Analytically-oriented approaches to nonlinear sloshing in moving smooth tanks

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Overview

- Motivation: coupling with rigid body dynamics
- Physical modelling and basic free boundary problems
- DFG Projects by talkers on “sloshing”
- Multimodal method
- Experience from 2D flow
- Generalization to 3D flow
- Perspectives

Industry Motivations

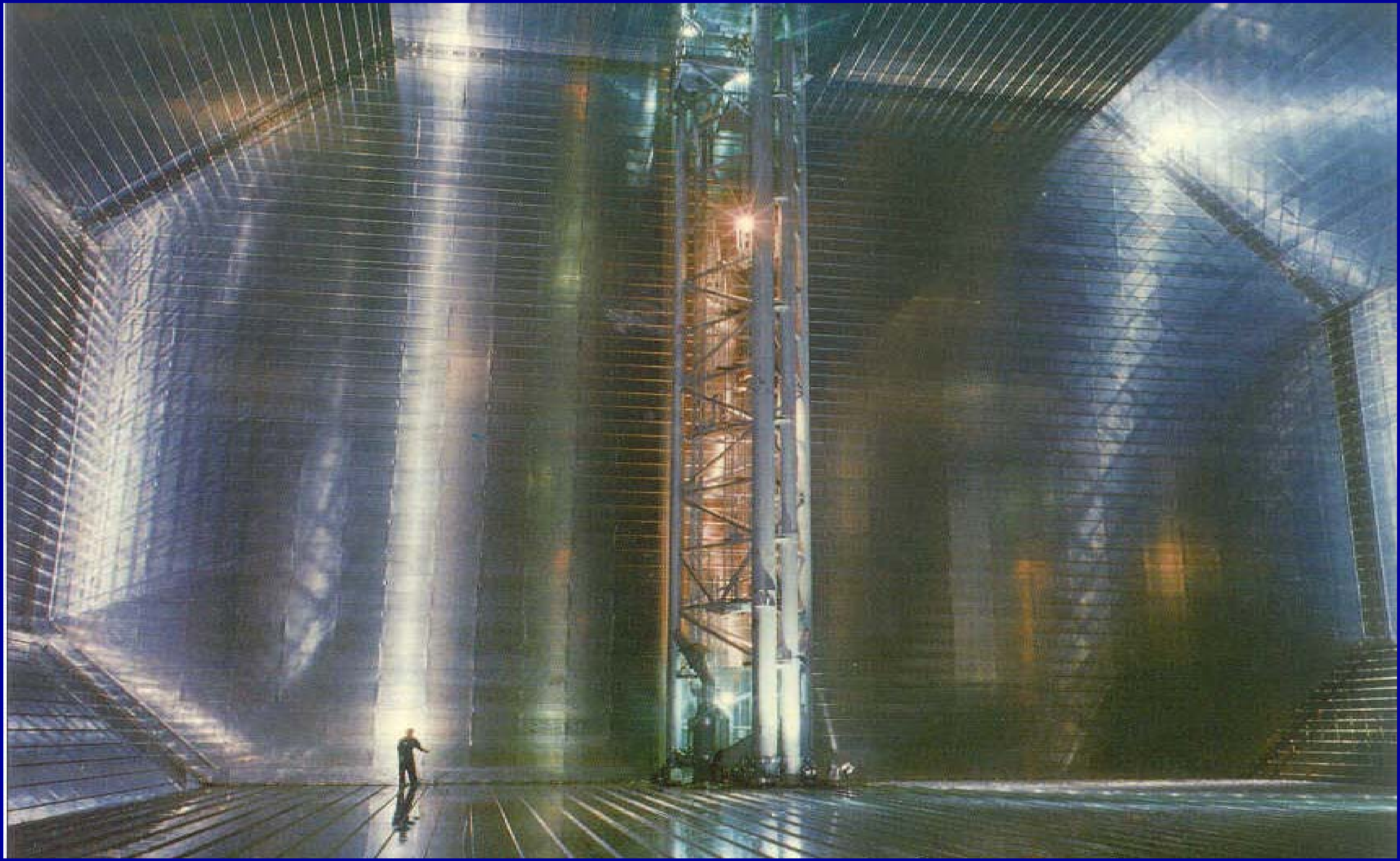
- Spacecraft, beginning from 50-60th
- Road tracks
- (PST) Petroleum Storage Tanks
- Oil Ship Tankers
- (LNG) Liquefied Natural Gas Carriers (smooth tanks, no baffles)
- (TLD, TSD) Tuned Liquid (Sloshing) Damper (smooth tanks, active device)

Unbaffled nonlinear fluid sloshing, coupled motions

Moss type
LNG Carriers

Membrane type
LNG Carriers

Gas Transport Containment (aft end)

