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One-Loop Calculations and Detailed Analysis of the Localized Non-Commutative p^{-2} $U(1)$ Gauge Model*

Daniel N. BLASCHKE †‡, *Arnold ROFNER* † and *René I.P. SEDMIK* †

† *Institute for Theoretical Physics, Vienna University of Technology, Wiedner Hauptstrasse 8-10, A-1040 Vienna, Austria*
E-mail: *blaschke@hep.itp.tuwien.ac.at, arofner@hep.itp.tuwien.ac.at, sedmik@hep.itp.tuwien.ac.at*

‡ *Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria*

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Abstract. This paper carries forward a series of articles describing our enterprise to construct a gauge equivalent for the θ-deformed non-commutative $\frac{1}{p^2}$ model originally introduced by Gurau et al. [*Comm. Math. Phys.* **287** (2009), 275–290]. It is shown that breaking terms of the form used by Vilar et al. [*J. Phys. A: Math. Theor.* **43** (2010), 135401, 13 pages] and ourselves [*Eur. Phys. J. C: Part. Fields* **62** (2009), 433–443] to localize the BRST covariant operator $\left(D^2\theta^2D^2\right)^{-1}$ lead to difficulties concerning renormalization. The reason is that this dimensionless operator is invariant with respect to any symmetry of the model, and can be inserted to arbitrary power. In the present article we discuss explicit one-loop calculations, and analyze the mechanism the mentioned problems originate from.

Key words: noncommutative field theory; gauge field theories; renormalization

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1 Introduction

Tackling the infamous UV/IR mixing problem [4, 5] plaguing Moyal-deformed QFTs has been one of the main research interests in the field for almost a decade (see [6, 7, 8] for reviews of the topic). It is accepted on a broad basis that non-commutativity necessitates additional terms in the action to reobtain renormalizability. Several interesting approaches have been worked out [9, 10], and proofs of renormalizability have been achieved mainly by utilizing Multiscale Analysis (MSA) [11, 12], or formally in the matrix base [13].

In the line of these developments Gurau et al. [1] introduced a term of type $\phi \star \frac{a}{\Box}\phi$ into the Lagrangian which, in a natural way, provides a counter term for the inevitable $\frac{1}{p^2}$ divergence inherently tied to the deformation of the product. In this way the theory is altered in the infrared region which breaks the UV/IR mixing and renders the theory renormalizable. This latter fact has been proven up to all orders by the authors using MSA. Motivated by the inherent translation invariance and simplicity of this model (referred to as $\frac{1}{p^2}$ model), a thorough study of the divergence structure and explicit renormalization at one-loop level [14], as well as a computation of the beta functions [15] have been carried out.

In the present article we work on Euclidean $\mathbb{R}^4_\theta$ with the Moyal-deformed product (also referred to as 'star product') $[x_\mu \stackrel{\star}{,} x_\nu] \equiv x_\mu \star x_\nu - x_\nu \star x_\mu = \mathrm{i}\theta_{\mu\nu}$ of regular commuting coordinates x_μ.

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In the simplest case, the real parameters $\theta_{\mu\nu} = -\theta_{\nu\mu}$ form the block-diagonal tensor

$$(\theta_{\mu\nu}) = \theta \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \qquad \text{with} \;\; \theta \in \mathbb{R},$$

obeying the practical relation $\theta_{\mu\rho}\theta_{\rho\nu} = -\theta^2\delta_{\mu\nu}$, where $\dim\theta = -2$. With these definitions we use the abbreviations $\tilde{v}_\mu \equiv \theta_{\mu\nu}v_\nu$ for vectors v and $\tilde{M} \equiv \theta_{\mu\nu}M_{\mu\nu}$ for matrices M.

Further research focused on the generalization of the scalar $\frac{1}{p^2}$ model to $U_\star(1)$ gauge theory[1] which was first proposed in [16] yielding the action

$$\begin{aligned} S &= S_{\text{inv}}[A] + S_{\text{gf}}[A, b, c, \bar{c}] \\ &= \int \mathrm{d}^4x \left[\frac{1}{4}F_{\mu\nu} \star F_{\mu\nu} + F_{\mu\nu} \star \frac{1}{D^2\widetilde{D}^2} \star F_{\mu\nu}\right] + \int \mathrm{d}^4x \left[b \star \partial \cdot A - \frac{\alpha}{2} b \star b - \bar{c} \star \partial_\mu D_\mu c\right], \end{aligned} \tag{1.1}$$

with the usual gauge boson A_μ, ghost and antighost fields c and $\bar{c}$ respectively, the Lagrange multiplier field b implementing the gauge fixing, and a real $U_\star(1)$ gauge parameter α. The antisymmetric field strength tensor $F_{\mu\nu}$ and the covariant derivative D_μ are defined by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g\left[A_\mu \stackrel{\star}{,} A_\nu\right], \qquad \text{and} \qquad D_\mu\varphi = \partial_\mu\varphi - \mathrm{i}g\left[A_\mu \stackrel{\star}{,} \varphi\right],$$

for arbitrary φ. The non-local term

$$S_{\text{nloc}} = \int \mathrm{d}^4x\, F_{\mu\nu} \star \frac{1}{D^2\widetilde{D}^2} \star F_{\mu\nu}, \tag{1.2}$$

implements the damping mechanism of the $\frac{1}{p^2}$ model by Gurau et al. [1] in a gauge covariant way. It has been described in [14] that the new operator can only be interpreted in a physically sensible way if it is cast into an infinite series which, however, corresponds to an infinite number of gauge vertices. A first attempt to localize the new operator by introducing a real valued auxiliary tensor field [3] led to additional degrees of freedom. However, this was considered to be dissatisfactory. Following the ideas of Vilar et al. [2] we enhanced our approach by coupling gauge and auxiliary sectors via complex conjugated pairs of fields together with associated pairs of ghosts in such a way, that BRST doublet structures were formed [17]. Such a mechanism has already been applied successfully for the Gribov–Zwanziger action of QCD [18, 19, 20] where a similar damping mechanism is applied.

Starting from a recapitulation of our recently presented localized model in Section 2 we give explicit one-loop calculations in Section 3, and undertake the attempt of one-loop renormalization. Subsequently, the results and their implications for higher loop orders are analyzed in Section 4, and finally we give a concluding discussion of the lessons learned in Section 5.

2 The localized $\frac{1}{p^2}$ $U(1)$ gauge model

2.1 Review: the construction of the model

As mentioned in Section 1, the non-local term of the action (1.1) leads to an infinite number of vertices: it formally consists of the inverse of covariant derivatives acting on field strength tensors, and therefore stands for an infinite power series (cf. [16]) making explicit calculations impossible. Considering only the first few orders of this power series is not an option as this

[1] Notice, that the star product modifies the initial $U(1)$ algebra in a way that it becomes non-Abelian. Hence, we call the resulting algebra $U_\star(1)$.

would destroy gauge invariance. Yet, the present problem can be circumvented by the localization of the term under consideration. In this sense, in a first approach described in [3], the introduction of an additional real antisymmetric field $\mathcal{B}_{\mu\nu}$ of mass dimension two led to the following localized version of the non-local term (1.2):

$$S_{\rm nloc} \to S_{\rm loc} = \int {\rm d}^4x \big[a'\mathcal{B}_{\mu\nu} \star F_{\mu\nu} - \mathcal{B}_{\mu\nu} \star \widetilde{D}^2 D^2 \star \mathcal{B}_{\mu\nu}\big]. \tag{2.1}$$

However, the $\mathcal{B}_{\mu\nu}$-field appears to have its own dynamical properties leading to new physical degrees of freedom which can only be avoided if the new terms in the action are written as an exact BRST variation. In order for such a mechanism to work, further unphysical fields are required.

Following the ideas of Vilar et al. [2], the localized action (2.1) was further developed in [17] by replacing $\mathcal{B}_{\mu\nu}$ with a complex conjugated pair of fields $(B_{\mu\nu}, \bar{B}_{\mu\nu})$ and by the introduction of an additional pair of ghost and antighost fields $\psi_{\mu\nu}$ and $\bar{\psi}_{\mu\nu}$ (all of mass dimension 1), thus leading to

$$S_{\rm loc} = \int {\rm d}^4x \left[\frac{\lambda}{2}\left(B_{\mu\nu} + \bar{B}_{\mu\nu}\right) F_{\mu\nu} - \mu^2 \bar{B}_{\mu\nu} D^2 \widetilde{D}^2 B_{\mu\nu} + \mu^2 \bar{\psi}_{\mu\nu} D^2 \widetilde{D}^2 \psi_{\mu\nu}\right]. \tag{2.2}$$

In this expression, as well as throughout the remainder of this section, all field products are considered to be star products. The new parameters λ and μ both have mass dimension 1 and replace the former dimensionless parameter a' by $a' = \lambda/\mu$. The proof of the equivalence between the non-local action (1.2) and equation (2.2) can be found in [17]. With the addition of a fixing term to the action one has BRST invariance, and for simplicity, we choose the Landau gauge

$$S_{\phi\pi} = \int {\rm d}^4x \left(b\partial^\mu A_\mu - \bar{c}\partial^\mu D_\mu c\right).$$

The BRST transformation laws for the fields read:

$$\begin{aligned}
&sA_\mu = D_\mu c, && sc = {\rm i}gcc,\\
&s\bar{c} = b, && sb = 0,\\
&s\bar{\psi}_{\mu\nu} = \bar{B}_{\mu\nu} + {\rm i}g\left\{c, \bar{\psi}_{\mu\nu}\right\}, && s\bar{B}_{\mu\nu} = {\rm i}g\left[c, \bar{B}_{\mu\nu}\right],\\
&sB_{\mu\nu} = \psi_{\mu\nu} + {\rm i}g\left[c, B_{\mu\nu}\right], && s\psi_{\mu\nu} = {\rm i}g\left\{c, \psi_{\mu\nu}\right\},\\
&s^2\varphi = 0 \quad \forall\, \varphi \in \left\{A_\mu, b, c, \bar{c}, B_{\mu\nu}, \bar{B}_{\mu\nu}, \psi_{\mu\nu}, \bar{\psi}_{\mu\nu}\right\}.
\end{aligned} \tag{2.3}$$

With (2.3) one can see that the localized part of the action can be written as the sum of a BRST exact and a so-called soft breaking term:

$$S_{\rm loc} = \int {\rm d}^4x \left[s\left(\frac{\lambda}{2}\bar{\psi}_{\mu\nu}F^{\mu\nu} - \mu^2 \bar{\psi}_{\mu\nu} D^2 \widetilde{D}^2 B^{\mu\nu}\right) + \frac{\lambda}{2} B_{\mu\nu}F^{\mu\nu}\right],$$

where

$$S_{\rm break} = \int {\rm d}^4x\, \frac{\lambda}{2} B_{\mu\nu}F^{\mu\nu}, \qquad \text{with} \qquad sS_{\rm break} = \int {\rm d}^4x\, \frac{\lambda}{2}\psi_{\mu\nu}F^{\mu\nu}. \tag{2.4}$$

As discussed in detail in [17], the breaking is considered to be soft, since the mass dimension of the field dependent part is $< D = 4$ and the term only modifies the infrared regime of the model. As has been shown by Zwanziger [19] terms of this type therefore do not spoil renormalizability. In order to restore BRST invariance in the UV region (as is a prerequisite for the application of algebraic renormalization) an additional set of sources

$$\begin{aligned}
&s\bar{Q}_{\mu\nu\alpha\beta} = \bar{J}_{\mu\nu\alpha\beta} + {\rm i}g\left\{c, \bar{Q}_{\mu\nu\alpha\beta}\right\}, && s\bar{J}_{\mu\nu\alpha\beta} = {\rm i}g\left[c, \bar{J}_{\mu\nu\alpha\beta}\right],\\
&sQ_{\mu\nu\alpha\beta} = J_{\mu\nu\alpha\beta} + {\rm i}g\left\{c, Q_{\mu\nu\alpha\beta}\right\}, && sJ_{\mu\nu\alpha\beta} = {\rm i}g\left[c, J_{\mu\nu\alpha\beta}\right],
\end{aligned}$$

is introduced, and coupled to the breaking term which then takes the (BRST exact) form

$$S_{\text{break}} = \int \mathrm{d}^4x s\left(\bar{Q}_{\mu\nu\alpha\beta}B^{\mu\nu}F^{\alpha\beta}\right) = \int \mathrm{d}^4x \left(\bar{J}_{\mu\nu\alpha\beta}B^{\mu\nu}F^{\alpha\beta} - \bar{Q}_{\mu\nu\alpha\beta}\psi^{\mu\nu}F^{\alpha\beta}\right).$$

