Group analysis of a class of fifth-order KdV-like equations

O. V. Kuriksha[1], S. Pošta[2], O. O. Vaneeva[3]

[1]Petro Mohyla Black Sea State University, Mykolaiv, Ukraine

[2]Czech Technical University in Prague, Prague, Czech Republic

[3]Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine

oksana.kuriksha@gmail.com, severin@km1.fjfi.cvut.cz, vaneeva@imath.kiev.ua

The classical Korteweg–de Vries (KdV) equation and its generalizations model various physical systems, including gravity waves, plasma waves and waves in lattices [1]. The simplest fifth-order KdV equation, $u_t + uu_x + \mu u_{xxxxx} = 0$ with $\mu = \mathrm{const}$, arises in the modeling of one-dimensional plane waves in cold quasi-neutral collision-free plasma propagating along the x-direction under the presence of a uniform magnetic field [2]. A number of generalizations of fifth-order KdV equations appeared later and were studied from different points of view. The study of Lie symmetries of differential equations is important task since such symmetries allow one not only to reduce a model partial differential equation to a partial differential equation with fewer number of independent variables or to an ordinary differential equation but also to derive cases that are potentially more interesting for applications [3].

We perform Lie symmetry classification of the variable coefficient generalized fifth-order KdV equations

$$u_t + u^n u_x + \alpha(t)u + \beta(t)u_{xxxxx} = 0, \quad \beta \neq 0. \tag{1}$$

Here n is an arbitrary nonzero integer, α and β are smooth functions of the variable t. The cases $n = 1$ and $n \neq 1$ are considered separately in [4] and [5], respectively. Firstly we study admissible point transformations in class (1). Criteria of reducibility of variable-coefficient equations (1) to constant coefficient equations from the same class are obtained. It appears that the Lie symmetry algebra of an equation from class (1) is of maximal dimension (which is equal to four) if $n = 1$ and either the equation is constant coefficient one or it is point-equivalent to equation from the same class with constant coefficients. All equations which possess two- and three-dimensional Lie symmetry algebras are also derived.

Then we perform all inequivalent Lie reductions of the equations (1) which possess Lie symmetry extensions to ordinary differential equations and give some examples on the construction of exact solutions.

1. Jeffrey A., Kakutani T. Weak nonlinear dispersive waves: a discussion centered around the Korteweg–de Vries equation. SIAM Review, 1972, **14**, No 4, P. 582–643.
2. Kakutani T., Ono H. Weak non-linear hydromagnetic waves in a cold collision-free plasma. J. Phys. Soc. Japan., 1969, **26**, No 5, P. 1305–1318.
3. Fushchich W. I., Nikitin A. G. Symmetries of Equations of Quantum Mechanics. — N.Y.: Allerton Press Inc., 1994, 480 p.
4. Kuriksha O., Pošta S., Vaneeva O. Group analysis of variable coefficient generalized fifth-order KdV equations. Physics of Particles and Nuclei Letters, 2014, **11**, No 7, P. 990–995.
5. Kuriksha O., Pošta S., Vaneeva O. Group analysis of generalized fifth-order Korteweg–de Vries equations with time-dependent coefficients. In V. Dobrev (ed.) Lie Theory and Its Application in Physics. Springer Proc. Math. Stat., **111**. — Tokyo: Springer, 2014, P. 311–321.