects [1]. These defects are topologically stable and whether their formation is possible or not and the type of their structure can be judged by the way of symmetry breaking at the phase transition. In addition to conventional topologically stable defects, another kind of cosmological defects are suggested as an interesting product in the early universe [2]. Even when the topologically charged configuration is not to be prevented from becoming a trivial one, such a field configuration can be constructed so that it satisfies the equation of motion although it is not energetically preferable. That is, embedded defects are unstable at zero temperature. In the early universe, however, the finite temperature plasma existed so that they can be stabilized because of the asymmetry between charged and neutral scalar components [3]. At the low temperature, the photon decoupling occurs on the other hand. Then defects undergo core phase transition and/or decay, which might bring primordial magnetic field generation. Although at present there exist magnetic fields in various astrophysical scales, the origin of galactic magnetic fields $\sim 10^{-6}$ G is not revealed completely [4]. We will see pion strings can provide seed magnetic fields which evolve to present large scale fields. Note that, in addition, some kinds of effects on cosmic microwave background radiation could be observed by the interaction between pion fields and cosmic background photons.

A cosmic string is a two-dimensional defect and it is widely investigated since it has various cosmological significances. If the effective potential has the form of a Mexican hat, a phase transition accompanied by some kind of symmetry breaking occurs and cosmic strings are produced.

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For example, suppose a complex scalar filed ϕ obeys

$$V(\phi) = \frac{\lambda}{4}\left(\phi^*\phi - \eta^2\right)^2,$$

where η is the energy scale of the symmetry breaking. At the region sufficiently far from the string core, the string configuration can be expressed as

$$\phi = \eta e^{in\theta},$$

where n is a winding number. The linear mass density of the string, μ, would be

$$\mu \approx \eta^2, \qquad \text{and} \qquad G\mu = \left(\frac{\eta}{M_{\rm Pl}}\right)^2 \tag{1}$$

is the parameter which determines the magnitude of the string effect where $M_{\rm Pl}$ is the Planck mass.

Even in the cases when the vacuum structure of a particle physics model is trivial, a string solution does exist. Although the configuration of embedded defects satisfies equations of motion, they are topologically, and in general also dynamically unstable. If three real scalar field components, $\phi = (\phi_1, \phi_2, \phi_3)$, obeys the effective potential as

$$V(\phi) = \frac{\lambda}{4}\left(\sum_{i=1}^{3}\phi_i^2 - \eta^2\right)^2,$$

then no string would be produced since the vacuum manifold is S^2 as

$$\sum_{i=1}^{3}\phi_i^2 = \eta^2.$$

However, by freezing out certain components such as $\phi_3 = 0$, the vacuum structure is modified to

$$\sum_{i=1}^{2}\phi_i^2 = \eta^2,$$

which is equal to S^1 and a string configuration can be realized.

In the next section, production and evolution of pion strings are briefly introduced. It is indicated that the pion string can be superconducting which means cosmic vortons may be formed and/or long horizon-scale strings can be observed as line-like objects in contrast to usual particle-like ones. Then in Section 3 we consider another possibility of electromagnetic interaction provided by pion strings, that is, the rotation of polarization axis of light, which should inevitably affect the microwave background radiation anisotropy also as a line-like signature. Moreover, the magnetic field generation by pion strings will be discussed in Section 4.

2 Pion string

One example of embedded global string is a pion string [5]. It is predicted in the context of the standard model of strong interaction and produced at the QCD phase transition. Below the confinement scale, this model is described by a sigma model as

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} - V_0, \qquad V_0 \equiv \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \eta^2\right)^2, \tag{2}$$

involving the sigma field σ and the three pions $\vec{\pi} = (\pi^0, \pi^1, \pi^2)$.