Equation (2.4) is reobtained if the sources $\bar{Q}$ and $\bar{J}$ take their 'physical values'

$$\begin{aligned} \bar{Q}_{\mu\nu\alpha\beta}\big|_{\text{phys}} &= 0, & \bar{J}_{\mu\nu\alpha\beta}\big|_{\text{phys}} &= \frac{\lambda}{4}\left(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}\right), \\ Q_{\mu\nu\alpha\beta}\big|_{\text{phys}} &= 0, & J_{\mu\nu\alpha\beta}\big|_{\text{phys}} &= \frac{\lambda}{4}\left(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}\right). \end{aligned} \tag{2.5}$$

Note that the Hermitian conjugate of the counter term S_{break} in equation (2.2) (i.e. the term $\int \mathrm{d}^4x\bar{B}_{\mu\nu}F^{\mu\nu}$) may also be coupled to external sources which, however, is not required for BRST invariance but restores Hermiticity of the action:

$$\frac{\lambda}{2}\int \mathrm{d}^4x\, \bar{B}_{\mu\nu}F^{\mu\nu} \;\longrightarrow\; \int \mathrm{d}^4x\, s\big(J_{\mu\nu\alpha\beta}\bar{\psi}^{\mu\nu}F^{\alpha\beta}\big) = \int \mathrm{d}^4x\, J_{\mu\nu\alpha\beta}\bar{B}^{\mu\nu}F^{\alpha\beta}.$$

Including external sources Ω^ϕ, $\phi \in \{A, c, B, \bar{B}, \psi, \bar{\psi}, J, \bar{J}, Q, \bar{Q}\}$ for the non-linear BRST transformations the complete action with Landau gauge $\partial^\mu A_\mu = 0$ and general $Q/\bar{Q}$ and $J/\bar{J}$ reads:

$$\begin{aligned} S &= S_{\text{inv}} + S_{\phi\pi} + S_{\text{new}} + S_{\text{break}} + S_{\text{ext}}, \qquad \text{with} \\ S_{\text{inv}} &= \int \mathrm{d}^4x \frac{1}{4}F_{\mu\nu}F_{\mu\nu}, \\ S_{\phi\pi} &= \int \mathrm{d}^4x\, s\left(\bar{c}\,\partial_\mu A_\mu\right) = \int \mathrm{d}^4x \left(b\,\partial_\mu A_\mu - \bar{c}\,\partial_\mu D_\mu c\right), \\ S_{\text{new}} &= \int \mathrm{d}^4x\, s\big(J_{\mu\nu\alpha\beta}\bar{\psi}_{\mu\nu}F_{\alpha\beta} - \mu^2\bar{\psi}_{\mu\nu}D^2\widetilde{D}^2B_{\mu\nu}\big) \\ &= \int \mathrm{d}^4x\big(J_{\mu\nu\alpha\beta}\bar{B}_{\mu\nu}F_{\alpha\beta} - \mu^2\bar{B}_{\mu\nu}D^2\widetilde{D}^2B_{\mu\nu} + \mu^2\bar{\psi}_{\mu\nu}D^2\widetilde{D}^2\psi_{\mu\nu}\big), \\ S_{\text{break}} &= \int \mathrm{d}^4x\, s\big(\bar{Q}_{\mu\nu\alpha\beta}B_{\mu\nu}F_{\alpha\beta}\big) = \int \mathrm{d}^4x\big(\bar{J}_{\mu\nu\alpha\beta}B_{\mu\nu}F_{\alpha\beta} - \bar{Q}_{\mu\nu\alpha\beta}\psi_{\mu\nu}F_{\alpha\beta}\big), \\ S_{\text{ext}} &= \int \mathrm{d}^4x \Big(\Omega^A_\mu D_\mu c + \mathrm{i}g\,\Omega^c cc + \Omega^B_{\mu\nu}\big(\psi_{\mu\nu} + \mathrm{i}g\,[c, B_{\mu\nu}]\big) + \mathrm{i}g\,\Omega^{\bar{B}}_{\mu\nu}\,[c, \bar{B}_{\mu\nu}] \\ &\quad + \mathrm{i}g\,\Omega^\psi_{\mu\nu}\,\{c, \psi_{\mu\nu}\} + \Omega^{\bar{\psi}}_{\mu\nu}\big(\bar{B}_{\mu\nu} + \mathrm{i}g\,\{c, \bar{\psi}_{\mu\nu}\}\big) + \Omega^Q_{\mu\nu\alpha\beta}\big(J_{\mu\nu\alpha\beta} + \mathrm{i}g\,\{c, Q_{\mu\nu\alpha\beta}\}\big) \\ &\quad + \mathrm{i}g\,\Omega^J_{\mu\nu\alpha\beta}\,[c, J_{\mu\nu\alpha\beta}] + \Omega^{\bar{Q}}_{\mu\nu\alpha\beta}\big(\bar{J}_{\mu\nu\alpha\beta} + \mathrm{i}g\,\{c, \bar{Q}_{\mu\nu\alpha\beta}\}\big) + \mathrm{i}g\,\Omega^{\bar{J}}_{\mu\nu\alpha\beta}\,[c, \bar{J}_{\mu\nu\alpha\beta}]\Big). \end{aligned} \tag{2.6}$$

Table 1 summarizes properties of the fields and sources contained in the model (2.6).

Notice that the mass μ is a physical parameter despite the fact that the variation of the action $\frac{\partial S}{\partial \mu^2} = s\big(\bar{\psi}_{\mu\nu}D^2\widetilde{D}^2B^{\mu\nu}\big)$ yields an exact BRST form. Following the argumentation in [21] this is a consequence of the introduction of a soft breaking term. For vanishing Gribov-like parameter λ the contributions to the path integral of the μ dependent sectors of S_{new} in (2.6) cancel each other. If $\lambda \neq 0$ one has to consider the additional breaking term which couples the gauge field A_μ to the auxiliary field $B_{\mu\nu}$ and the associated ghost $\psi_{\mu\nu}$. This mixing is reflected by the appearance of $a' = \lambda/\mu$ in the damping factor $\left(k^2 + \frac{a'^2}{k^2}\right)$ featured by all field propagators (2.7c)–(2.7f) below.

Table 1. Properties of fields and sources.

Field	A_μ	c	$\bar{c}$	$B_{\mu\nu}$	$\bar{B}_{\mu\nu}$	$\psi_{\mu\nu}$	$\bar{\psi}_{\mu\nu}$	$J_{\alpha\beta\mu\nu}$	$\bar{J}_{\alpha\beta\mu\nu}$	$Q_{\alpha\beta\mu\nu}$	$\bar{Q}_{\alpha\beta\mu\nu}$
$g_\sharp$	0	1	−1	0	0	1	−1	0	0	−1	−1
Mass dim.	1	0	2	1	1	1	1	1	1	1	1
Statistics	b	f	f	b	b	f	f	b	b	f	f
Source	Ω^A_μ	Ω^c	b	$\Omega^B_{\mu\nu}$	$\Omega^{\bar{B}}_{\mu\nu}$	$\Omega^\psi_{\mu\nu}$	$\Omega^{\bar{\psi}}_{\mu\nu}$	$\Omega^J_{\alpha\beta\mu\nu}$	$\Omega^{\bar{J}}_{\alpha\beta\mu\nu}$	$\Omega^Q_{\alpha\beta\mu\nu}$	$\Omega^{\bar{Q}}_{\alpha\beta\mu\nu}$
$g_\sharp$	−1	−2	0	−1	−1	−2	0	−1	−1	0	0
Mass dim.	3	4	2	3	3	3	3	3	3	3	3
Statistics	f	b	b	f	f	b	b	f	f	b	b

2.2 Feynman rules

2.2.1 Propagators

From the action (2.6) with $J/\bar{J}$ and $Q/\bar{Q}$ set to their physical values given by (2.5) one finds the propagators

$$G^{\bar{c}c}(k) = -\frac{1}{k^2}, \tag{2.7a}$$

$$G^{\bar{\psi}\psi}_{\mu\nu,\rho\sigma}(k) = \frac{(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})}{2\mu^2 k^2 \tilde{k}^2}, \tag{2.7b}$$

$$G^{AA}_{\mu\nu}(k) = \frac{1}{\left(k^2 + \frac{a'^2}{\tilde{k}^2}\right)}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right), \tag{2.7c}$$

$$G^{AB}_{\mu,\rho\sigma}(k) = \frac{\mathrm{i}a'}{2\mu}\frac{(k_\rho\delta_{\mu\sigma} - k_\sigma\delta_{\mu\rho})}{k^2\tilde{k}^2\left(k^2 + \frac{a'^2}{\tilde{k}^2}\right)} = G^{A\bar{B}}_{\mu,\rho\sigma}(k) = -G^{\bar{B}A}_{\rho\sigma,\mu}(k), \tag{2.7d}$$

$$G^{\bar{B}B}_{\mu\nu,\rho\sigma}(k) = \frac{-1}{2\mu^2 k^2\tilde{k}^2}\left[\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho} - a'^2\frac{k_\mu k_\rho\delta_{\nu\sigma} + k_\nu k_\sigma\delta_{\mu\rho} - k_\mu k_\sigma\delta_{\nu\rho} - k_\nu k_\rho\delta_{\mu\sigma}}{2k^2\tilde{k}^2\left(k^2 + \frac{a'^2}{\tilde{k}^2}\right)}\right], \tag{2.7e}$$

$$G^{BB}_{\mu\nu,\rho\sigma}(k) = \frac{a'^2}{2\mu^2 k^2\tilde{k}^2}\left[\frac{k_\mu k_\rho\delta_{\nu\sigma} + k_\nu k_\sigma\delta_{\mu\rho} - k_\mu k_\sigma\delta_{\nu\rho} - k_\nu k_\rho\delta_{\mu\sigma}}{2k^2\tilde{k}^2\left(k^2 + \frac{a'^2}{\tilde{k}^2}\right)}\right] = G^{\bar{B}\bar{B}}_{\mu\nu,\rho\sigma}(k), \tag{2.7f}$$

where the abbreviation $a' \equiv \lambda/\mu$ is used. Notice, that they obey the following symmetries and relations:

$$G^{AB}_{\mu,\rho\sigma}(k) = G^{A\bar{B}}_{\mu,\rho\sigma}(k) = -G^{BA}_{\rho\sigma,\mu}(k) = -G^{\bar{B}A}_{\rho\sigma,\mu}(k), \tag{2.8a}$$

$$G^{\phi}_{\mu\nu,\rho\sigma}(k) = -G^{\phi}_{\nu\mu,\rho\sigma} = -G^{\phi}_{\mu\nu,\sigma\rho}(k) = G^{\phi}_{\nu\mu,\sigma\rho}(k), \qquad \text{for } \phi \in \{\bar{\psi}\psi, \bar{B}B, BB, \bar{B}\bar{B}\}, \tag{2.8b}$$

$$2k^2\tilde{k}^2 G^{AB}_{\rho,\mu\nu}(k) = \mathrm{i}\frac{a'}{\mu}\left(k_\mu G^{AA}_{\rho\nu}(k) - k_\nu G^{AA}_{\rho\mu}(k)\right), \tag{2.8c}$$

$$\frac{1}{\mu^2}(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}) = \mathrm{i}\frac{a'}{\mu}\left(k_\mu G^{BA}_{\rho\sigma,\nu}(k) - k_\nu G^{BA}_{\rho\sigma,\mu}(k)\right) - 2k^2\tilde{k}^2 G^{B\bar{B}}_{\mu\nu,\rho\sigma}(k), \tag{2.8d}$$

$$0 = \mathrm{i}\frac{a'}{\mu}\left(k_\mu G^{BA}_{\rho\sigma,\nu}(k) - k_\nu G^{BA}_{\rho\sigma,\mu}(k)\right) - 2k^2\tilde{k}^2 G^{BB}_{\mu\nu,\rho\sigma}(k), \tag{2.8e}$$

$$G^{B\bar{B}}_{\mu\nu,\rho\sigma}(k) = G^{\bar{\psi}\psi}_{\mu\nu,\rho\sigma}(k) + G^{BB}_{\mu\nu,\rho\sigma}(k). \tag{2.8f}$$

2.2.2 Vertices

The action (2.6) leads to 13 tree level vertices whose rather lengthy expressions are listed in Appendix A. One immediately finds the following vertex relation:

$$\widetilde{V}^{\bar{\psi}\psi(n\times A)}_{\mu\nu,\rho\sigma,\xi_1\dots\xi_n}(q_1,q_2,k_{\xi_1},\dots,k_{\xi_n}) = -\widetilde{V}^{\bar{B}B(n\times A)}_{\mu\nu,\rho\sigma,\xi_1\dots\xi_n}(q_1,q_2,k_{\xi_1},\dots,k_{\xi_n}),$$

i.e. all vertices with one B, one $\bar{B}$ and an arbitrary number of A legs have exactly the same form as the ones with one ψ, one $\bar{\psi}$ and an arbitrary number of A legs. This is due to the fact that the $\bar{\psi}\psi nA$ and $\bar{B}BnA$ vertices stem from terms in the action which are of the same structure, and are thus equal in their form.