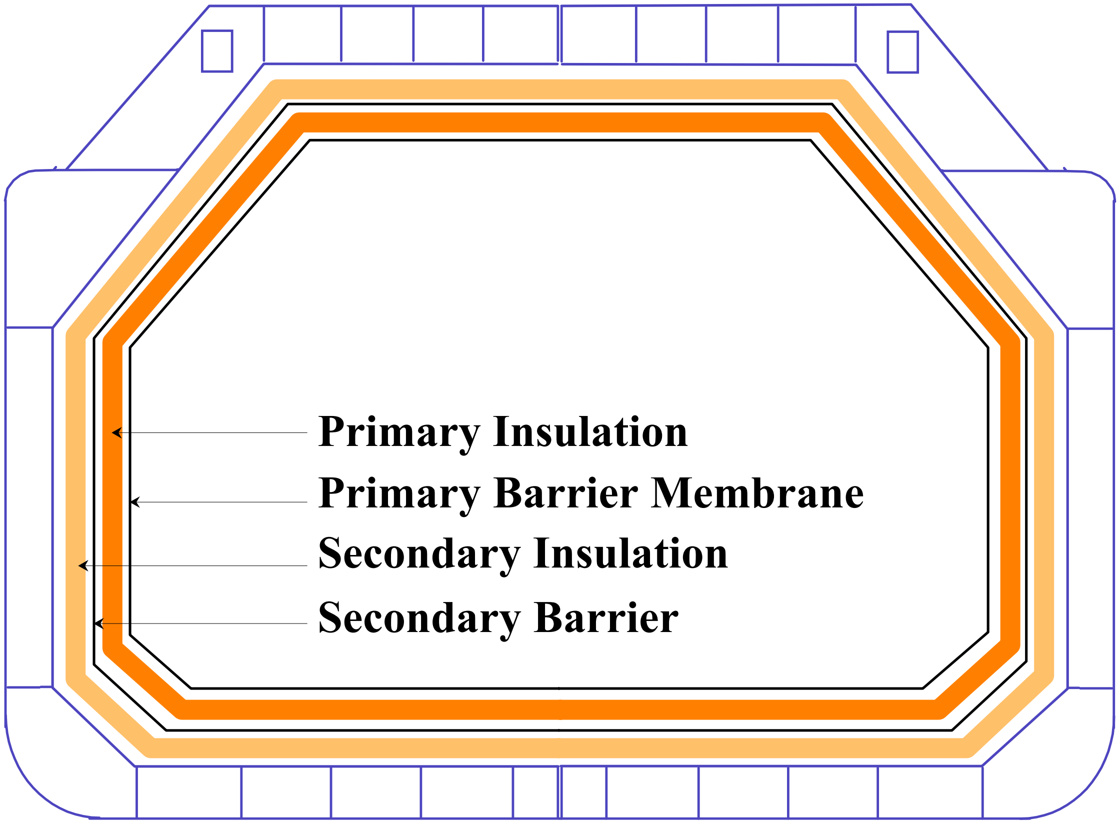


LNG Tank — by Containment System

Gas Transport (GT) No. 96

Technigaz (TGZ) Mark III CS1

Membrane



Tuned Liquid Dampers

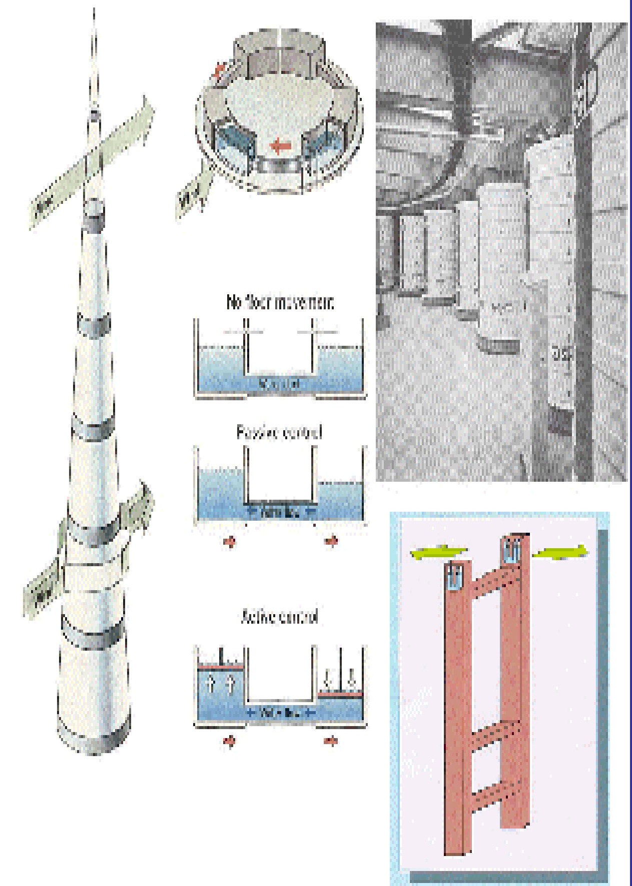
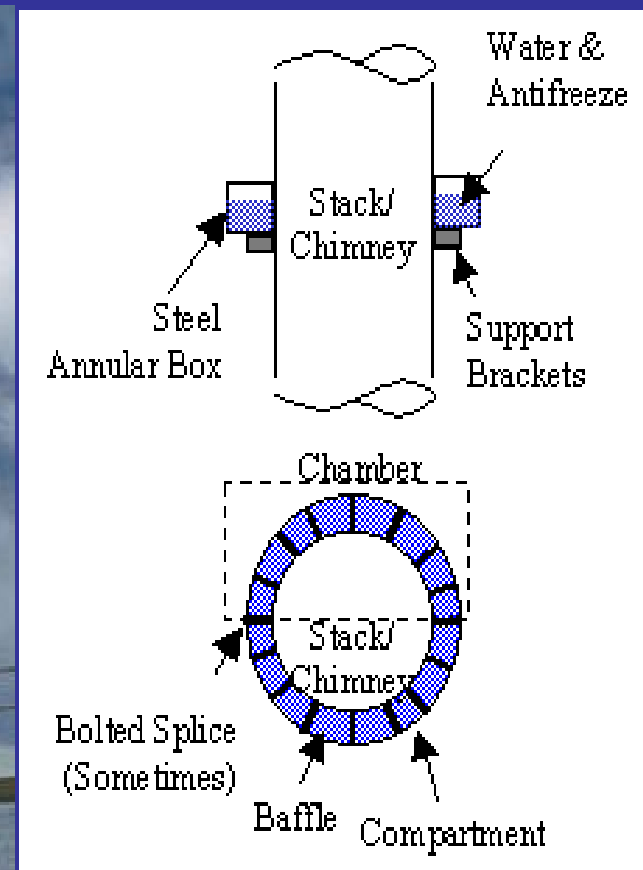


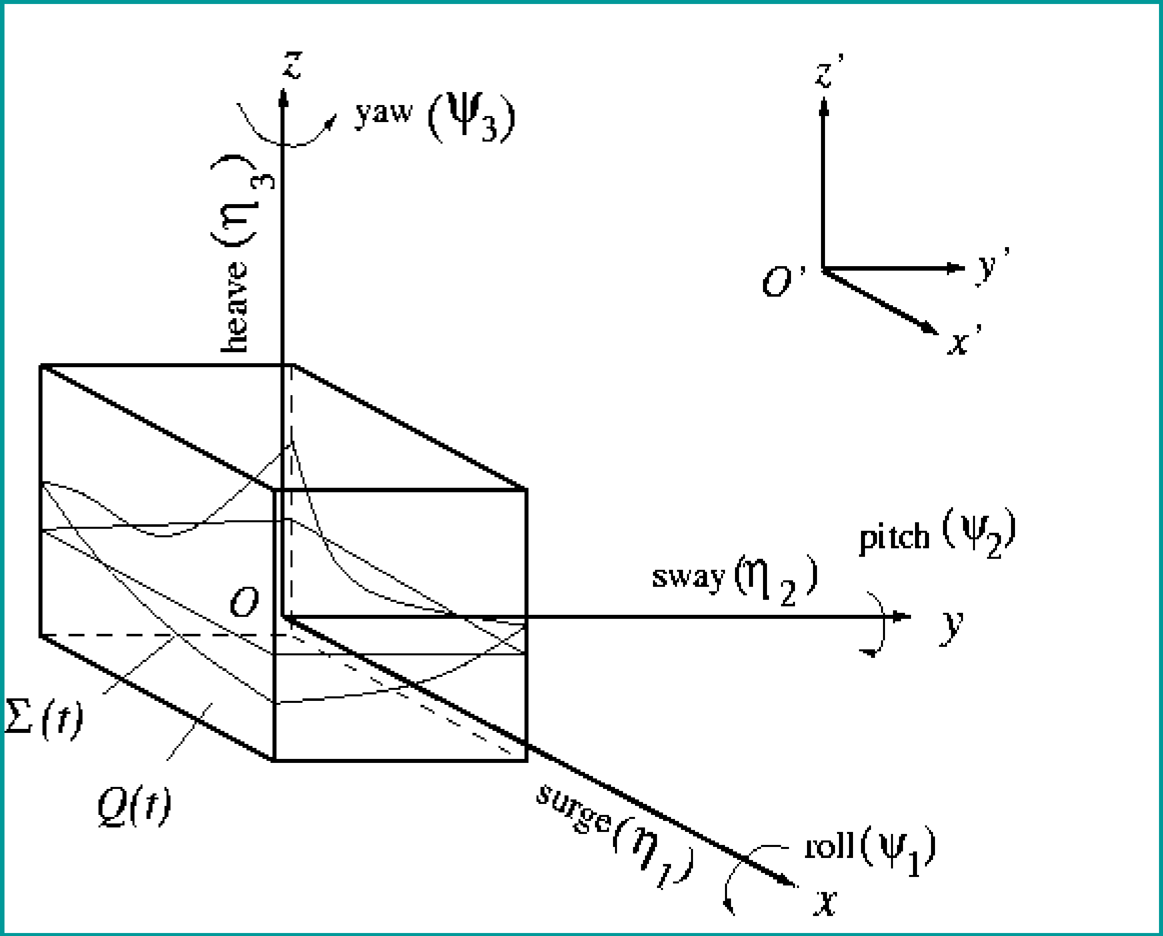
Fig 5: (a) Millennium Tower (b) TLDs installation (c) TLCDs in bridges