Here we briefly introduce how finite temperature effect stabilizes pion strings [6]. When the background photon plasma in the early universe can be regarded as a thermal bath, the interaction between charged fields and photon could be included into the effective potential [3] as

$$V_T = V_0 + \frac{1}{2}e^2\kappa T^2\{(\pi^1)^2 + (\pi^2)^2\}, \tag{3}$$

where T is the temperature of the universe, e is the elementary electric charge and $\kappa \sim \mathcal{O}(1)$ is a constant. Hence in contrast to the neutral field that still obeys a double-well type potential, the charged field is compelled to stay on the symmetric state. The destabilization temperature can be calculated [3] as

$$T_D = 2\lambda^{1/2}\kappa^{-1/2}e^{-1}\eta,$$

by analyzing the stability of the pion string solution using a variational method for an unstable mode of the cylindrically symmetric configuration. At the destabilization temperature, some kinds of embedded defects undergo a core phase transition [7]. When the finite temperature effect dominates, the neutral fields have a string configuration and the charged ones are frozen out so that $\sqrt{(\pi^1)^2 + (\pi^2)^2} = 0$ everywhere. In contrast to that, after a core phase transition, the scalar field has a finite expectation value also at the string core, that is, it settles in the ground state in the whole universe. However, the neutral field configuration is not destroyed. The charged fields have a finite expectation value only at the string core in order to compensate the neutral fields as $\sqrt{\sigma^2 + (\vec{\pi})^2} = \eta$. On the contrary, the charged fields keep zero expectation value far from the string core since the winding number is a kind of topological charge and it must be conserved. In the pion string case, it has been shown a core phase transition should occur by the results of numerical simulations [6] in which pion strings are formed at $T \sim \eta$ and $T_D = 0.04\eta - 0.05\eta$.

For the pion string, since

$$\eta = \Lambda_{\mathrm{QCD}} \approx 200 \text{ MeV},$$

gravitational effect should be negligible during the cosmic evolution as you can see in the equation (1). Thus the most distinct evidence of pion strings could be given by the electromagnetic interaction. Note that the pion strings are expected to be formed in LHC Pb–Pb collision experiments following the Kibble–Zurek mechanism, although it is different from the cosmological scenario. These effects could be observable and bring distinction compared to conventional predictions [8].

One promising electromagnetic effect by pion strings comes from the superconductivity [9]. After the core phase transition, charged fields have finite expectation value as

$$\sqrt{(\pi^1)^2 + (\pi^2)^2} = \phi_c(x, y)e^{i\varphi_c(z,t)},$$

where xy-plain lies perpendicular to the string and the phase has a spatial gradient along the string so that the electric current will be generated whose amplitude, J_c, will be

$$J_c \sim e\frac{d\varphi_c}{dz}. \tag{4}$$

The evidence of the existence of superconducting strings can be found by cosmological observations [10]. In addition, superconducting sting loops may become vortons [11] in some cases and have potential cosmological consequences [12]. However, since the linear mass density is small compared to, for example, GUT scale strings, the particle emission and cosmic microwave radiation distortion caused by superconducting pion strings will be overcome by other effects.

Thus the possible detection of a string might be obtained by the spatial distribution pattern of emission or distortion. This is very similar to the case of a gravitational lensing [13]. Although the lensing effect itself occurs frequently in the universe, a line-like spatial distribution feature must be characteristic. Also in the case of pion strings, infinitely long strings would show a distinct pattern compared to other various fluctuation sources and could directly prove the existence of horizon scale long strings.