Finally, the vertices obey the following additional relations:

$$\begin{aligned}
&\widetilde{V}^{\bar{\psi}\psi(n\times A)}_{\mu\nu,\rho\sigma,\xi_1\dots\xi_n}(q_1,q_2,k_{\xi_1},\dots,k_{\xi_n}) = -\widetilde{V}^{\psi\bar{\psi}(n\times A)}_{\rho\sigma,\mu\nu,\xi_1\dots\xi_n}(q_2,q_1,k_{\xi_1},\dots,k_{\xi_n})\\
&\qquad = -\widetilde{V}^{\bar{\psi}\psi(n\times A)}_{\nu\mu,\rho\sigma,\xi_1\dots\xi_n}(q_1,q_2,k_{\xi_1},\dots,k_{\xi_n}) = -\widetilde{V}^{\bar{\psi}\psi(n\times A)}_{\mu\nu,\sigma\rho,\xi_1\dots\xi_n}(q_1,q_2,k_{\xi_1},\dots,k_{\xi_n}),
\end{aligned}$$

and

$$\begin{aligned}
&\widetilde{V}^{\bar{B}B(n\times A)}_{\mu\nu,\rho\sigma,\xi_1\dots\xi_n}(q_1,q_2,k_{\xi_1},\dots,k_{\xi_n}) = +\widetilde{V}^{B\bar{B}(n\times A)}_{\rho\sigma,\mu\nu,\xi_1\dots\xi_n}(q_2,q_1,k_{\xi_1},\dots,k_{\xi_n})\\
&= -\widetilde{V}^{\bar{B}B(n\times A)}_{\nu\mu,\rho\sigma,\xi_1\dots\xi_n}(q_1,q_2,k_{\xi_1},\dots,k_{\xi_n}) = -\widetilde{V}^{\bar{B}B(n\times A)}_{\mu\nu,\sigma\rho,\xi_1\dots\xi_n}(q_1,q_2,k_{\xi_1},\dots,k_{\xi_n}), \quad \text{for } n \in 1,2,3,4.
\end{aligned}$$

2.3 Symmetries

Before moving on to explicit one-loop calculations, let us briefly discuss the symmetries of our action equation (2.6). The Slavnov–Taylor identity is given by

$$\begin{aligned}
\mathcal{B}(S) = \int \mathrm{d}^4x \Bigg[&\frac{\delta S}{\delta\Omega^A_\mu}\frac{\delta S}{\delta A_\mu} + \frac{\delta S}{\delta\Omega^c}\frac{\delta S}{\delta c} + b\frac{\delta S}{\delta\bar{c}} + \frac{\delta S}{\delta\Omega^B_{\mu\nu}}\frac{\delta S}{\delta B_{\mu\nu}} + \frac{\delta S}{\delta\Omega^{\bar{B}}_{\mu\nu}}\frac{\delta S}{\delta\bar{B}_{\mu\nu}}\\
&+ \frac{\delta S}{\delta\Omega^{\psi}_{\mu\nu}}\frac{\delta S}{\delta\psi_{\mu\nu}} + \frac{\delta S}{\delta\Omega^{\bar{\psi}}_{\mu\nu}}\frac{\delta S}{\delta\bar{\psi}_{\mu\nu}} + \frac{\delta S}{\delta\Omega^{Q}_{\mu\nu\alpha\beta}}\frac{\delta S}{\delta Q_{\mu\nu\alpha\beta}} + \frac{\delta S}{\delta\Omega^{J}_{\mu\nu\alpha\beta}}\frac{\delta S}{\delta J_{\mu\nu\alpha\beta}}\\
&+ \frac{\delta S}{\delta\Omega^{\bar{Q}}_{\mu\nu\alpha\beta}}\frac{\delta S}{\delta\bar{Q}_{\mu\nu\alpha\beta}} + \frac{\delta S}{\delta\Omega^{\bar{J}}_{\mu\nu\alpha\beta}}\frac{\delta S}{\delta\bar{J}_{\mu\nu\alpha\beta}}\Bigg] = 0.
\end{aligned} \tag{2.9}$$

Furthermore we have the gauge fixing condition

$$\frac{\delta S}{\delta b} = \partial_\mu A_\mu = 0,$$

the ghost equation

$$\mathcal{G}(S) = \partial_\mu\frac{\delta S}{\delta\Omega^A_\mu} + \frac{\delta S}{\delta\bar{c}} = 0,$$

and the antighost equation

$$\bar{\mathcal{G}}(S) = \int \mathrm{d}^4x \frac{\delta S}{\delta c} = 0.$$

Following the notation of [2] the identity associated to the BRST doublet structure is given by

$$\mathcal{U}^{(1)}_{\alpha\beta\mu\nu}(S) = \int \mathrm{d}^4x \Bigg(\bar{B}_{\alpha\beta}\frac{\delta S}{\delta\bar{\psi}_{\mu\nu}} + \Omega^{\bar{\psi}}_{\mu\nu}\frac{\delta S}{\delta\Omega^{\bar{B}}_{\alpha\beta}} + \psi_{\mu\nu}\frac{\delta S}{\delta B_{\alpha\beta}} - \Omega^{B}_{\alpha\beta}\frac{\delta S}{\delta\Omega^{\psi}_{\mu\nu}}$$

$$+ J_{\mu\nu\rho\sigma}\frac{\delta S}{\delta Q_{\alpha\beta\rho\sigma}} + \Omega^{Q}_{\alpha\beta\rho\sigma}\frac{\delta S}{\delta\Omega^{J}_{\mu\nu\rho\sigma}} + \bar{J}_{\alpha\beta\rho\sigma}\frac{\delta S}{\delta\bar{Q}_{\mu\nu\rho\sigma}} + \Omega^{\bar{Q}}_{\mu\nu\rho\sigma}\frac{\delta S}{\delta\Omega^{\bar{J}}_{\alpha\beta\rho\sigma}}\Bigg) = 0.$$

Note that the first two terms of the second line,

$$\int \mathrm{d}^4x \left(J_{\mu\nu\rho\sigma}\frac{\delta S}{\delta Q_{\alpha\beta\rho\sigma}} + \Omega^{Q}_{\alpha\beta\rho\sigma}\frac{\delta S}{\delta\Omega^{J}_{\mu\nu\rho\sigma}}\right) = 0,$$

constitute a symmetry by themselves. These terms stem from the insertion of conjugated field partners J and Q for $\bar{J}$ and $\bar{Q}$, respectively, which are not necessarily required as discussed above in Section 2.1.

Furthermore, we have the linearly broken symmetries $\mathcal{U}^{(0)}$ and $\tilde{\mathcal{U}}^{(0)}$:

$$\mathcal{U}^{(0)}_{\alpha\beta\mu\nu}(S) = -\Theta^{(0)}_{\alpha\beta\mu\nu} = -\tilde{\mathcal{U}}^{(0)}_{\alpha\beta\mu\nu}(S),$$

with

$$\begin{aligned}
\mathcal{U}^{(0)}_{\alpha\beta\mu\nu}(S) &= \int \mathrm{d}^4x \Bigg[B_{\alpha\beta}\frac{\delta S}{\delta B_{\mu\nu}} - \bar{B}_{\mu\nu}\frac{\delta S}{\delta \bar{B}_{\alpha\beta}} - \Omega^{B}_{\mu\nu}\frac{\delta S}{\delta\Omega^{B}_{\alpha\beta}} + \Omega^{\bar{B}}_{\alpha\beta}\frac{\delta S}{\delta\Omega^{\bar{B}}_{\mu\nu}} \\
&\quad + J_{\alpha\beta\rho\sigma}\frac{\delta S}{\delta J_{\mu\nu\rho\sigma}} - \bar{J}_{\mu\nu\rho\sigma}\frac{\delta S}{\delta\bar{J}_{\alpha\beta\rho\sigma}} - \Omega^{J}_{\mu\nu\rho\sigma}\frac{\delta S}{\delta\Omega^{J}_{\alpha\beta\rho\sigma}} + \Omega^{\bar{J}}_{\alpha\beta\rho\sigma}\frac{\delta S}{\delta\Omega^{\bar{J}}_{\mu\nu\rho\sigma}}\Bigg], \\
\tilde{\mathcal{U}}^{(0)}_{\alpha\beta\mu\nu}(S) &= \int \mathrm{d}^4x \Bigg[\psi_{\alpha\beta}\frac{\delta S}{\delta \psi_{\mu\nu}} - \bar{\psi}_{\mu\nu}\frac{\delta S}{\delta \bar{\psi}_{\alpha\beta}} - \Omega^{\psi}_{\mu\nu}\frac{\delta S}{\delta\Omega^{\psi}_{\alpha\beta}} + \Omega^{\bar{\psi}}_{\alpha\beta}\frac{\delta S}{\delta\Omega^{\bar{\psi}}_{\mu\nu}} \\
&\quad + Q_{\alpha\beta\rho\sigma}\frac{\delta S}{\delta Q_{\mu\nu\rho\sigma}} - \bar{Q}_{\mu\nu\rho\sigma}\frac{\delta S}{\delta\bar{Q}_{\alpha\beta\rho\sigma}} - \Omega^{Q}_{\mu\nu\rho\sigma}\frac{\delta S}{\delta\Omega^{Q}_{\alpha\beta\rho\sigma}} + \Omega^{\bar{Q}}_{\alpha\beta\rho\sigma}\frac{\delta S}{\delta\Omega^{\bar{Q}}_{\mu\nu\rho\sigma}}\Bigg], \\
\Theta^{(0)}_{\alpha\beta\mu\nu} &= \int \mathrm{d}^4x \left[\bar{B}_{\mu\nu}\Omega^{\bar{\psi}}_{\alpha\beta} - \psi_{\alpha\beta}\Omega^{B}_{\mu\nu} + \bar{J}_{\mu\nu\rho\sigma}\Omega^{\bar{Q}}_{\alpha\beta\rho\sigma} - J_{\alpha\beta\rho\sigma}\Omega^{Q}_{\mu\nu\rho\sigma}\right].
\end{aligned}$$

The above relations would, if applicable, form the starting point for the algebraic renormalization procedure. In order to assure the completeness of the set of symmetries it has to be assured that the algebra generated by them closes. From the Slavnov–Taylor identity (2.9) one derives the linearized Slavnov operator

$$\begin{aligned}
\mathcal{B}_S = \int \mathrm{d}^4x \Bigg[& \frac{\delta S}{\delta\Omega^{A}_{\mu}}\frac{\delta}{\delta A_\mu} + \frac{\delta S}{\delta A_\mu}\frac{\delta}{\delta\Omega^{A}_{\mu}} + \frac{\delta S}{\delta c}\frac{\delta}{\delta\Omega^{c}} + \frac{\delta S}{\delta\Omega^{c}}\frac{\delta}{\delta c} + b\frac{\delta S}{\delta\bar{c}} + \frac{\delta S}{\delta\Omega^{B}_{\mu\nu}}\frac{\delta}{\delta B_{\mu\nu}} + \frac{\delta S}{\delta B_{\mu\nu}}\frac{\delta}{\delta\Omega^{B}_{\mu\nu}} \\
& + \frac{\delta S}{\delta\Omega^{\bar{B}}_{\mu\nu}}\frac{\delta}{\delta\bar{B}_{\mu\nu}} + \frac{\delta S}{\delta\bar{B}_{\mu\nu}}\frac{\delta}{\delta\Omega^{\bar{B}}_{\mu\nu}} + \frac{\delta S}{\delta\Omega^{\psi}_{\mu\nu}}\frac{\delta}{\delta\psi_{\mu\nu}} + \frac{\delta S}{\delta\psi_{\mu\nu}}\frac{\delta}{\delta\Omega^{\psi}_{\mu\nu}} + \frac{\delta S}{\delta\Omega^{\bar{\psi}}_{\mu\nu}}\frac{\delta}{\delta\bar{\psi}_{\mu\nu}} + \frac{\delta S}{\delta\bar{\psi}_{\mu\nu}}\frac{\delta}{\delta\Omega^{\bar{\psi}}_{\mu\nu}} \\
& + \frac{\delta S}{\delta\Omega^{Q}_{\mu\nu\alpha\beta}}\frac{\delta}{\delta Q_{\mu\nu\alpha\beta}} + \frac{\delta S}{\delta Q_{\mu\nu\alpha\beta}}\frac{\delta}{\delta\Omega^{Q}_{\mu\nu\alpha\beta}} + \frac{\delta S}{\delta\Omega^{J}_{\mu\nu\alpha\beta}}\frac{\delta}{\delta J_{\mu\nu\alpha\beta}} + \frac{\delta S}{\delta J_{\mu\nu\alpha\beta}}\frac{\delta}{\delta\Omega^{J}_{\mu\nu\alpha\beta}} \\
& + \frac{\delta S}{\delta\Omega^{\bar{Q}}_{\mu\nu\alpha\beta}}\frac{\delta}{\delta\bar{Q}_{\mu\nu\alpha\beta}} + \frac{\delta S}{\delta\bar{Q}_{\mu\nu\alpha\beta}}\frac{\delta}{\delta\Omega^{\bar{Q}}_{\mu\nu\alpha\beta}} + \frac{\delta S}{\delta\Omega^{\bar{J}}_{\mu\nu\alpha\beta}}\frac{\delta}{\delta\bar{J}_{\mu\nu\alpha\beta}} + \frac{\delta S}{\delta\bar{J}_{\mu\nu\alpha\beta}}\frac{\delta}{\delta\Omega^{\bar{J}}_{\mu\nu\alpha\beta}}\Bigg].
\end{aligned}$$

Furthermore, the $\mathcal{U}^{(0)}$ and $\tilde{\mathcal{U}}^{(0)}$ symmetries are combined to define the operator $\mathcal{Q}$ as

$$\mathcal{Q} \equiv \delta_{\alpha\mu}\delta_{\beta\nu}\big(\mathcal{U}^{(0)}_{\alpha\beta\mu\nu} + \tilde{\mathcal{U}}^{(0)}_{\alpha\beta\mu\nu}\big).$$

Notice that the action is invariant under $\mathcal{Q}$, i.e. $\mathcal{Q}(S) = 0$ because of $\mathcal{U}^{(0)}_{\alpha\beta\mu\nu}(S) = -\tilde{\mathcal{U}}^{(0)}_{\alpha\beta\mu\nu}(S)$.