Physical effects in modelling nonlinear sloshing

	Parameters	Importance	Comments
1	Froude number	Yes	
2	Reynolds number (Viscosity)	No	Second order for large amplitude sloshing
3	Liquid compressibility	No	
4	Surface tension	No	
5	Ullage Space compressibility	Not clear	Further investigation
6	Gas cushioning / cavitation	Not clear	Further investigation
7	Ullage vapour condensation	N/A	No exp. should use water/ saturated-water-vapour at room temp.
8	Wall flexibility (hydroelasticity)	Yes ?	Further investigation

Fluid sloshing in a moving tank



Free boundary problem

A mobile rigid tank partly filled by an inviscid incompressible fluid is considered. The flow is irrotational. The fluid volume bounded by the free surface $\Sigma(t)$ and the wetted tank surface $S(t)$ is denoted $Q(t)$. Let $O'x'y'z'$ be an absolute coordinate system and $Oxyz$ be a moving coordinate system fixed with respect to the rigid tank. The origin of $Oxyz$ is in the unperturbed free surface and moves with the velocity $\boldsymbol{v}_0$ relative to $O'x'y'z'$. The tank has an angular velocity $\boldsymbol{\omega}$ relative to $O'x'y'z'$. The gravity field has the potential

$$U(x, y, z, t) = -\boldsymbol{g} \cdot \boldsymbol{r}', \qquad \boldsymbol{r}' = \boldsymbol{r}'_0 + \boldsymbol{r},$$

where $\boldsymbol{r}'$ is the radius-vector of a point of the body–fluid system with respect to O', $\boldsymbol{r}'_0$ is the radius-vector of the point O with respect to O', $\boldsymbol{r}$ is the radius-vector with respect to O and $\boldsymbol{g}$ is the gravity acceleration vector.

Since the flow is irrotational, the fluid velocity can be represented as $\boldsymbol{v}_a = \nabla\Phi$, where $\boldsymbol{v}_a$ is the fluid velocity vector at the point (x, y, z) in the moving coordinate system and $\Phi(x, y, z, t)$ is the velocity potential. The velocity potential and the free surface $\Sigma(t)$ can be found from the following nonlinear free boundary problem:

$$\left.\begin{aligned}
&\Delta\Phi = 0 \quad \text{in } Q(t), \qquad \frac{\partial\Phi}{\partial\boldsymbol{\nu}} = \boldsymbol{v}_0 \cdot \boldsymbol{\nu} + \boldsymbol{\omega} \cdot [\boldsymbol{r} \times \boldsymbol{\nu}] \quad \text{on } S(t),\\
&\frac{\partial\Phi}{\partial\boldsymbol{\nu}} = \boldsymbol{v}_0 \cdot \boldsymbol{\nu} + \boldsymbol{\omega} \cdot [\boldsymbol{r} \times \boldsymbol{\nu}] + \frac{\xi_t}{|\nabla\xi|} \quad \text{on } \Sigma(t),\\
&\frac{\partial\Phi}{\partial t} + \tfrac{1}{2}(\nabla\Phi)^2 - \nabla\Phi \cdot (\boldsymbol{v}_0 + \boldsymbol{\omega} \times \boldsymbol{r}) + U = 0 \quad \text{on } \Sigma(t), \qquad \int_{Q(t)} dQ = \text{const.}
\end{aligned}\right\}$$

Here $\boldsymbol{\nu}$ is the outer normal to the boundary of $Q(t)$ and $\xi(x, y, z, t) = 0$ is the equation of the free surface $\Sigma(t)$. The last integral condition in (2.2) implies fluid volume conservation and is also the well-known solvability condition for the Neumann boundary value problem.

“Sloshing” in DFG projects:

- “Combined numerical-analytical methods for solving non-classical evolutional and spectral problems arising in fluid sloshing analysis” (1996-1999)
- “Analytische, numerische und geometrische Methoden bei der Evolution freier Flüssigkeitsränder” (2000-2002)
- “Numerisch-analytische Methoden für oszillierende Grenzfläschen in Mikrogravitation und Hyperelastisitaet” (2003-2005)

involving Institute of Mathematics (Kiev, Ukraine)
Leipzig Universität & Berufsakademie Thüringen

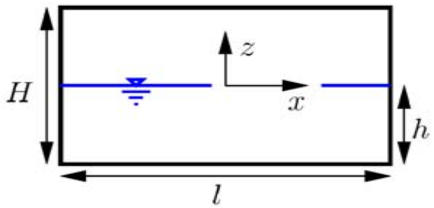
Tendency of these projects:

- from classical “sloshing” problems to more complicated mathematical statements;
- concentration on mathematical aspects;
- minor attention to mechanical conclusions and physical phenomena.

Multimodal methods as analytically-oriented approach to sloshing and coupling

- Potential flow, no overturning waves (statement described)
- Vertical wall and no roof impact
- Free surface elevation as a Fourier series by natural modes (example for 2D flow)

$$\zeta = \sum_{i=1}^{N} \beta_i(t) \cos\left(\frac{\pi i(x+0.5l)}{l}\right)$$



β_i represent free surface modes

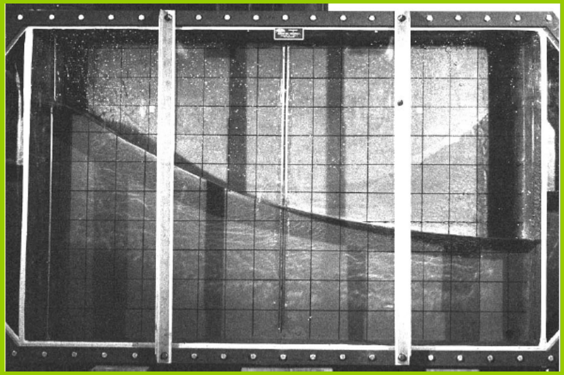
- Variational formulation to derive a discrete model, which
- Leads to system of nonlinear ordinary differential equations in time for β_i

Importance of the tank shape

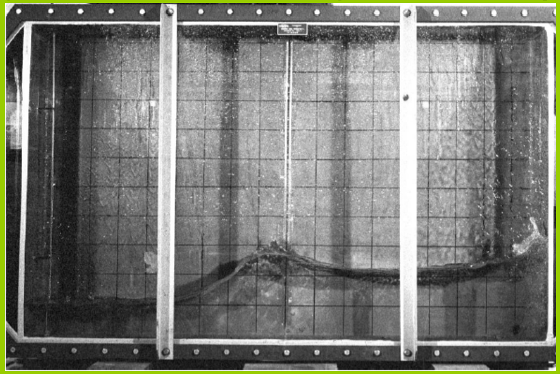
- Natural modes to be known analytically or numerically-analytically
- Distribution of the energy between lower and higher modes to be known (dispersion relationship)
- Spacecraft applications/TLD – tanks of revolution (vertical circular cylinder, conical, spherical tanks)
- Ship tanks - prismatic or spherical geometry