Here we do not discuss the observational consequences in detail but just refer the result of 3-dimensional numerical simulations. Their results have confirmed that the pion string has a superconducting mode after it experiences a core phase transition. We have simulated the evolution of a pion string using a code based on the one employed in [6]. In calculations, all dimensional quantities are rescaled by appropriate powers of η to make them dimensionless. The spatial resolution is $\Delta x = 0.5\eta^{-1}$, and the time steps are chosen as $\Delta t = \frac{1}{10}\Delta x$. The size of box is $300^2 \times 600$ with periodic boundary in the z direction. For x and y directions, Neumann boundary conditions are employed but we have checked that the result is insensitive to this particular choice. First we set up the initial configuration as an infinitely long straight global string with a width, $100\eta^{-1}$, along the z-axis which is formed by the neutral components of the scalar fields σ and π_0 whose center resides in the midpoint of the xy-plane. For charged components, $\pi_1 = \pi_2 = 0$. Although such a highly symmetric configuration might be too ideal, it would be appropriate to see whether the superconducting can occur or not for the horizon scale string. We then add thermal energy to the configuration in the form of kinetic energy, that is, the time derivative of the scalar fields. Its amplitude is $0.1 \times T^2$, and the allocation to the four components of the scalar field is chosen at random. The initial temperature is well below T_D, and equal to 0.01η. Then, four scalar fields σ and $\vec{\pi}$ are evolved numerically on a three-dimensional lattice by means of the equations of motion derived from (2). During each simulation, the background temperature is constant and the cosmic expansion is not taken into account. The results of 20 times simulations with different initial settings of the phases of the time derivative of the scalar fields show that the distribution of φ_c shows the winding number appears in some cases. The probability of winding number appearance has been derived to be 20% which coincides with the analytical estimation as about 16% based on the random phase distribution of $600\eta^{-1}/T^{-1}$ steps along the string. Thus it is confirmed that the pion string after the core phase transition can have superconductivity. Note that we have calculated for various patterns of the numerical values of parameters and confirmed they do not affect the essential results.

3 Interaction of scalar field with electromagnetic field

Here we deal with one of the possibilities of observing the line-like signature by horizon scale pion strings. In general, the following type of interaction $\mathcal{L}_{\rm int}$ between the electromagnetic filed and a certain kind of field, $\mathcal{O}_\mu$, would appear in the Lagrangian when the anomaly or the Chern–Simons term is taken into account [14]. For example,

$$\mathcal{L}_{\rm int} = -\frac{1}{2}\mathcal{O}_\mu A_\nu \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta},$$

where A_μ is the electromagnetic field and $F_{\mu\nu}$ is its strength. Then the equations of motion for electromagnetic field should be modified as

$$\partial_\mu F^{\mu\nu} = 4\pi J^\nu + \mathcal{O}_\mu \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta},$$

so that the polarization of background radiation would occur because of the field, $\mathcal{O}_\mu$.

In case of the pion string, there exists an interaction between the pion field π^0 and the electromagnetic field as

$$\mathcal{L}_{\rm int} = -\frac{N_c\alpha}{24\pi}\frac{\pi^0}{\eta}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}, \tag{5}$$

where $N_c = 3$ and α is a fine structure constant [15]. We can estimate the effect on the light propagation based on modified Maxwell equations using this Lagrangian.

Since we would like to know how the propagation of the cosmic background radiation can be affected by pion strings, we assume the pion string field configuration is not disturbed by electromagnetic field and pion fields contribute just as background current sources. Moreover, the time evolution of string distribution is neglected so that the string spatial position can be regarded as fixed during the photon propagation. Such assumptions can be applied when the string motion is sufficiently slow. Then the equations without an ordinary electromagnetic current under the string background will be written as

$$\nabla\mathbf{E} = -\mathbf{\Phi}\cdot\mathbf{B}, \qquad -\frac{\partial\mathbf{E}}{\partial t} + \nabla\times\mathbf{B} = \mathbf{\Phi}\times\mathbf{E}, \qquad \mathbf{\Phi} \equiv \frac{N_c\alpha}{3\pi}\nabla\frac{\pi^0}{\eta},$$

and the dispersion relation becomes

$$k^2 = \omega^2 \pm \omega\Phi\cos\varphi\left(1 - \frac{\Phi^2\sin^2\varphi}{\omega^2 - k^2}\right)^{-1/2},$$

where $\Phi \equiv |\mathbf{\Phi}|$ and φ is the angle between $\mathbf{\Phi}$ and the wave number vector $\mathbf{k}$. Under the approximation that Φ is sufficiently small,