Having defined the operators $\mathcal{B}_S$, $\bar{\mathcal{G}}$, $\mathcal{Q}$ and $\mathcal{U}^{(1)}$ we may derive the following set of graded commutators:

$$
\begin{aligned}
&\{\bar{\mathcal{G}},\bar{\mathcal{G}}\}=0, && \{\mathcal{B}_S,\mathcal{B}_S\}=0, && \{\bar{\mathcal{G}},\mathcal{B}_S\}=0,\\
&[\bar{\mathcal{G}},\mathcal{Q}]=0, && [\mathcal{Q},\mathcal{Q}]=0, && \{\bar{\mathcal{G}},\mathcal{U}^{(1)}_{\mu\nu\alpha\beta}\}=0,\\
&\{\mathcal{B}_S,\mathcal{U}^{(1)}_{\mu\nu\alpha\beta}\}=0, && \{\mathcal{U}^{(1)}_{\mu\nu\alpha\beta},\mathcal{U}^{(1)}_{\mu'\nu'\alpha'\beta'}\}=0, && [\mathcal{U}^{(1)}_{\mu\nu\alpha\beta},\mathcal{Q}]=0,\\
&[\mathcal{B}_S,\mathcal{Q}]=0,
\end{aligned}
$$

which shows that the algebra of symmetries closes.

Having derived the symmetry content of the model, we would now be ready to apply the method of Algebraic Renormalization (AR). The latter requires locality which, however, is not given in the present case and generally for all non-commutative QFTs, due to the inherent non-locality of the star product. Hence, before the application of AR it would be required to establish the foundations of this method also for non-commutative theories. For a detailed discussion we would like to refer to our recent article [22].

3 One-loop calculations

In this section we shall present the calculations relevant for the one-loop correction to the gauge boson propagator. Due to the existence of the mixed propagators G^{AB}, $G^{\mathrm{A}\bar{B}}$, and their mirrored counterparts, the two point function $\langle A_\mu A_\nu\rangle$ receives contributions not only from graphs with external gauge boson legs, but also from those featuring external B and/or $\bar{B}$ fields.

In the following (i.e. in Sections 3.1–3.4), we will present a detailed analysis of all truncated two-point functions relevant for the calculation of the full one-loop AA-propagator. Every type of correction, being characterized by its amputated external legs (i.e. A, B or $\bar{B}$), is discussed in a separate subsection. Finally, in Section 3.5 the dressed AA-propagator and the attempt for its one-loop renormalization will be given explicitly.

3.1 Vacuum polarization

The model (2.6) gives rise to 23 graphs contributing to the two-point function $G^{AA}_{\mu\nu}(p)$. Omitting convergent expressions, there are 11 graphs left depicted in Fig. 1. Being interested in the divergent contributions one can apply the expansion [3]

$$
\begin{aligned}
\Pi_{\mu\nu} = \int d^4k\, \mathcal{I}_{\mu\nu}(p,k)\sin^2\left(\tfrac{k\tilde{p}}{2}\right) \approx \int d^4k\, \sin^2\left(\tfrac{k\tilde{p}}{2}\right)\Big\{\mathcal{I}_{\mu\nu}(0,k) + p_\rho\left[\partial_{p_\rho}\mathcal{I}_{\mu\nu}(p,k)\right]_{p=0}\\
+\frac{p_\rho p_\sigma}{2}\left[\partial_{p_\rho}\partial_{p_\sigma}\mathcal{I}_{\mu\nu}(p,k)\right]_{p=0} + \mathcal{O}\left(p^3\right)\Big\},
\end{aligned}
\tag{3.1}
$$

where the integrand $\mathcal{I}_{\mu\nu}(p,k)$ has been separated from the phase factor in order to keep the regularizing effects in the non-planar parts due to rapid oscillations for large k. Summing up the contributions of the graphs in Fig. 1 and denoting the result at order i for the planar (p) part by $\Pi^{(i),\mathrm{p}}_{\mu\nu}$, one is left with

$$
\begin{aligned}
\Pi^{(0),\mathrm{p}}_{\mu\nu}(p) &= \frac{g^2}{16\pi^2}\Lambda^2\delta_{\mu\nu}\left(-10s_{\mathrm{c}} - 96s_{\mathrm{h}} - 96s_{\mathrm{j}} + 12s_{\mathrm{a}} + s_{\mathrm{b}} + 96s_{\mathrm{d}} + 96s_{\mathrm{f}}\right) = 0,\\
\Pi^{(2),\mathrm{p}}_{\mu\nu}(p) &= -\frac{1}{3}\frac{g^2}{16\pi^2}\Big[\delta_{\mu\nu}p^2\left(22s_{\mathrm{a}} + s_{\mathrm{b}} + 48(s_{\mathrm{d}}+s_{\mathrm{f}})\right)\\
&\qquad + 2p_\mu p_\nu\left(72(s_{\mathrm{h}}+s_{\mathrm{j}}) - 8s_{\mathrm{a}} + s_{\mathrm{b}} - 96(s_{\mathrm{d}}+s_{\mathrm{f}})\right)\Big]\mathrm{K}_0\left(2\sqrt{\frac{M^2}{\Lambda^2}}\right)
\end{aligned}
$$

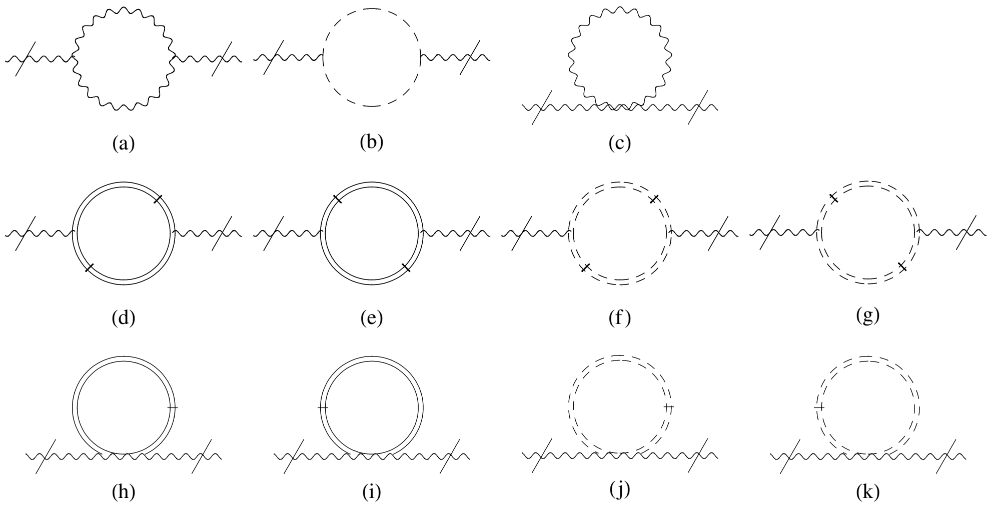


Figure 1. One loop corrections for the gauge boson propagator.

Table 2. Symmetry factors for the one loop vacuum polarization (where the factor (-1) for fermionic loops has been included).

s_{a}	$\frac{1}{2}$	s_{e}	1	s_{i}	1
s_{b}	-1	s_{f}	-1	s_{j}	-1
s_{c}	$\frac{1}{2}$	s_{g}	-1	s_{k}	-1
s_{d}	1	s_{h}	1		

$$
\begin{aligned}
&= -\frac{5g^2}{12\pi^2}\left(p^2\delta_{\mu\nu} - p_\mu p_\nu\right) K_0\left(2\sqrt{\frac{M^2}{\Lambda^2}}\right) \\
&\approx -\frac{5g^2}{24\pi^2}\left(p^2\delta_{\mu\nu} - p_\mu p_\nu\right) \ln\left(\frac{\Lambda^2}{M^2}\right) + \text{finite},
\end{aligned}
$$

where the symmetry factors in Table 2 have been inserted and the approximation

$$
\mathrm{K}_0(x) \underset{x\ll 1}{\approx} \ln\tfrac{2}{x} - \gamma_E + \mathcal{O}\left(x^2\right),
$$

for the modified Bessel function K_0 can be utilized for small arguments, i.e. vanishing regulator cutoffs[2] $\Lambda \to \infty$ and $M \to 0$. Finally, γ_E denotes the Euler–Mascheroni constant. Note that the first order vanishes identically due to an odd power of k in the integrand which leads to a cancellation under the symmetric integration over the momenta.

Of particular interest is the non-planar part (np) which for small p results to:

$$
\begin{aligned}
\Pi^{(0),\mathrm{np}}_{\mu\nu}(p) &= \frac{g^2}{4\pi^2\tilde{p}^2}\Big[\delta_{\mu\nu}\left(96(s_{\mathrm{h}}+s_{\mathrm{j}}-s_{\mathrm{d}}-s_{\mathrm{f}}) - 12s_{\mathrm{a}} - s_{\mathrm{b}} + 10s_{\mathrm{c}}\right) \\
&\quad - 2\frac{\tilde{p}_\mu\tilde{p}_\nu}{\tilde{p}^2}\left(48(s_{\mathrm{h}}+s_{\mathrm{j}}) - 96(s_{\mathrm{d}}+s_{\mathrm{f}}) - 12s_{\mathrm{a}} - s_{\mathrm{b}} + 2s_{\mathrm{c}}\right)\Big] = \frac{2g^2}{\pi^2}\frac{\tilde{p}_\mu\tilde{p}_\nu}{(\tilde{p}^2)^2}, \qquad (3.2\mathrm{a}) \\
\Pi^{(2),\mathrm{np}}_{\mu\nu}(p) &= \frac{g^2}{48\pi^2\tilde{p}^2}\Big\{2\theta^2 p_\mu p_\nu p^2\left(72(s_{\mathrm{h}}+s_{\mathrm{j}}) - 8s_{\mathrm{a}} + s_{\mathrm{b}} - 96(s_{\mathrm{d}}+s_{\mathrm{f}})\right)\mathrm{K}_0\left(\sqrt{M^2\tilde{p}^2}\right) \\
&\quad + \sqrt{\frac{\tilde{p}^2}{M^2}}p^2\Big[\sqrt{\frac{\tilde{p}^2}{M^2}}\left(22s_{\mathrm{a}} + s_{\mathrm{b}} + 48(s_{\mathrm{d}}+s_{\mathrm{f}})\right)M^2\delta_{\mu\nu}\,\mathrm{K}_0\left(\sqrt{M^2\tilde{p}^2}\right)
\end{aligned}
$$

[2]The cutoffs are introduced via a factor $\exp\left[-M^2\alpha - \frac{1}{\Lambda^2\alpha}\right]$ to regularize parameter integrals $\int_0^\infty \mathrm{d}\alpha$. See [3] for a more extensive description of the mathematical details underlying these computations.