Importance of filling level

- Finite water depth sloshing (filling height/tank length>0.24)
  - Resembles standing wave



- Shallow water sloshing (filling height/tank length<0.1)
  - Hydraulic jump / bore
  - Thin vertical jet – run-up



- Intermediate depth

General modal system by Miles-Lukovsky

$$\frac{\mathrm{d}}{\mathrm{d}t}A_n - \sum_k R_k A_{nk} = 0, \quad n = 1, 2, \ldots,$$

$$\begin{aligned}
&\sum_n \dot{R}_n \frac{\partial A_n}{\partial \beta_i} + \frac{1}{2}\sum_n \sum_k \frac{\partial A_{nk}}{\partial \beta_i} R_n R_k + \dot{\omega}_1 \frac{\partial l_{1\omega}}{\partial \beta_i} + \dot{\omega}_2 \frac{\partial l_{2\omega}}{\partial \beta_i} + \dot{\omega}_3 \frac{\partial l_{3\omega}}{\partial \beta_i} + \omega_1 \frac{\partial l_{1\omega t}}{\partial \beta_i} + \omega_2 \frac{\partial l_{2\omega t}}{\partial \beta_i} \\
&\quad + \omega_3 \frac{\partial l_{3\omega t}}{\partial \beta_i} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\omega_1 \frac{\partial l_{1\omega t}}{\partial \dot{\beta}_i} + \omega_2 \frac{\partial l_{2\omega t}}{\partial \dot{\beta}_i} + \omega_3 \frac{\partial l_{3\omega t}}{\partial \dot{\beta}_i}\right) + (\dot{v}_{01} - g_1 + \omega_2 v_{03} - \omega_3 v_{02}) \frac{\partial l_1}{\partial \beta_i} \\
&\quad + (\dot{v}_{02} - g_2 + \omega_3 v_{01} - \omega_1 v_{03}) \frac{\partial l_2}{\partial \beta_i} + (\dot{v}_{03} - g_3 + \omega_1 v_{02} - \omega_2 v_{01}) \frac{\partial l_3}{\partial \beta_i} - \frac{1}{2}\omega_1^2 \frac{\partial J_{11}^1}{\partial \beta_i} \\
&\quad - \frac{1}{2}\omega_2^2 \frac{\partial J_{22}^1}{\partial \beta_i} - \frac{1}{2}\omega_3^2 \frac{\partial J_{33}^1}{\partial \beta_i} - \omega_1 \omega_2 \frac{\partial J_{12}^1}{\partial \beta_i} - \omega_1 \omega_3 \frac{\partial J_{13}^1}{\partial \beta_i} - \omega_2 \omega_3 \frac{\partial J_{23}^1}{\partial \beta_i} = 0.
\end{aligned}$$

May be reduced to ODE in the modal functions β_i, finite-dimensional when ordering modal functions

Hydrodynamic Forces

force calculations are based on the formula derived by Lukovsky (1990):

$$\boldsymbol{F} = m\boldsymbol{g} - m[\dot{\boldsymbol{v}}_0 + \boldsymbol{\omega} \times \boldsymbol{v}_0 + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_C) + \dot{\boldsymbol{\omega}} \times \boldsymbol{r}_C + 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_C + \ddot{\boldsymbol{r}}_C].$$

Here m is the fluid mass, $\boldsymbol{r}_C$ is the radius-vector of the centre of mass in the moving coordinate system and $m\boldsymbol{g}$ is the fluid weight. The terms in square brackets are: $\dot{\boldsymbol{v}}_0$ the acceleration of the origin O, $\boldsymbol{\omega} \times \boldsymbol{v}_0$ the tangential acceleration, $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_C)$ the centripetal acceleration, $2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_C$ Coriolis acceleration, $\ddot{\boldsymbol{r}}_C$ the relative acceleration.

and Moments

The hydrodynamic moment relative to axis Oy can also be calculated by

$$\boldsymbol{M}_0 = m\boldsymbol{r}_C \times (\boldsymbol{g} - \boldsymbol{\omega} \times \boldsymbol{v}_0 - \dot{\boldsymbol{v}}_0) - \boldsymbol{J}^1 \cdot \dot{\boldsymbol{\omega}} - \dot{\boldsymbol{J}}^1 \cdot \boldsymbol{\omega} - \boldsymbol{\omega} \times (\boldsymbol{J}^1 \cdot \boldsymbol{\omega}) - \ddot{\boldsymbol{l}}_\omega + \dot{\boldsymbol{l}}_{\omega t} - \boldsymbol{\omega} \times (\dot{\boldsymbol{l}}_\omega - \boldsymbol{l}_{\omega t}),$$

where $\boldsymbol{J}^1$ is the inertia tensor

This makes hydrodynamic forces/moments by function of generalised coordinates and provides a efficient coupling (dynamic system for the coupled object)

Multimodal models for 2D sloshing

- The ordering reflects a response of lower order than the forced oscillation amplitude ε
- Finite depth: $\beta_1=O(\varepsilon^{1/3})$, $\beta_2=O(\varepsilon^{2/3})$, $\beta_3=O(\varepsilon)$
- Increased importance of secondary (internal) resonance with decreasing depth and increasing excitation implies more than one dominant mode
- Boussinesq ordering for intermediate and shallow depth
- System truncation based on assumed order of different modes

Extensive experimental validation

- Transient and steady-state phase of 2D flow in rectangular or close to rectangular tank
- Wave elevation, horizontal force and roll moment
- Fluid depth-tank length ratios h/L between 0.08 and 0.5
- Include shallow depth,intermediate depth and critical depth h/L=0.34
- Sway amplitude-tank length ratios up to 0.1
- Roll angles up to 0.2 rad

Finite depth ordering

- Forcing frequency is in the vicinity of lowest natural frequency and lowest mode is assumed dominant
- Excitation amplitude-tank length ratio is $O(\varepsilon)$
- Generalized free surface coordinate $\beta_1 = O(\varepsilon^{1/3})$
- $\beta_2 = O(\varepsilon^{2/3})$
- $\beta_3 = O(\varepsilon)$

Differential equations for finite depth

$$(\ddot{\beta}_1 + \sigma_1^2\beta_1) + d_1(\ddot{\beta}_1\beta_2 + \dot{\beta}_1\dot{\beta}_2) + d_2(\ddot{\beta}_1\beta_1^2 + \dot{\beta}_1^2\beta_1) + d_3\ddot{\beta}_2\beta_1 + P_1(\dot{v}_{0x} - S_1\dot{\omega} - g\psi) + Q_1\dot{v}_{0z}\beta_1 = 0,$$