$$k \cong \omega \pm \frac{\Phi}{2}\cos\varphi.$$

Thus the rotation angle difference of polarization axis between left-handed polarization and right-handed one, $\Delta\psi$, can be estimated dependent on the direction of the string axis to the line of sight as

$$\Delta\psi = \frac{\Phi}{2}\cos\varphi d = \frac{N_c\alpha}{6\pi}\left|\nabla\frac{\pi^0}{\eta}\right| d\cos\varphi,$$

where d is the distance a photon travels from the source to the observer. When the light travels perpendicular to the string and sufficiently close to the string core where $\pi_0 = 0$, $\left|\nabla\frac{\pi^0}{\eta}\right|d \sim 1$ if d is sufficiently large since $-\eta \le \pi_0 \le \eta$ far away from the string. Then

$$\Delta\psi \approx 10^{-3}, \tag{6}$$

which is the maximum value since $\cos\varphi = 1$. Note that the sign depends on which side the light passes. On the other hand, when the light travels in parallel with the string,

$$\Delta\psi \approx 0.$$

In any cases, the distance between the string and the light path is not so significant.

It has been shown that similarly to other defects [16] pion strings also cause the polarization rotation angle difference. What is new in the present work is the application to the pion string and the quantitative estimation in (6). Its amplitude is very small and hard to be detected. Hence the string signature could be claimed when the polarization rotation angle has a line-like spatial pattern, which is the same situation as one in the last section.

4 Generation of magnetic field helicity from pion strings

In this section, we consider how the same interaction as (5) affects the generation of magnetic field. Due to this additional interaction zero mode current appears within the string core and the azimuthal magnetic field as

$$B_\theta = -N_c \frac{en_c}{2\pi} \delta_s^{-\alpha/\pi} r^{-1+\alpha/\pi}, \tag{7}$$

is produced[17] where n_c is a line number density of charge carriers on the string, δ_s is the string core length and r is the distance from the string. This formula was derived [18] by solving the classical equation of motion for electromagnetic fields with a source term added by the interaction with a scalar field (5). The solution was calculated under the condition that the background scalar field has a configuration obtained from an infinitely long straight pion string along the z-axis. The magnetic field strength generated by a pion string at the recombination epoch has been derived [17] by substituting appropriate values as

$$B_\theta \approx 10^{-23}\ \mathrm{G} \left(\frac{r_i}{\delta_s}\right)^{\alpha/\pi} \frac{1\ \mathrm{kpc}}{r_p}, \tag{8}$$

where r_i is r measured at the string formation and r_p is r of the present scale. Although this numerical value might be sufficient for the seed magnetic field amplitude, we will see it can be enhanced when the helicity conservation is taken into account.

If the twist and tangle of strings are biased when the CP violation exists, then the helicity of magnetic field is also biased so that its conservation leads to the generation of larger magnetic field amplitude [19]. First let us estimate the magnetic helicity density, $\mathcal{H}$, produced by pion strings. Since it can be written as

$$\mathcal{H} = \frac{1}{V} \int_V d^3x A_\mu B^\mu,$$

within a certain volume V and the vector potential can be written by the same calculation as one in (7) as

$$A_z = \frac{\pi}{\alpha} N_c \frac{en_c}{2\pi} \delta_s^{-\alpha/\pi} r^{\alpha/\pi},$$

we can calculate $\mathcal{H}$ if we know the complete configuration of all the pion string within V. However, it would be too difficult to obtain such a distribution so that only a simple estimation is performed in the following.