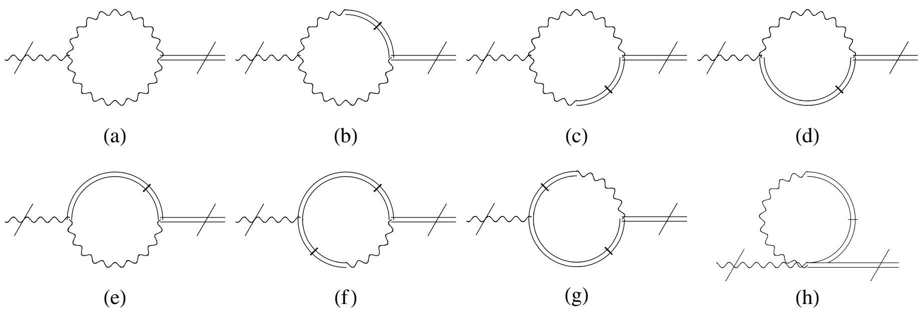


Figure 2. One loop corrections for $\langle A_\mu B_{\nu_1\nu_2}\rangle$ (with amputated external legs).

Table 3. Symmetry factors for the graphs depicted in Fig. 2.

(a)	$1/2$	(e)	1
(b)	1	(f)	1
(c)	1	(g)	1
(d)	1	(h)	1

$$
\begin{aligned}
&+ 2M^2\left(13s_{\mathrm{a}} + s_{\mathrm{b}} + 120(s_{\mathrm{d}} + s_{\mathrm{f}})\right)\tilde{p}_\mu\tilde{p}_\nu\,\mathrm{K}_1\left(\sqrt{M^2\tilde{p}^2}\right)\\
&- 3\sqrt{\frac{M^2}{\tilde{p}^2}}\left(16s_{\mathrm{a}} + s_{\mathrm{b}} + 96(s_{\mathrm{d}} + s_{\mathrm{f}})\right)\tilde{p}_\mu\tilde{p}_\nu\Bigg]\Bigg\}\\
=& -\frac{g^2}{48\pi^2}\left[\tilde{p}_\mu\tilde{p}_\nu\left(\frac{21}{\theta^2} - 11p^2\sqrt{\frac{M^2}{\tilde{p}^2}}\,\mathrm{K}_1\left(\sqrt{M^2\tilde{p}^2}\right)\right)\right.\\
&\left. - 10\,\mathrm{K}_0\left(\sqrt{M^2\tilde{p}^2}\right)\left(p^2\delta_{\mu\nu} - p_\mu p_\nu\right)\right]. \qquad (3.2b)
\end{aligned}
$$

Considering the limit $\tilde{p}^2 \to 0$ rectifies application of the approximation

$$
\mathrm{K}_1(x) \underset{x\ll 1}{\approx} \tfrac{1}{x} + \tfrac{x}{2}\left(\gamma_E - \tfrac{1}{2} + \ln\tfrac{x}{2}\right) + \mathcal{O}\left(x^2\right),
$$

which reveals that the second order is IR finite (which is immediately clear from the fact that the terms of lowest order in p are $\mathcal{O}\left(p^2\right)$), apart from a $\ln(M^2)$-term which cancels in the sum of planar and non-planar contributions. Hence, collecting all divergent terms one is left with (in the limit $M \to 0$ and $\Lambda \to \infty$),

$$
\Pi_{\mu\nu}(p) = \frac{2g^2}{\pi^2}\frac{\tilde{p}_\mu\tilde{p}_\nu}{(\tilde{p}^2)^2} - \lim_{\Lambda\to\infty}\frac{5g^2}{24\pi^2}\left(p^2\delta_{\mu\nu} - p_\mu p_\nu\right)\ln\left(\Lambda^2\right) + \text{finite terms}, \qquad (3.3)
$$

which is independent of the IR-cutoff M. As expected[3], equation (3.3) exhibits a quadratic IR divergence in $\tilde{p}^2$ and a logarithmic divergence in the cutoff Λ. Furthermore, the transversality condition $p_\mu\Pi_{\mu\nu}(p) = 0$ is fulfilled, which serves as a consistency check for the symmetry factors.

3.2 Corrections to the AB propagator

The action (2.6) gives rise to eight divergent graphs with one external A_μ and one $B_{\mu\nu}$ which are depicted in Fig. 2. Applying an expansion of type (3.1) for small external momenta p and

[3]In fact, equation (3.3) qualitatively resembles the result of the "naïve" non-commutative gauge model discussed e.g. in [23, 24, 25, 26]. The different numerical factor in front of the logarithmic UV divergence is a consequence of the contribution of additional fields in the current model.

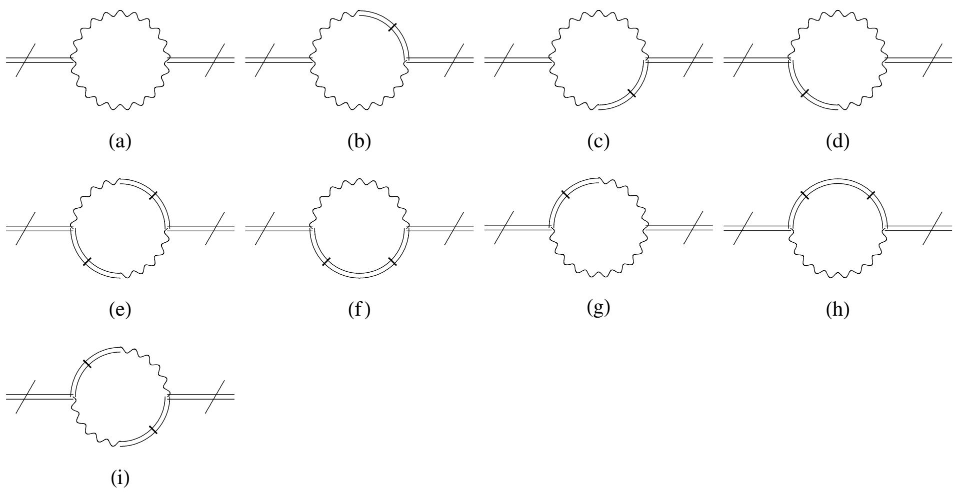


Figure 3. One loop corrections for $\langle B_{\mu_1\mu_2} B_{\nu_1\nu_2} \rangle$ (with amputated external legs).

Table 4. Symmetry factors for the graphs depicted in Fig. 3.

(a)	$1/2$	(d)	1	(g)	1
(b)	1	(e)	1	(h)	1
(c)	1	(f)	1	(i)	1

summing up the divergent contributions of all graphs (all orders of an expansion similar to equation (3.1)) one ends up with,

$$\Sigma^{\mathrm{p,AB}}_{\mu1,\nu1\nu2}(p) = -\frac{3\mathrm{i}g^2}{32\pi^2}\lambda \left(p_{\nu1}\delta_{\mu1\nu2} - p_{\nu2}\delta_{\mu1\nu1}\right) \mathrm{K}_0\left(2\sqrt{\frac{M^2}{\Lambda^2}}\right) + \text{finite},$$

$$\Sigma^{\mathrm{np,AB}}_{\mu1,\nu1\nu2}(p) = \frac{3\mathrm{i}g^2}{32\pi^2}\lambda \, \mathrm{K}_0\left(\sqrt{M^2\tilde{p}^2}\right) \left(p_{\nu1}\delta_{\mu1\nu2} - p_{\nu2}\delta_{\mu1\nu1}\right) + \text{finite}.$$

Approximating the Bessel functions as in Section 3.1 and summing up planar and non-planar parts one finds the expression

$$\Sigma^{\mathrm{AB}}_{\mu1,\nu1\nu2}(p) = \frac{3\mathrm{i}g^2}{32\pi^2}\lambda \left(p_{\nu1}\delta_{\mu1\nu2} - p_{\nu2}\delta_{\mu1\nu1}\right) \left(\ln\Lambda + \ln|\tilde{p}|\right) + \text{finite},$$

where the IR cutoff M has cancelled, and which shows a logarithmic divergence for $\Lambda \to \infty$.

Due to the symmetry between B and $\bar{B}$ in the sense that both have identical interactions with the gauge field, it is obvious that $\Sigma^{\mathrm{AB}}_{\mu1,\nu1\nu2} \equiv \Sigma^{\mathrm{A\bar{B}}}_{\mu1,\nu1\nu2}$ and as implied by equation (2.8a) it also holds that $\Sigma^{\mathrm{BA}}_{\mu1\mu2,\nu1} \equiv -\Sigma^{\mathrm{AB}}_{\nu1,\mu1\mu2}$.

3.3 Corrections to the BB propagator

The set of divergent graphs contributing to $\langle B_{\mu1\mu2} B_{\nu1\nu2} \rangle$ consists of those depicted in Fig. 3. Making an expansion of type (3.1) for small external momenta p and summing up the contributions of all nine graphs yields

$$\Sigma^{\mathrm{p,BB}}_{\mu1\mu2,\nu1\nu2}(p) = \frac{g^2\lambda^2}{32\pi^2}\left(\delta_{\mu1\nu1}\delta_{\mu2\nu2} - \delta_{\mu2\nu1}\delta_{\mu1\nu2}\right) \mathrm{K}_0\left(2\sqrt{\frac{M^2}{\Lambda^2}}\right) + \text{finite},$$

$$\Sigma^{\mathrm{np,BB}}_{\mu1\mu2,\nu1\nu2}(p) = \frac{g^2\lambda^2}{64\pi^2}\Bigg(\frac{\delta_{\mu1\nu2}\tilde{p}_{\mu2}\tilde{p}_{\nu1} - \delta_{\mu1\nu1}\tilde{p}_{\mu2}\tilde{p}_{\nu2} - \delta_{\mu2\nu2}\tilde{p}_{\mu1}\tilde{p}_{\nu1} + \delta_{\mu2\nu1}\tilde{p}_{\mu1}\tilde{p}_{\nu2}}{\tilde{p}^2}$$

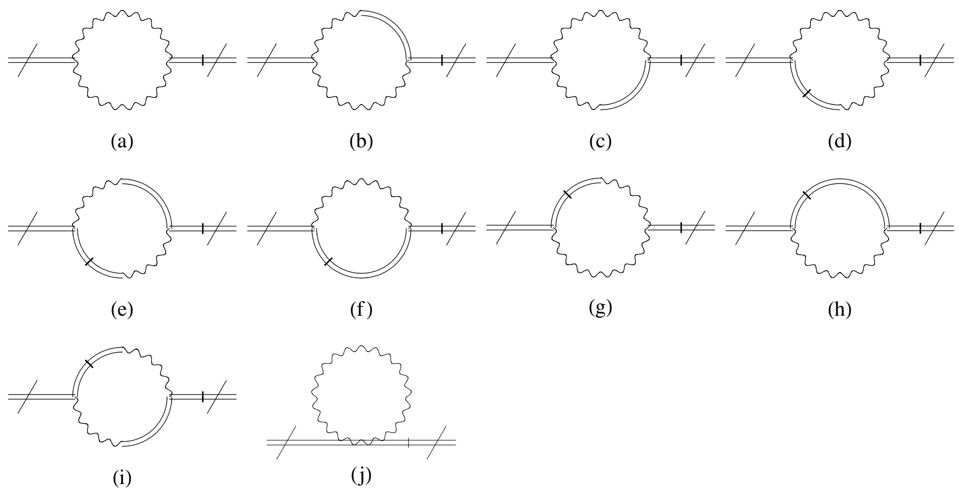


Figure 4. One loop corrections for $\langle B_{\mu_1\mu_2}\bar{B}_{\nu_1\nu_2}\rangle$ (with amputated external legs).

Table 5. Symmetry factors for the graphs depicted in Fig. 4.