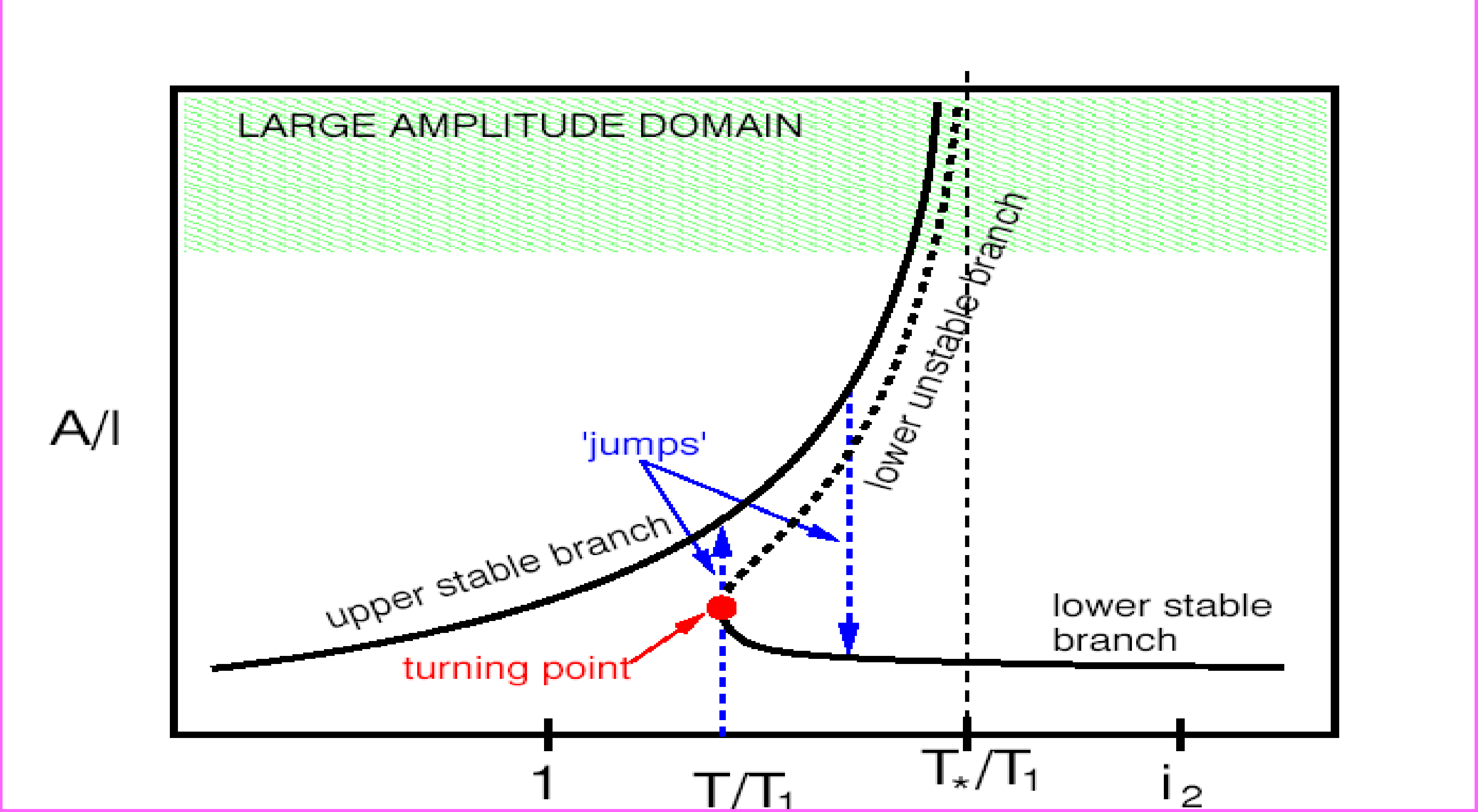
$$(\ddot{\beta}_2 + \sigma_2^2\beta_2) + d_4\ddot{\beta}_1\beta_1 + d_5\dot{\beta}_1^2 + Q_2\dot{v}_{0z}\beta_2 = 0,$$

$$(\ddot{\beta}_3 + \sigma_3^2\beta_3) + d_6\ddot{\beta}_1\beta_2 + d_7\ddot{\beta}_1\beta_1^2 + d_8\ddot{\beta}_2\beta_1 + d_9\dot{\beta}_1\dot{\beta}_2 + d_{10}\dot{\beta}_1^2\beta_1 + P_3(\dot{v}_{0x} - S_3\dot{\omega} - g\psi) + Q_3\dot{v}_{0z}\beta_3 = 0.$$

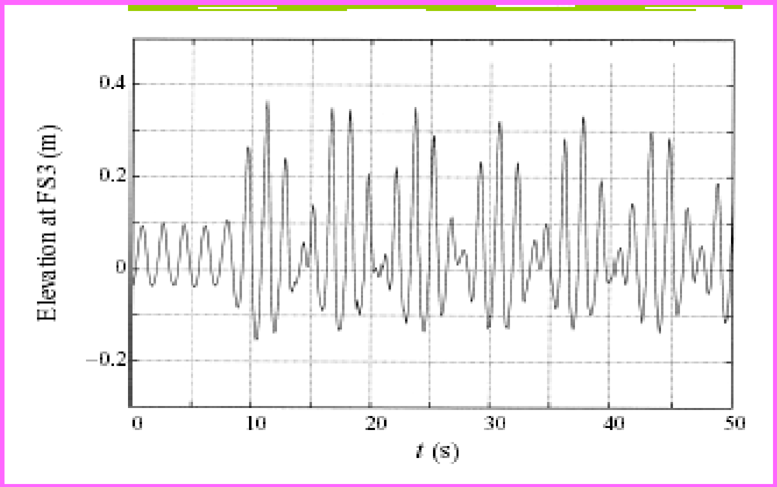
$$\ddot{\beta}_i + \sigma_i^2\beta_i + P_i(\dot{v}_{0x} - S_i\dot{\omega} - g\psi) + Q_i\dot{v}_{0z}\beta_i = 0, \quad i \geqslant 4.$$

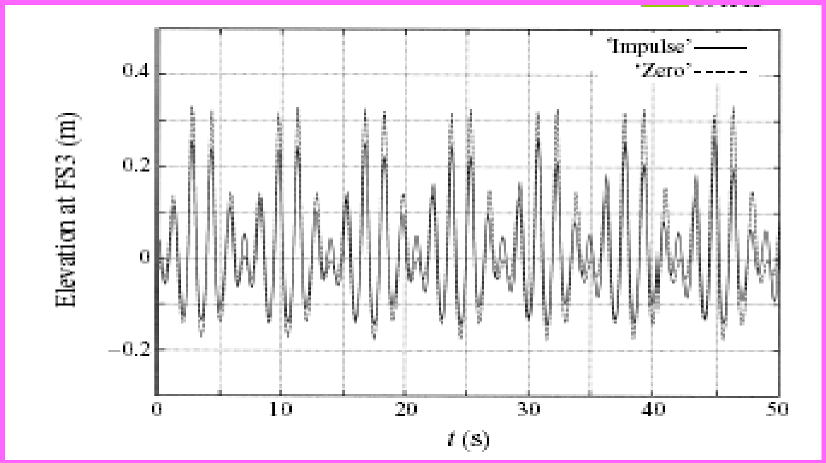
Forces and moments are functions of β_i

Steady-state response for 2D flow with finite depth



Transient response





Comparison of experimental data (elevation near the wall) and the theory for resonant sway forcing