Using the helicity amplitude per unit string length realized by a pair of pion strings, $\mathcal{H}_1$, the total helicity can be written as

$$\mathcal{H} \simeq \epsilon_{\mathrm{CP}} n_s l_s 2\mathcal{H}_1,$$

where ϵ_{CP} is the CP violation strength which determines how much the twist and tangle of pion strings should be biased, n_s is the string number density and l_s is the typical string length. In order to derive $\mathcal{H}_1$, here we employ a very rough estimation as

$$\mathcal{H}_1 \sim B(\delta_s) A(L_s) \delta_s^2, \tag{9}$$

where L_s is the mean separation length between pion strings. This estimation is the most optimistic one which corresponds to the case when pion strings are so arranged that A field and B field are parallel. Then

$$\mathcal{H} \sim -\epsilon_{\mathrm{CP}} n_s \frac{N_c^2 e^2}{2\pi\alpha} n_c^2 l_s \delta_s \left(\frac{L_s}{\delta_s}\right)^{\alpha/\pi}.$$

It would be reasonable if we assume

$$\delta_s \sim \Lambda_{\rm QCD}^{-1},$$

and

$$L_s \sim l_s \sim n_s^{-1/3}.$$

Here we introduce a parameter β as

$$L_s = \beta T^{-1},$$

then the helicity density can be expressed as

$$\mathcal{H} \sim -\epsilon_{\rm CP} 2N_c^2 \beta^{-2+\alpha/\pi} \left(\frac{\Lambda_{\rm QCD}}{T}\right)^{1+\alpha/\pi} T^3,$$

where $n_c \sim \Lambda_{\rm QCD}$ has been substituted. If we normalize this value by the cosmic entropy density, s,

$$\frac{\mathcal{H}}{s} \sim -\epsilon_{\rm CP} \frac{45 N_c^2}{\pi^2 \mathcal{N}} \beta^{-2+\alpha/\pi} \left(\frac{\Lambda_{\rm QCD}}{T}\right)^{1+\alpha/\pi},$$

where $\mathcal{N}$ is the number of gauge freedom at T. The most efficient case is realized when $\beta \sim 1$ and such a situation would be probable at the string formation epoch, that is, $T \sim \Lambda_{\rm QCD}$. On the other hand, at much lower temperature, the coherence scale of the string distribution may be larger so that the resulting helicity density would be insignificant. For example, at the temperature scale ~ 1 MeV, β could be 10^{-21} if about one string exists within the horizon scale.

Next we have to consider the evolution of the magnetic field and whether the magnetic helicity would be conserved or not. At the formation of the magnetic field, the magnetic Reynolds number should be sufficiently large since the condition as

$$v \gg \frac{e^2}{4\pi},$$

seems to be satisfied where v is the characteristic fluid velocity. Thus the generated magnetic helicity should not be erased and the following evolution would be worth to be analyzed. There exist two important scales in this stage. One is the frozen-in scale, L_f, above which the helicity is conserved. It can be calculated by the condition that the effect of the dissipation by the surrounding plasma in the classical field equation of motion for the magnetic field is negligible compared to the damping due to the cosmological expansion as

$$L_f \sim \sqrt{\frac{t}{4\pi\sigma_c}},$$

where t is the cosmic time and $\sigma_c \sim T/e^2$ is the electrical conductivity of the plasma. Magnetic fields whose coherent scales are larger than L_f are not dissipated and are frozen in. The other scale is the magnetic coherence scale, $L(t)$. In one expansion time after the magnetic field generation, L, can be written as

$$L(t_d) \sim L_i \left(\frac{t}{t_i}\right)^{\xi},$$

where L_i is the initial coherence scale, $t_i \sim L_i/v$ is the time scale of magnetohydrodynamic turbulence and the subscript d depicts the decay of the pion string. It is assumed that L grows as $\propto t^{\xi}$. Although the value of ξ is not completely determined, it is of the order of unity. For example, $\xi = 1/2$ was derived in some of MHD simulation results [20] and $\xi = 2/3$ was the case when the magnetic energy density is scale free [21]. These two values are employed as typical ones in the following estimation. Although these particular choices are important when the helicity conservation condition is examined, they have little effect on the quantitative estimation of the resulting magnetic field strength. We take L_i as a simple thermal one, $L_i \sim 1/T_d$, and the fluid is relativistic, $v \sim 1$, then