(a)	$1/2$	(e)	1	(i)	1
(b)	1	(f)	1	(j)	$1/2$
(c)	1	(g)	1		
(d)	1	(h)	1		

$$+\, 2\,\mathrm{K}_0\left(\sqrt{M^2\tilde{p}^2}\right)\left(\delta_{\mu1\nu2}\delta_{\mu2\nu1}-\delta_{\mu1\nu1}\delta_{\mu2\nu2}\right)\Bigg) + \text{finite},$$

for the planar/non-planar part, respectively. Approximating the Bessel functions as in Section 3.1 reveals cancellations of contributions depending on M in the final sum. Hence, the divergent part boils down to

$$\Sigma^{\mathrm{BB}}_{\mu1\mu2,\nu1\nu2}(p) = \frac{g^2\lambda^2}{64\pi^2}\left(\delta_{\mu1\nu1}\delta_{\mu2\nu2}-\delta_{\mu2\nu1}\delta_{\mu1\nu2}\right)\left(\ln\Lambda^2+\ln\tilde{p}^2\right) + \text{finite}, \tag{3.4}$$

leaving a logarithmic divergence for both the planar and the non-planar part. Due to symmetry reasons this result is also equal to the according correction to the $\bar{B}\bar{B}$ propagator, i.e.

$$\Sigma^{\bar{\mathrm{B}}\bar{\mathrm{B}}}_{\mu1\mu2,\nu1\nu2}(p) = \Sigma^{\mathrm{BB}}_{\mu1\mu2,\nu1\nu2}(p).$$

3.4 Corrections to the $B\bar{B}$ propagator

For the correction to $\langle B_{\mu1\mu2}\bar{B}_{\nu1\nu2}\rangle$ one finds the ten divergent graphs depicted in Fig. 4. Expansion for small external momenta p and summation of the integrated results yields

$$\begin{aligned}
\Sigma^{\mathrm{p,B\bar{B}}}_{\mu1\mu2,\nu1\nu2}(p) &= \frac{g^2}{2\pi^2}\Lambda^2\mu^2\tilde{p}^2\left(\delta_{\mu2\nu1}\delta_{\mu1\nu2}-\delta_{\mu1\nu1}\delta_{\mu2\nu2}\right)\\
&\quad+\frac{g^2\lambda^2}{32\pi^2}\left(\delta_{\mu1\nu1}\delta_{\mu2\nu2}-\delta_{\mu2\nu1}\delta_{\mu1\nu2}\right)\mathrm{K}_0\left(2\sqrt{\frac{M^2}{\Lambda^2}}\right)+\text{finite},\\
\Sigma^{\mathrm{np,B\bar{B}}}_{\mu1\mu2,\nu1\nu2}(p) &= \frac{g^2\lambda^2}{64\pi^2}\Bigg(\frac{\delta_{\mu1\nu2}\tilde{p}_{\mu2}\tilde{p}_{\nu1}-\delta_{\mu1\nu1}\tilde{p}_{\mu2}\tilde{p}_{\nu2}-\delta_{\mu2\nu2}\tilde{p}_{\mu1}\tilde{p}_{\nu1}+\delta_{\mu2\nu1}\tilde{p}_{\mu1}\tilde{p}_{\nu2}}{\tilde{p}^2}\\
&\quad+2\,\mathrm{K}_0\left(\sqrt{M^2\tilde{p}^2}\right)\left(\delta_{\mu1\nu2}\delta_{\mu2\nu1}-\delta_{\mu1\nu1}\delta_{\mu2\nu2}\right)\Bigg)+\text{finite},
\end{aligned}$$

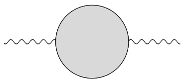
Hence, the divergent part is given by

$$\begin{aligned}\Sigma^{\mathrm{B}\bar{B}}_{\mu1\mu2,\nu1\nu2}(p) &= \frac{g^2}{2\pi^2}\Lambda^2\mu^2\tilde{p}^2\left(\delta_{\mu2\nu1}\delta_{\mu1\nu2}-\delta_{\mu1\nu1}\delta_{\mu2\nu2}\right)\\ &\quad+\frac{g^2\lambda^2}{64\pi^2}\left(\delta_{\mu1\nu1}\delta_{\mu2\nu2}-\delta_{\mu2\nu1}\delta_{\mu1\nu2}\right)\left(\ln\Lambda^2+\ln\tilde{p}^2\right)+\text{finite},\end{aligned}$$

which is logarithmically divergent in $\tilde{p}^2$ and quadratically in Λ. Once more, M has dropped out in the sum of planar and non-planar contributions. Furthermore, note that $\Sigma^{\mathrm{B\bar{B}}}_{\mu1\mu2,\nu1\nu2}\equiv\Sigma^{\mathrm{\bar{B}B}}_{\nu1\nu2,\mu1\mu2}$ as is obvious from the result (3.4).

3.5 Dressed gauge boson propagator and analysis

In the standard renormalization procedure, the dressed propagator at one-loop level is given by



$$\equiv\Delta'(p)=\frac{1}{\mathcal{A}}+\frac{1}{\mathcal{A}}\Sigma(\Lambda,p)\frac{1}{\mathcal{A}},\tag{3.5}$$

where

$$\frac{1}{\mathcal{A}}\equiv G^{\mathrm{AA}}_{\mu\nu}(p),\qquad \Sigma(\Lambda,p)\equiv\left(\Pi^{\text{plan}}\right)_{\text{regul.}}(\Lambda,p)+\Pi^{\text{n-pl}}(p).$$

For $\mathcal{A}\neq0$, one can apply the formula

$$\frac{1}{\mathcal{A}+\mathcal{B}}=\frac{1}{\mathcal{A}}-\frac{1}{\mathcal{A}}\mathcal{B}\frac{1}{\mathcal{A}+\mathcal{B}}=\frac{1}{\mathcal{A}}-\frac{1}{\mathcal{A}}\mathcal{B}\frac{1}{\mathcal{A}}+\mathcal{O}(\mathcal{B}^2),\tag{3.6}$$

which allows one to rewrite expression (3.5) to order Σ as

$$\Delta'(p)=\frac{1}{\mathcal{A}-\Sigma(\Lambda,p)},$$

and thus (in the case of renormalizability) to absorb any divergences in the appropriate parameters of the theory present in $\mathcal{A}$ (see [14] for an example).

However, in our case (3.6) cannot be applied directly, as the complete one loop correction to the gauge boson propagator is given by the sum of all the results of Sections 3.1–3.4 after multiplication with *appropriate, i.e. different* external legs:

$$\begin{aligned}G^{\mathrm{AA},1\mathrm{l-ren}}_{\mu\nu}(p) &= G^{\mathrm{AA}}_{\mu\nu}(p)+G^{\mathrm{AA}}_{\mu\rho}(p)\Pi_{\rho\sigma}(p)G^{\mathrm{AA}}_{\sigma\nu}(p)+G^{\mathrm{AA}}_{\mu\rho}(p)2\Sigma^{\mathrm{AB}}_{\rho,\sigma1\sigma2}(p)G^{\mathrm{BA}}_{\sigma1\sigma2,\nu}(p)\\ &\quad+G^{\mathrm{AA}}_{\mu\rho}(p)2\Sigma^{\mathrm{A\bar{B}}}_{\rho,\sigma1\sigma2}(p)G^{\mathrm{\bar{B}A}}_{\sigma1\sigma2,\nu}(p)+G^{\mathrm{AB}}_{\mu,\rho1\rho2}(p)\Sigma^{\mathrm{BB}}_{\rho1\rho2,\sigma1\sigma2}(p)G^{\mathrm{BA}}_{\sigma1\sigma2,\nu}(p)\\ &\quad+G^{\mathrm{AB}}_{\mu,\rho1\rho2}(p)2\Sigma^{\mathrm{B\bar{B}}}_{\rho1\rho2,\sigma1\sigma2}(p)G^{\mathrm{\bar{B}A}}_{\sigma1\sigma2,\nu}(p)+G^{\mathrm{A\bar{B}}}_{\mu,\rho1\rho2}(p)\Sigma^{\mathrm{\bar{B}\bar{B}}}_{\rho1\rho2,\sigma1\sigma2}(p)G^{\mathrm{\bar{B}A}}_{\sigma1\sigma2,\nu}(p)\\ &\quad+\mathcal{O}\left(g^4\right).\end{aligned}\tag{3.7}$$

Note, that the factors 2 stem from the (not explicitly written) mirrored contributions $AB\leftrightarrow BA$, $A\bar{B}\leftrightarrow\bar{B}A$, and $B\bar{B}\leftrightarrow\bar{B}B$. Since the factor $\mathcal{A}$ must be the same for all summands we have to use the Ward Identities (2.8a) and (2.8c), i.e.

$$\begin{aligned}&G^{AB}_{\mu,\rho\sigma}(k)=G^{A\bar{B}}_{\mu,\rho\sigma}(k)=-G^{BA}_{\rho\sigma,\mu}(k)=-G^{\bar{B}A}_{\rho\sigma,\mu}(k),\\ &2k^2\tilde{k}^2G^{AB}_{\rho,\mu\nu}(k)=\mathrm{i}\frac{a'}{\mu}\left(k_\mu G^{AA}_{\rho\nu}(k)-k_\nu G^{AA}_{\rho\mu}(k)\right),\end{aligned}\tag{3.8}$$

which allow us to express the (tree level) AB and $A\bar{B}$ propagators uniquely in terms of AA-propagators. This leads (in analogy to (3.6)) to the following representation for the dressed one-loop gauge boson propagator:

$$G^{\mathrm{AA},1\mathrm{l-ren}}_{\mu\nu}(p)=\frac{1}{\mathcal{A}}-\frac{1}{\mathcal{A}}\Big(\sum\mathcal{B}_i\Big)\frac{1}{\mathcal{A}},$$

where $1/\mathcal{A}$ once more stands for the tree level gauge boson propagator. The $\mathcal{B}_i$'s are given by the one-loop corrections (with amputated external legs) of the two-point functions relevant for the dressed gauge boson propagator, multiplied by any prefactors coming from (3.8) and the factor 2 where needed (c.f. (3.7)). Thus, the full propagator is given by

$$\begin{aligned}
G^{\mathrm{AA,1l-ren}}_{\mu\nu}(p) &= G^{\mathrm{AA}}_{\mu\nu}(p) + G^{\mathrm{AA}}_{\mu\rho}(p)\Pi_{\rho\sigma}(p)G^{\mathrm{AA}}_{\sigma\nu}(p) \\
&+ \left(\frac{\mathrm{i}a'}{\mu p^2\tilde{p}^2}\right)\Bigg\{2G^{\mathrm{AA}}_{\mu\rho}(p)\left(\Sigma^{\mathrm{AB}}_{\rho,\sigma1\sigma2}(p) + \Sigma^{\mathrm{A\bar{B}}}_{\rho,\sigma1\sigma2}(p)\right)p_{\sigma2}G^{\mathrm{AA}}_{\nu\sigma1}(p) \\
&+ \left(\tfrac{\mathrm{i}a'}{\mu p^2\tilde{p}^2}\right)p_{\rho1}G^{\mathrm{AA}}_{\mu\rho2}(p)\left(\Sigma^{\mathrm{BB}}_{\rho1\rho2,\sigma1\sigma2}(p) + 2\Sigma^{\mathrm{B\bar{B}}}_{\rho1\rho2,\sigma1\sigma2}(p) + \Sigma^{\mathrm{\bar{B}\bar{B}}}_{\rho1\rho2,\sigma1\sigma2}(p)\right)p_{\sigma2}G^{\mathrm{AA}}_{\nu\sigma1}(p)\Bigg\}.
\end{aligned}$$

The expression $\mathcal{B} = \sum\limits_i \mathcal{B}_i$ for $M \to 0$ is explicitly given by

$$\begin{aligned}
\mathcal{B} = \frac{g^2}{8\pi^2\mu^4}\Bigg\{&\tilde{p}_\mu\tilde{p}_\nu\left(\frac{16\mu^4}{(\tilde{p}^2)^2} + \frac{\theta^4\lambda^4}{2(\tilde{p}^2)^4}\right) - 7\lambda^2\mu^2\frac{\theta^4}{(\tilde{p}^2)^4}\left(p^2\delta_{\mu\nu} - p_\mu p_\nu\right)\left(4 - \tilde{p}^2\Lambda^2\right) \\
&+ \left(p^2\delta_{\mu\nu} - p_\mu p_\nu\right)\left[\ln 2 - \ln|\tilde{p}| - \ln\Lambda\right]\left(\frac{5}{3}\mu^4 + \frac{3\lambda^2\mu^2\theta^2}{(\tilde{p}^2)^2} + \frac{\lambda^4\theta^4}{(\tilde{p}^2)^4}\right)\Bigg\} + \text{finite},
\end{aligned}$$

and shows us two things: In contrast to commutative gauge models and even though the vacuum polarization tensor $\Pi_{\mu\nu}$ only had a logarithmic UV divergence, the full $\mathcal{B}$ diverges quadratically in the UV cutoff Λ. Secondly, despite the fact that $\Pi_{\mu\nu}$ exhibited the usual quadratic IR divergence, $\mathcal{B}$ behaves like $\frac{1}{(\tilde{p}^2)^3}$ in the IR limit. Both properties arise due to the existence (and the form) of the mixed AB and $A\bar{B}$ propagators, and seem problematic concerning renormalization for two reasons: On the one hand, the form of the propagator is modified implying new counter terms in the effective action. On the other hand, higher loop insertions of this expression can lead to IR divergent integrals, as will be discussed in the next section.

4 Higher loop calculations

In the light of higher loop calculations it is important to investigate the IR behaviour of expected integrands with insertions of the one-loop corrections being discussed in Section 3. The aim is to identify possible poles at $\tilde{p}^2 = 0$. Hence, we consider a chain of n non-planar insertions denoted by $\Xi^{\phi_1\phi_2}(p, n)$, which may be part of a higher loop graph. Every insertion Ξ represents the sum of all divergent one-loop contributions with external fields ϕ_1 and ϕ_2 (cf. Sections 3.1–3.4). Due to the numerous possibilities of constructing such graphs, we will examine only a few exemplary configurations in this section – especially those for which one expects the worst IR behaviour.