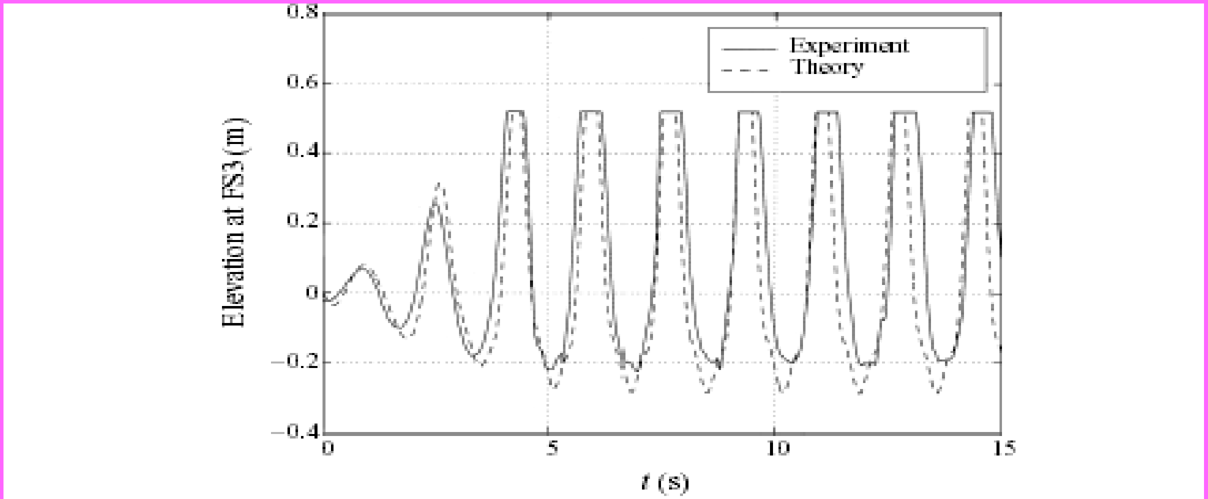
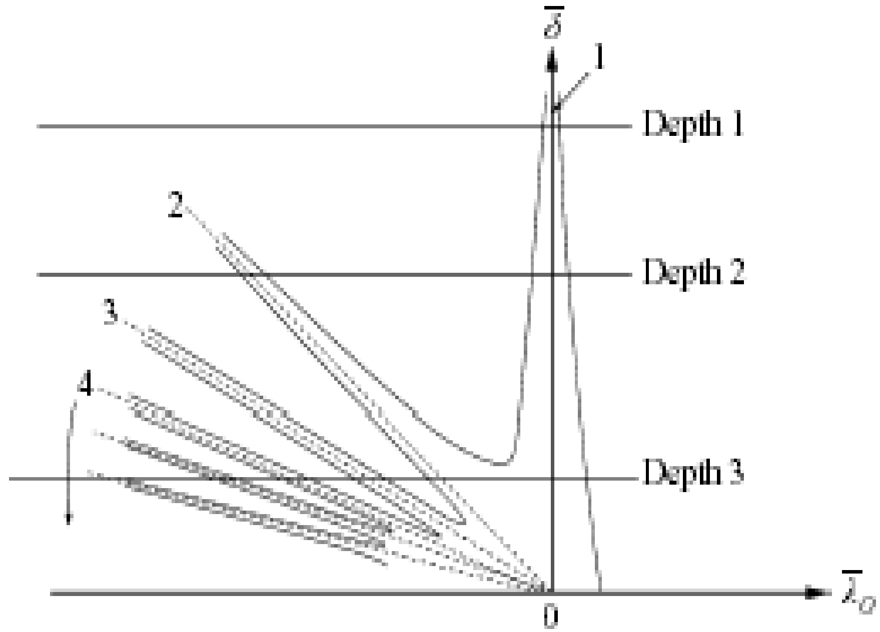


FIGURE 11. Measured and calculated free surface elevation at wave probe FS3 for $T = 1.71$ s, $h = 0.5$ m and $H = 0.05$ m. Calculations account for wave impact on tank ceiling.

Secondary resonance and nonlinear modal systems

- Nonlinear multiple frequency effects excite higher natural frequencies
- Increased importance with decreasing depth and increasing forcing amplitude
- Reason for decreasing depth importance is that $\sigma_n = n\sigma_1$ when depth goes to zero
- Implies that more then one mode is dominant

Secondary resonance and fluid depth



Possible secondary resonances in the detuning/depth $(\bar{\lambda}_O, \bar{\delta})$ plane. Here, the numbers of the 'fingers' correspond to various amplification modes: 1 (near the line $\bar{\lambda}_O = 0$ implies the primary resonance, 2 (near the line $\bar{\delta} = -\bar{\lambda}_O$) corresponds to the secondary resonance of the second mode, 3 (near the line $\bar{\delta} = -\frac{3}{8}\bar{\lambda}_O$) corresponds to the secondary resonance of the third mode and so on.

Different ordering of modes due to secondary resonance for large amplitude forcing with finite depth