$$L(t_d) \sim \frac{1}{T_d}\left(\frac{T_{pl}}{T_d}\right)^{\xi}, \tag{10}$$

where $T_{pl} \sim 10^{19}$ GeV is the Planck temperature. When $T_d = 100$ MeV,

$$L_f \sim 10^9 T_d^{-1}, \qquad \text{and} \qquad L(t_d) \sim 10^{10} T_d^{-1},$$

for the case $\xi = 1/2$,

$$L(t_d) \sim 2 \times 10^{13} T_d^{-1},$$

for the case $\xi = 2/3$. When $T_d = 1$ MeV,

$$L_f \sim 10^{10} T_d^{-1}, \qquad \text{and} \qquad L(t_d) \sim 10^{11} T_d^{-1},$$

for the case $\xi = 1/2$,

$$L(t_d) \sim 5 \times 10^{14} T_d^{-1},$$

for the case $\xi = 2/3$. In both cases,

$$L_f < L(t_d),$$

which means the helicity conservation condition is satisfied. This condition will be also satisfied in the following evolution since

$$L_f \propto \left(\frac{T_d}{T}\right)^{3/2}, \qquad \text{and} \qquad L(t) = L(t_d)\left(\frac{T_d}{T}\right)^{1+\xi}.$$

At $T \sim 0.1$ MeV, the electron pair annihilation occurs and the electric conductivity decreases suddenly as

$$\sigma_c \sim 10^{-10}\frac{m_e}{e^2},$$

where m_e is the electron mass so that the frozen-in scale exceeds the coherence scale as

$$\frac{L(t)}{L_f} \sim 2 \times 10^{-4},$$

when $\xi = 1/2$ independent of the value of T_d. On the other hand, when $\xi = 2/3$, $L(t)$ is comparable to L_f and this is more interesting case.

Hence we can calculate the coherence scale at the recombination when $T \sim 0.1$ eV using the formula as

$$L_{\rm rec} = L(t_d)\left(\frac{T_d}{T_{\rm eq}}\right)^{1+\xi}\left(\frac{T_{\rm eq}}{T_{\rm rec}}\right)^{1+\xi/2}, \tag{11}$$

where $T_{\rm eq} \sim 1$ eV is the matter-radiation equality time. Then $L_{\rm rec}$ can be calculated as

$$L_{\rm rec} \sim 10^{15} \text{ cm}.$$

Note that although it is 10^{14} cm in the paper [19], it comes simply from the different treatment of numerical factors since from the equations (10) and (11) we can see that $L_{\rm rec}$ does not depend on T_d. Finally we show the numerical value of the magnetic field strength for the extreme case when $\mathcal{H}/s \sim 1$ as

$$B \sim 10^{-9} \text{ G}, \qquad \text{on } \sim 1 \text{ pc},$$

at the recombination epoch. This value is remarkably larger than that in (8) and much more promising for the explanation of the origin of the astrophysical magnetic field. Although we employ the optimistic estimation (9), the actual value cannot be smaller than 10^{-10} G if we consider the average over randomly oriented A and B fields. However, its coherence scale is smaller compared to the galactic or intergalactic scales. In order to obtain the field amplitude on larger scales, we have to determine the detailed initial distribution of magnetic helicity, that is, the twist and tangle of pion strings. Further investigation would be interesting.

5 Summary

In this paper, the magnetic field generation by pion strings produced at the QCD phase transition and their possible detection by astronomical observation are considered.

For the latter purpose, the superconductivity of pion strings are demonstrated. In addition, their interaction with the cosmic background radiation is investigated and it is shown that the rotation of light polarization axis is caused by the string field. In both cases, the line-like structure of strings itself is essential and the spatial distribution of microwave background distortion or rotation angle difference of polarization traces the arrangement of the strings in our universe.

Since the observational consequences are just suggestions, our main conclusion is that pion strings can generate primordial magnetic fields which may be seeds of present cosmic magnetic fields. Its amplitude can exceed $B \sim 10^{-10}$ G and this is much higher than that in the preceding work [17]. Such an enhancement becomes possible by the consideration of the helicity conservation which has been shown to be reasonable for the pion string.

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