To start with, let us state that amongst all types of two point functions, the vacuum polarization shows the highest, namely a quadratic divergence. Amongst the propagators those with two external double-indexed legs, e.g. B or $\bar{B}$ feature the highest (quartic) divergence in the limit of vanishing external momenta. A chain of n vacuum polarizations $\Pi^{\mathrm{np}}_{\mu\nu}(p)$ (see equations (3.2a) and (3.2b)) with $(n+1)$ AA-propagators ($(n-1)$ between the individual vacuum polarization graphs, and one at each end) leads to the following expression (for a graphical representation, see Fig. 5):

$$\Xi^{AA}_{\mu\nu}(p, n) = \left(G^{AA}(p)\Pi^{\mathrm{np}}(p)\right)^n_{\mu\rho}G^{AA}_{\rho\nu}(p) = \left(\frac{2g^2}{\pi^2}\right)^n \frac{1}{\left(p^2 + \frac{a'^2}{\tilde{p}^2}\right)^{n+1}}\frac{\tilde{p}_\mu\tilde{p}_\nu}{(\tilde{p}^2)^{n+1}}.$$

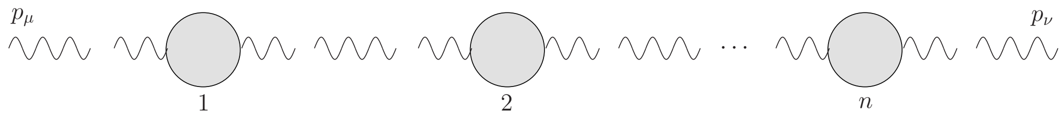


Figure 5. A chain of n non-planar insertions, concatenated by gauge field propagators.

Note that due to transversality, from the propagator (2.7c) only the term with the Kronecker delta enters the calculation. For vanishing momenta, i.e. in the limit $\tilde{p}^2 \to 0$ the expression reduces to

$$\lim_{\tilde{p}^2 \to 0} \Xi^{AA}_{\mu\nu}(p,n) = \left(\frac{2g^2}{\pi^2}\right)^n \frac{\tilde{p}_\mu \tilde{p}_\nu}{a'^{2(n+1)}},$$

exhibiting IR finiteness which is independent from the number of inserted loops.

Another representative is the chain

$$\Xi^{A\phi}(p,n) \equiv G^{A\phi}(p) \left(\Sigma^{\mathrm{np},\phi\mathrm{A}}(p) G^{A\phi}(p)\right)^n, \qquad \text{where} \quad \phi \in \{B, \bar{B}\},$$

which could replace any single G^{AB} (or $G^{A\bar{B}}$) line. Obviously, one has

$$\Xi^{A\phi}_{\mu,\nu1\nu2}(p,n) = \frac{\mathrm{i}a'}{2\mu}\left(-\frac{3g^2}{32\pi^2}a'^2\right)^n \frac{(p_{\nu1}\delta_{\mu\,\nu2} - p_{\nu2}\delta_{\mu\,\nu1})}{p^2\left[\tilde{p}^2\left(p^2 + \frac{a'^2}{\tilde{p}^2}\right)\right]^{n+1}} n \ln \tilde{p}^2,$$

which for $\tilde{p}^2 \ll 1$ (and neglecting dimensionless prefactors) behaves like

$$\Xi^{A\phi}_{\mu,\nu1\nu2}(p,n) \approx n \frac{(p_{\nu1}\delta_{\mu\,\nu2} - p_{\nu2}\delta_{\mu\,\nu1})}{\mu p^2} \ln \tilde{p}^2.$$

The latter insertion can be regularized since the pole at $p = 0$ is independent of n. In contrast, higher divergences are expected for chain graphs being concatenated by propagators with four indices, i.e. $G^{\bar{B}B}_{\mu\nu,\rho\sigma}$, $G^{BB}_{\mu\nu,\rho\sigma}$, $G^{\bar{\psi}\psi}_{\mu\nu,\rho\sigma}$, due to the inherent quartic IR singularities. Let us start with the combination $\Xi^{\bar{B}B}(p,n) \equiv \left(G^{\bar{B}B}(p)\Sigma^{\mathrm{p,B\bar{B}}}(p)\right)^n G^{\bar{B}B}(p)$. As before, we can approximate for $\tilde{p}^2 \ll 1$ and, omitting dimensionless prefactors and indices, find

$$\Xi^{A\phi}(p,n) \underset{\tilde{p}^2 \ll 1}{\propto} \frac{n}{\mu^2} \frac{\ln \tilde{p}^2}{(p^2\tilde{p}^2)^n},$$

which represents a singularity $\forall\, n > 1$ (since in any graph, at $n = 0$, the divergence is regularized by the phase factor being a sine function which behaves like p for small momenta). Regarding the index structures, no cancellations can be expected since the product of an arbitrary number of contracted, completely antisymmetric tensors is again an antisymmetric tensor with the outermost indices of the chain being free.

Exactly the same result is obtained for $\Xi^{BB}(p) \equiv \left(G^{BB}(p)\Sigma^{\mathrm{p,BB}}(p)\right)^n G^{BB}(p)$. From this it is clear that the damping mechanism seen in $\Xi^{AA}(p,n)$ fails for higher insertions of $B/\bar{B}$ (and also $\psi/\bar{\psi}$) fields).

5 Discussion

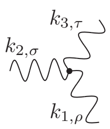
We have elaborated on our recently introduced non-commutative gauge model [17]. Initially, the intent was to apply Algebraic Renormalization (AR), as was suggested by Vilar et al. [2]. In the light of that renormalization scheme it is most important to maximise the symmetry content of the theory which is the basis for the generation of constraints to potential counter terms. Therefore, after recapitulating general properties of our model, we studied the resulting algebra of symmetries. However, as we exposed recently [22], the foundations of AR are only proved to be valid in *local* QFTs so far, and hence may not be applicable in non-commutative field theories, as the deformation inherently implies non-locality. In order to find a way out of this dilemma, explicit loop-calculations were presented, and our hope was to show renormalizability – at least at the one-loop level. In this respect, unexpected difficulties appeared. The soft breaking term, being required to implement the IR damping behaviour of the $1/p^2$ model in a way being compatible with the Quantum Action Principle of AR, gives rise to mixed propagators G^{AB} and $G^{A\bar{B}}$. These, in turn, allow the insertion of one-loop corrections with external B-fields into the dressed AA propagator (see Section 4) and, therefore, enter the renormalization. Despite all corrections featuring the expected $\frac{1}{p^2}$ IR behaviour, the dressed propagators with external AB or $A\bar{B}$ legs multiplicatively receive higher poles due to the inherent quadratic divergences in $G^{AB}(p)$ (and $G^{A\bar{B}}(p)$) for $p \to 0$. As a consequence, the resulting corrections cannot be absorbed in a straightforward manner.

However, renormalizability of the non-local model (1.1) cannot depend on how it is localized due to equivalence of the respective path integrals (see [17]). Therefore, we expect the same problems to appear in *all* localized versions of (1.1), including the one of Vilar et al. [2]. In fact, from the discussion in Appendix B, one notices that the propagators (B.2f)–(B.2h) and (B.2s) of their action all exhibit the same quartic IR divergences as those of our present model (2.6), even though the operator D_μ appears at most quadratically as D^2 in the according action (B.1). Nonetheless, the authors claim to have shown renormalizability using Algebraic Renormalization, which as we have discussed in [22] may not be applicable in non-commutative theories.

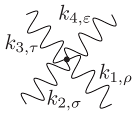
In this respect it has to be noted that in commutative space the model of Vilar et al. [2] should indeed be renormalizable, since the action, apart from the star product, is completely local and provides the necessary symmetries for the Quantum Action Principle. Since the propagators are the same in both spaces, and hence show the same quartic IR divergences, one may expect related IR problems to cancel when considering the sum of bosonic and fermionic sectors (i.e. B/χ and ψ/ξ). These cancellations should also take place in non-commutative space (in both models), but the problem of proving renormalization remains (cf. Section 3.5).

Coming back to the problem of IR divergent propagators we have also investigated the structure of singularities in higher-loop integrands by studying chain graphs consisting of interleaving tree-level propagators, and one-loop corrections of various types. It turned out that chains containing gauge fields benefit from the damping of the propagator (2.7c) while those consisting (solely) of concatenated B and $\bar{B}$ fields and insertions do (expectedly) not. Hence, at first sight, there exist divergences which increase order by order, which would indicate non-renormalizability. However, we may point out that, due to the symmetry between the $B/\bar{B}$ and $\psi/\bar{\psi}$ sectors, cancellations can be expected. These already appear in our one-loop calculations, and there is strong evidence that they appear to all orders. An intuitive argument can be given when considering the action (2.2) for $\lambda \to 0$, i.e. vanishing damping. In this case, the $B/\bar{B}$ and $\psi/\bar{\psi}$ fields may simply be integrated out in the path integral formalism (see [17]), and the contributions cancel exactly. An alternative approach which avoids these uncertainties is in preparation.

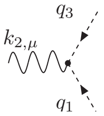
A Vertices



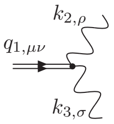
$$
\begin{aligned}
&= \widetilde{V}^{3A}_{\rho\sigma\tau}(k_1,k_2,k_3) = 2\mathrm{i}g(2\pi)^4\delta^4(k_1+k_2+k_3)\sin\left(\tfrac{k_1\tilde{k}_2}{2}\right)\\
&\quad\times\left[(k_3-k_2)_\rho\delta_{\sigma\tau}+(k_1-k_3)_\sigma\delta_{\rho\tau}+(k_2-k_1)_\tau\delta_{\rho\sigma}\right],
\end{aligned}
$$



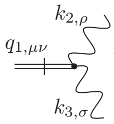
$$= \widetilde{V}^{4A}_{\rho\sigma\tau\epsilon}(k_1,k_2,k_3,k_4) = -4g^2(2\pi)^4\delta^4(k_1+k_2+k_3+k_4)$$
$$\times \Big[(\delta_{\rho\tau}\delta_{\sigma\epsilon} - \delta_{\rho\epsilon}\delta_{\sigma\tau}) \sin\left(\tfrac{k_1\tilde{k}_2}{2}\right)\sin\left(\tfrac{k_3\tilde{k}_4}{2}\right)$$
$$+ (\delta_{\rho\sigma}\delta_{\tau\epsilon} - \delta_{\rho\epsilon}\delta_{\sigma\tau}) \sin\left(\tfrac{k_1\tilde{k}_3}{2}\right)\sin\left(\tfrac{k_2\tilde{k}_4}{2}\right)$$
$$+ (\delta_{\rho\sigma}\delta_{\tau\epsilon} - \delta_{\rho\tau}\delta_{\sigma\epsilon}) \sin\left(\tfrac{k_2\tilde{k}_3}{2}\right)\sin\left(\tfrac{k_1\tilde{k}_4}{2}\right)\Big],$$



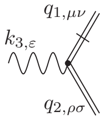
$$= \widetilde{V}^{\bar{c}Ac}_{\mu}(q_1,k_2,q_3) = -2\mathrm{i}g(2\pi)^4\delta^4(q_1+q_2+k_3)q_{1\mu}\sin\left(\tfrac{q_1\tilde{q}_3}{2}\right),$$



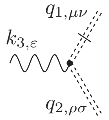
$$= \widetilde{V}^{BAA}_{\mu\nu,\rho\sigma}(q_1,k_2,k_3) =$$



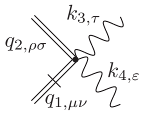
$$\widetilde{V}^{\bar{B}AA}_{\mu\nu,\rho\sigma}(q_1,k_2,k_3)$$
$$= \lambda g(2\pi)^4\delta^4(q_1+k_2+k_3)\left(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}\right)\sin\left(\tfrac{k_2\tilde{k}_3}{2}\right),$$



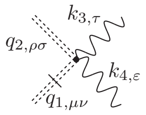
$$= \widetilde{V}^{\bar{B}BA}_{\mu\nu,\rho\sigma\epsilon}(q_1,q_2,k_3) =$$



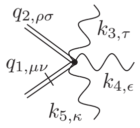
$$= -\widetilde{V}^{\bar{\psi}\psi A}_{\mu\nu,\rho\sigma\epsilon}(q_1,q_2,k_3)$$
$$= -\mathrm{i}\mu^2 g(2\pi)^4\delta^4(q_1+q_2+k_3)\left(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}\right)$$
$$\times \left((\tilde{q}_1)^2 + (\tilde{q}_2)^2\right)(q_1-q_2)_\epsilon \sin\left(\tfrac{q_1\tilde{q}_2}{2}\right),$$



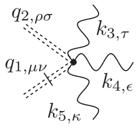
$$= \widetilde{V}^{\bar{B}B2A}_{\mu\nu,\rho\sigma,\tau\epsilon}(q_1,q_2,k_3,k_4) =$$