Forcing frequency slightly smaller of the lowest natural frequency

- Model I

$$\beta_i = \mathcal{O}\left(\epsilon^{\frac{1}{3}}\right), \quad i = 1, 3$$

$$\beta_i = \mathcal{O}\left(\epsilon^{\frac{2}{3}}\right), \quad i = 2, 6$$

$$\beta_i = \mathcal{O}\left(\epsilon\right), \quad i = 4, 5, 7 - 9$$

Forcing frequency in vicinity of the lowest natural frequency

- Model II

$$\beta_i = \mathcal{O}\left(\epsilon^{\frac{1}{3}}\right), \quad i = 1, 2$$

$$\beta_i = \mathcal{O}\left(\epsilon\right), \quad i = 3 - 6$$

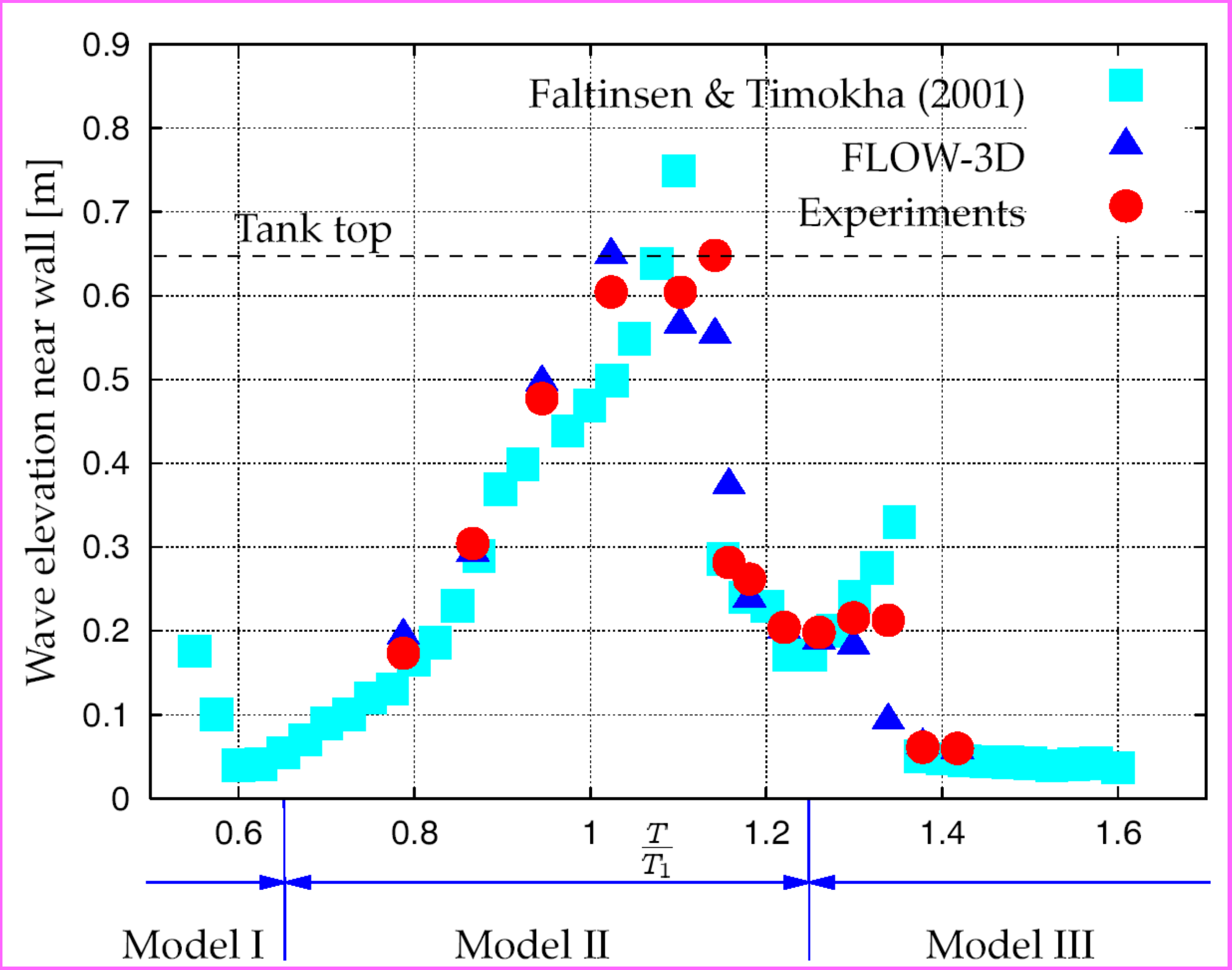
Forcing frequency slightly larger of the lowest natural frequency

- Model III

$$\beta_i = \mathcal{O}\left(\epsilon^{\frac{1}{3}}\right), \quad i = 1, 2, 3$$

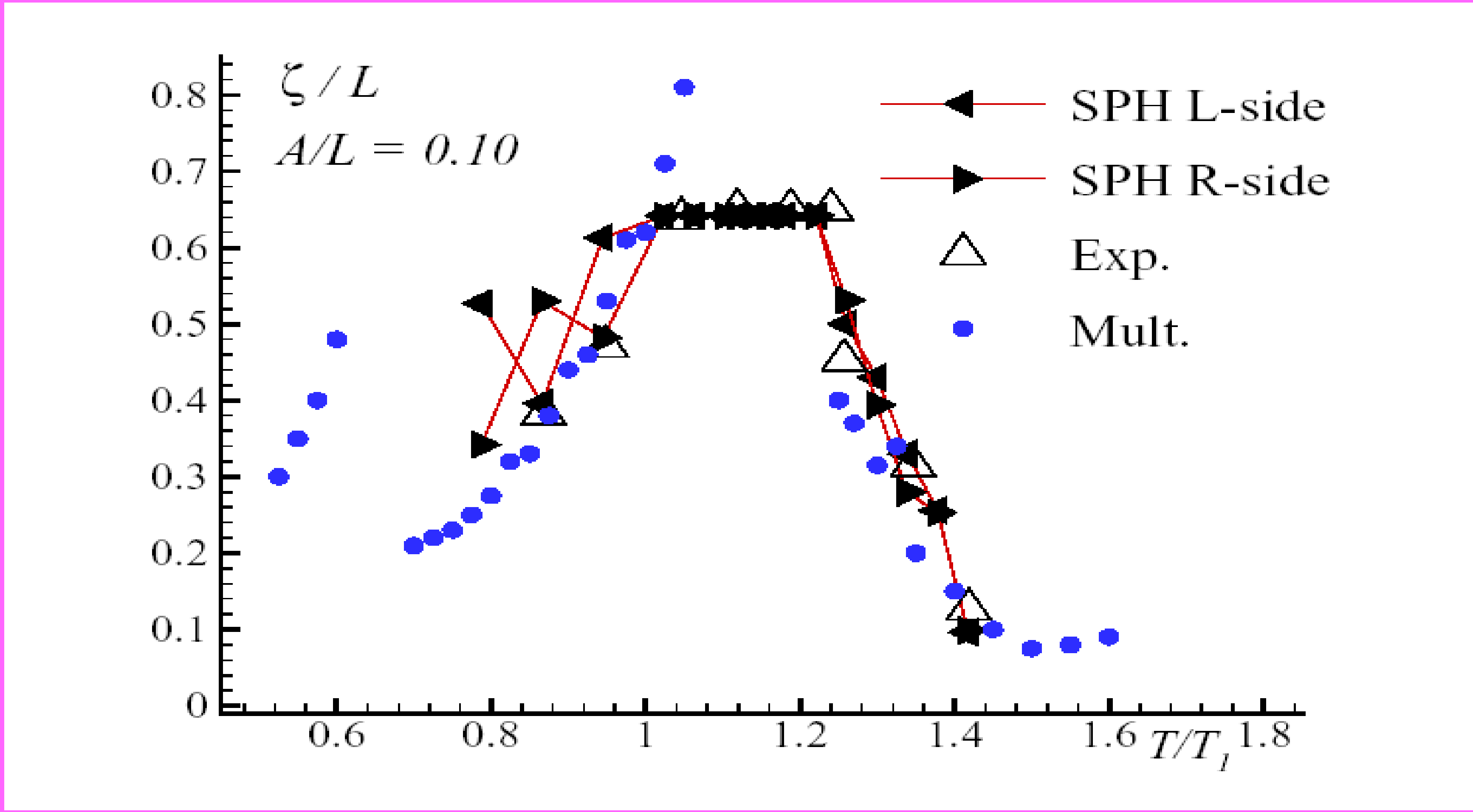
$$\beta_i = \mathcal{O}\left(\epsilon\right), \quad i = 4 - 5$$

Comparison with experiments



Rectangular tank, 1x1m. Small-ampliude forced sway motion with period T. h/l=0.35