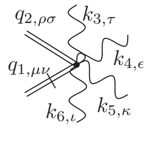
$$= -\widetilde{V}^{\bar{\psi}\psi 2A}_{\mu\nu,\rho\sigma,\tau\epsilon}(q_1,q_2,k_3,k_4)$$
$$= 2\mu^2 g^2\theta^2(2\pi)^4\delta^4(q_1+q_2+k_3+k_4)\left(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}\right)$$
$$\times \Big\{ \left[k_{3,\tau}k_{4,\epsilon} + 2(q_{1,\tau}k_{4,\epsilon} + q_{2,\epsilon}k_{3,\tau}) + 4q_{1,\tau}q_{2,\epsilon} - \delta_{\epsilon\tau}\left({q_1}^2 + {q_2}^2\right)\right]\sin\left(\tfrac{q_1\tilde{k}_3}{2}\right)\sin\left(\tfrac{q_2\tilde{k}_4}{2}\right)$$
$$+ \left[k_{3,\tau}k_{4,\epsilon} + 2(q_{2,\tau}k_{4,\epsilon} + q_{1,\epsilon}k_{3,\tau}) + 4q_{1,\epsilon}q_{2,\tau} - \delta_{\epsilon\tau}\left({q_1}^2 + {q_2}^2\right)\right]\sin\left(\tfrac{q_1\tilde{k}_4}{2}\right)\sin\left(\tfrac{q_2\tilde{k}_3}{2}\right)\Big\},$$



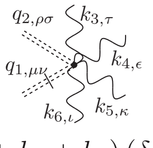
$$= \widetilde{V}^{\bar{B}B3A}_{\mu\nu,\rho\sigma,\tau\epsilon\kappa}(q_1,q_2,k_3,k_4,k_5) =$$



$$= -\widetilde{V}^{\bar{\psi}\psi 3A}_{\mu\nu,\rho\sigma,\tau\epsilon\kappa}(q_1,q_2,k_3,k_4,k_5)$$
$$= -4\mathrm{i}g^3\mu^2\theta^2(2\pi)^4\delta^4(q_1+q_2+k_3+k_4+k_5)\left(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}\right)\times$$
$$\times \Bigg\{ [k_3+2q_1]_\tau\,\delta_{\epsilon\kappa}\sin\left(\tfrac{k_3\tilde{q}_1}{2}\right)\left[\sin\left(\tfrac{k_5\tilde{q}_2}{2}\right)\sin\left(\tfrac{k_4(\tilde{k}_5+\tilde{q}_2)}{2}\right) + (k_4\leftrightarrow k_5)\right]$$
$$+ [k_4+2q_1]_\epsilon\,\delta_{\tau\kappa}\sin\left(\tfrac{k_4\tilde{q}_1}{2}\right)\left[\sin\left(\tfrac{k_5\tilde{q}_2}{2}\right)\sin\left(\tfrac{k_3(\tilde{k}_5+\tilde{q}_2)}{2}\right) + (k_5\leftrightarrow k_3)\right]$$
$$+ [k_5+2q_1]_\kappa\,\delta_{\tau\epsilon}\sin\left(\tfrac{k_5\tilde{q}_1}{2}\right)\left[\sin\left(\tfrac{k_3\tilde{q}_2}{2}\right)\sin\left(\tfrac{k_4(\tilde{k}_3+\tilde{q}_2)}{2}\right) + (k_3\leftrightarrow k_4)\right] + (q_1\leftrightarrow q_2)\Bigg\},$$



$$= \widetilde{V}^{\bar{B}B4A}_{\mu\nu,\rho\sigma,\tau\epsilon\kappa\iota}(q_1,q_2,k_3,k_4,k_5,k_6) =$$



$$= -\widetilde{V}^{\bar{\psi}\psi 4A}_{\mu\nu,\rho\sigma,\tau\epsilon\kappa\iota}(q_1,q_2,k_{3-6})$$
$$= 2g^4\mu^2\theta^2(2\pi)^4\delta^4(q_1+q_2+k_3+k_4+k_5+k_6)\left(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}\right)$$

$$\times\Bigg\{2\delta_{\tau\epsilon}\delta_{\kappa\iota}\Big[\sin\big(\tfrac{k_4\tilde{q}_1}{2}\big)\sin\big(\tfrac{k_3(\tilde{k}_4+\tilde{q}_1)}{2}\big)\sin\big(\tfrac{k_6\tilde{q}_2}{2}\big)\sin\big(\tfrac{k_5(\tilde{k}_6+\tilde{q}_2)}{2}\big)+(k_3\leftrightarrow k_4)+(k_5\leftrightarrow k_6)\Big]$$
$$+\delta_{\tau\kappa}\delta_{\epsilon\iota}\Big[\sin\big(\tfrac{k_5\tilde{q}_1}{2}\big)\sin\big(\tfrac{k_3(\tilde{k}_5+\tilde{q}_1)}{2}\big)\sin\big(\tfrac{k_6\tilde{q}_2}{2}\big)\sin\big(\tfrac{k_4(\tilde{k}_6+\tilde{q}_2)}{2}\big)+(k_3\leftrightarrow k_5)+(k_4\leftrightarrow k_6)\Big]$$
$$+\delta_{\tau\iota}\delta_{\kappa\epsilon}\Big[\sin\big(\tfrac{k_6\tilde{q}_1}{2}\big)\sin\big(\tfrac{k_3(\tilde{k}_6+\tilde{q}_1)}{2}\big)\sin\big(\tfrac{k_4\tilde{q}_2}{2}\big)\sin\big(\tfrac{k_5(\tilde{k}_4+\tilde{q}_2)}{2}\big)+(k_3\leftrightarrow k_6)+(k_5\leftrightarrow k_4)\Big]$$
$$+(q_1\leftrightarrow q_2)\Bigg\}.$$

B Propagators of the model by Vilar et al.

The tree level action of [2] is given by:

$$\begin{aligned}
S &= S_0 + S_{\mathrm{break}} + S_{\mathrm{G}} + S_{\mathrm{gf}},\\
S_0 &= \int \mathrm{d}^4x \left[\frac{1}{4}F_{\mu\nu}\star F^{\mu\nu} + \bar{\chi}_{\mu\nu}\star D^2 B^{\mu\nu} + \bar{B}_{\mu\nu}\star D^2\chi^{\mu\nu} + \gamma^2\bar{\chi}_{\mu\nu}\star\chi^{\mu\nu}\right],\\
S_{\mathrm{break}} &= \int \mathrm{d}^4x \left[-\mathrm{i}\frac{\gamma}{2}B_{\mu\nu}\star F^{\mu\nu} + \mathrm{i}\frac{\gamma}{2}\bar{B}_{\mu\nu}\star F^{\mu\nu}\right],\\
S_{\mathrm{G}} &= \int \mathrm{d}^4x \left[-\bar{\psi}_{\mu\nu}\star D^2\star\xi^{\mu\nu} - \bar{\xi}_{\mu\nu}\star D^2\psi^{\mu\nu} - \gamma^2\bar{\psi}_{\mu\nu}\star\psi^{\mu\nu}\right],\\
S_{\mathrm{gf}} &= \int \mathrm{d}^4x \left[\mathrm{i}b\star\partial^\mu A_\mu + \bar{c}\star\partial^\mu D_\mu c\right],
\end{aligned}\tag{B.1}$$

where the complex conjugated pairs $(B_{\mu\nu},\, \bar{B}_{\mu\nu})$, $(\chi_{\mu\nu},\, \bar{\chi}_{\mu\nu})$ are bosonic auxiliary fields of mass dimension 1, and $(\psi_{\mu\nu},\, \bar{\psi}_{\mu\nu})$, $(\xi_{\mu\nu},\, \bar{\xi}_{\mu\nu})$ are their associated ghost fields. From the bilinear parts of this action one derives the following 19 propagators:

$$G^{A}_{\mu\nu}(k) = \frac{-1}{\left(k^2+\frac{\gamma^4}{k^2}\right)}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right),\tag{B.2a}$$

$$G^{BA}_{\rho,\sigma\tau}(k) = \frac{-\gamma^3}{\left(k^2+\frac{\gamma^4}{k^2}\right)}\frac{(k_\sigma\delta_{\rho\tau} - k_\tau\delta_{\rho\sigma})}{2(k^2)^2},\tag{B.2b}$$

$$G^{\bar{B}A}_{\rho,\sigma\tau}(k) = -G^{BA}_{\rho,\sigma\tau}(k),\tag{B.2c}$$

$$G^{\chi A}_{\rho,\sigma\tau}(k) = \frac{\mathrm{i}\gamma}{\left(k^2+\frac{\gamma^4}{k^2}\right)}\frac{(k_\sigma\delta_{\rho\tau} - k_\tau\delta_{\rho\sigma})}{2k^2},\tag{B.2d}$$

$$G^{\bar{\chi}A}_{\rho,\sigma\tau}(k) = -G^{\chi A}_{\rho,\sigma\tau}(k),\tag{B.2e}$$

$$G^{\bar{B}\bar{B}}_{\rho\sigma,\tau\epsilon}(k) = \frac{\gamma^4}{(k^2)^2}\frac{(k_\rho k_\tau\delta_{\sigma\epsilon} + k_\sigma k_\epsilon\delta_{\rho\tau} - k_\rho k_\epsilon\delta_{\sigma\tau} - k_\sigma k_\tau\delta_{\rho\epsilon})}{4(k^2)^2\left(k^2+\frac{\gamma^4}{k^2}\right)},\tag{B.2f}$$

$$G^{BB}_{\rho\sigma,\tau\epsilon}(k) = G^{\bar{B}\bar{B}}_{\rho\sigma,\tau\epsilon}(k),\tag{B.2g}$$

$$G^{B\bar{B}}_{\rho\sigma,\tau\epsilon}(k) = \gamma^2\frac{(\delta_{\rho\tau}\delta_{\sigma\epsilon} - \delta_{\rho\epsilon}\delta_{\sigma\tau})}{2(k^2)^2} - G^{\bar{B}\bar{B}}_{\rho\sigma,\tau\epsilon}(k),\tag{B.2h}$$

$$G^{\bar{\chi}\bar{\chi}}_{\rho\sigma,\tau\epsilon}(k) = -\gamma^2\frac{(k_\rho k_\tau\delta_{\sigma\epsilon} + k_\sigma k_\epsilon\delta_{\rho\tau} - k_\rho k_\epsilon\delta_{\sigma\tau} - k_\sigma k_\tau\delta_{\rho\epsilon})}{4(k^2)^2\left(k^2+\frac{\gamma^4}{k^2}\right)},\tag{B.2i}$$

$$G^{\chi\chi}_{\rho\sigma,\tau\epsilon}(k) = G^{\bar{\chi}\bar{\chi}}_{\rho\sigma,\tau\epsilon}(k),\tag{B.2j}$$

$$G^{\chi\bar{\chi}}_{\rho\sigma,\tau\epsilon}(k) = -G^{\bar{\chi}\bar{\chi}}_{\rho\sigma,\tau\epsilon}(k),\tag{B.2k}$$

$$G^{\chi B}_{\rho\sigma,\tau\epsilon}(k) = \frac{\gamma^4}{k^2}\frac{(k_\rho k_\tau\delta_{\sigma\epsilon} + k_\sigma k_\epsilon\delta_{\rho\tau} - k_\rho k_\epsilon\delta_{\sigma\tau} - k_\sigma k_\tau\delta_{\rho\epsilon})}{4(k^2)^2\left(k^2+\frac{\gamma^4}{k^2}\right)}, \tag{B.2l}$$

$$G^{\bar\chi\bar B}_{\rho\sigma,\tau\epsilon}(k) = G^{\chi B}_{\rho\sigma,\tau\epsilon}(k), \tag{B.2m}$$

$$G^{\chi\bar B}_{\rho\sigma,\tau\epsilon}(k) = \frac{(\delta_{\rho\tau}\delta_{\sigma\epsilon} - \delta_{\rho\epsilon}\delta_{\sigma\tau})}{2k^2} - G^{\chi B}_{\rho\sigma,\tau\epsilon}(k), \tag{B.2n}$$

$$G^{\bar\chi B}_{\rho\sigma,\tau\epsilon}(k) = G^{\chi\bar B}_{\rho\sigma,\tau\epsilon}(k), \tag{B.2o}$$

$$G^{\bar c c}(k) = -\frac{1}{k^2}, \tag{B.2p}$$

$$G^{\xi,\bar\psi}_{\mu\nu,\rho\sigma}(k) = \frac{(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})}{2k^2}, \tag{B.2q}$$

$$G^{\bar\xi,\psi}_{\mu\nu,\rho\sigma}(k) = -G^{\xi,\bar\psi}_{\mu\nu,\rho\sigma}, \tag{B.2r}$$

$$G^{\bar\xi\xi}_{\mu\nu,\rho\sigma}(k) = -\gamma^2\frac{(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})}{2(k^2)^2}. \tag{B.2s}$$

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