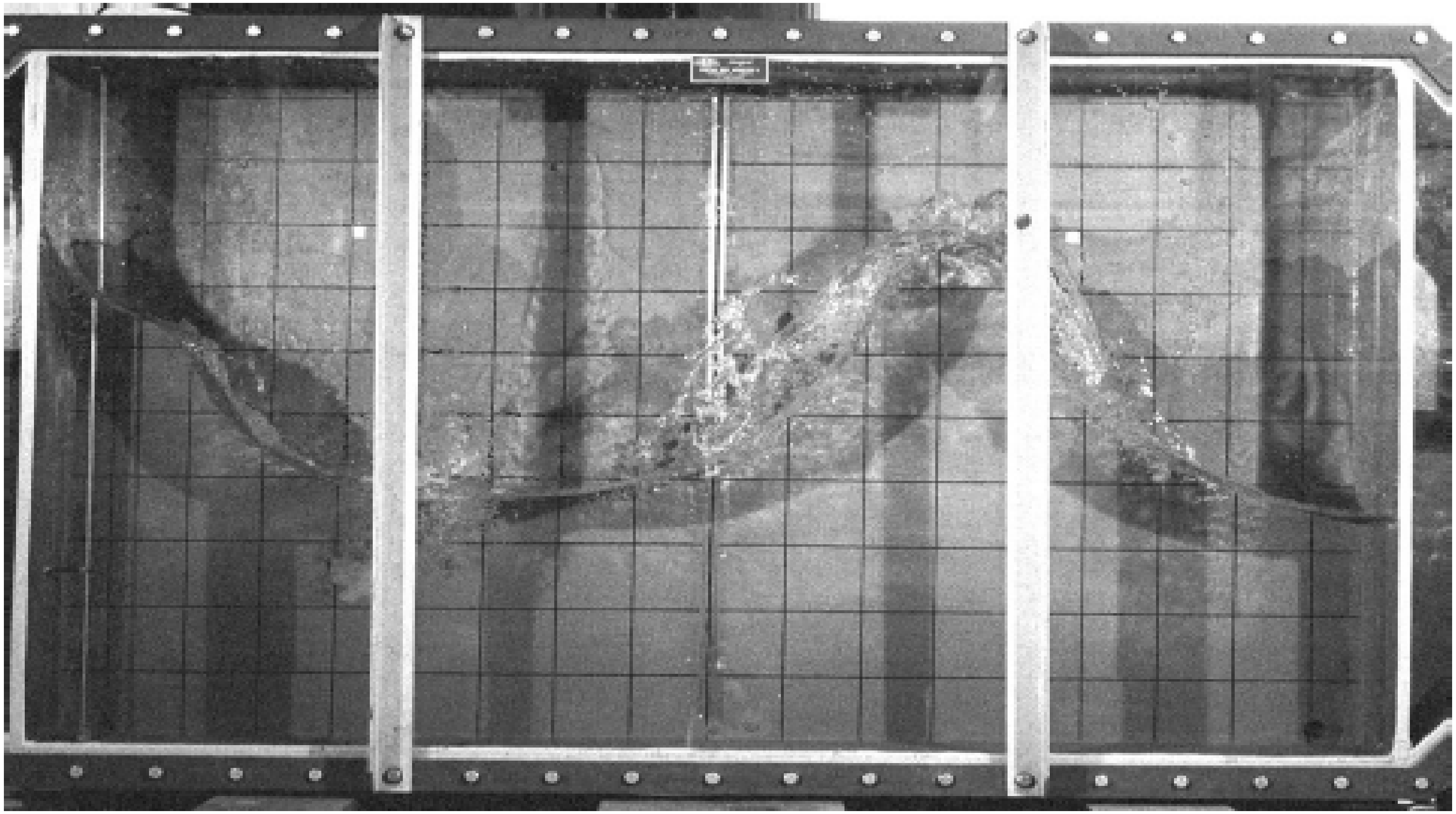
Comparison with experiments and CFD calculations (Smoothed Particles, Flow3D fails) for large-amplitude forcing; fluid depth/tank length=0.35



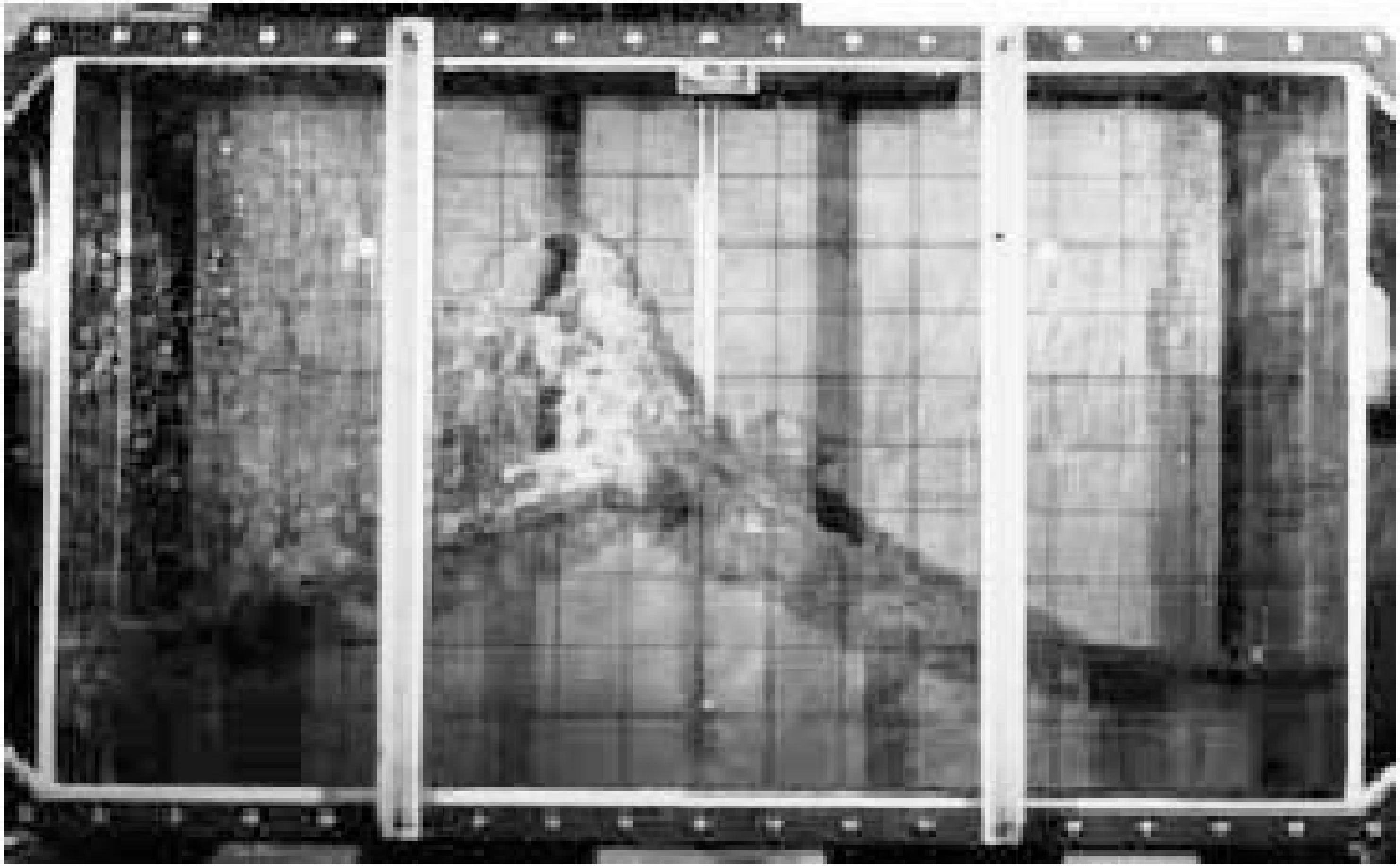
Intermediate-to-shallow depth ordering

- Boussinesq type ordering:
- All generalized coordinates β_i**=O(**$\varepsilon^{1/4}$)
- Fluid depth –tank length ratio **h/L=O(**$\varepsilon^{1/4}$**)**
- Also applied to shallow depth
- Many modes needed when depth goes to zero

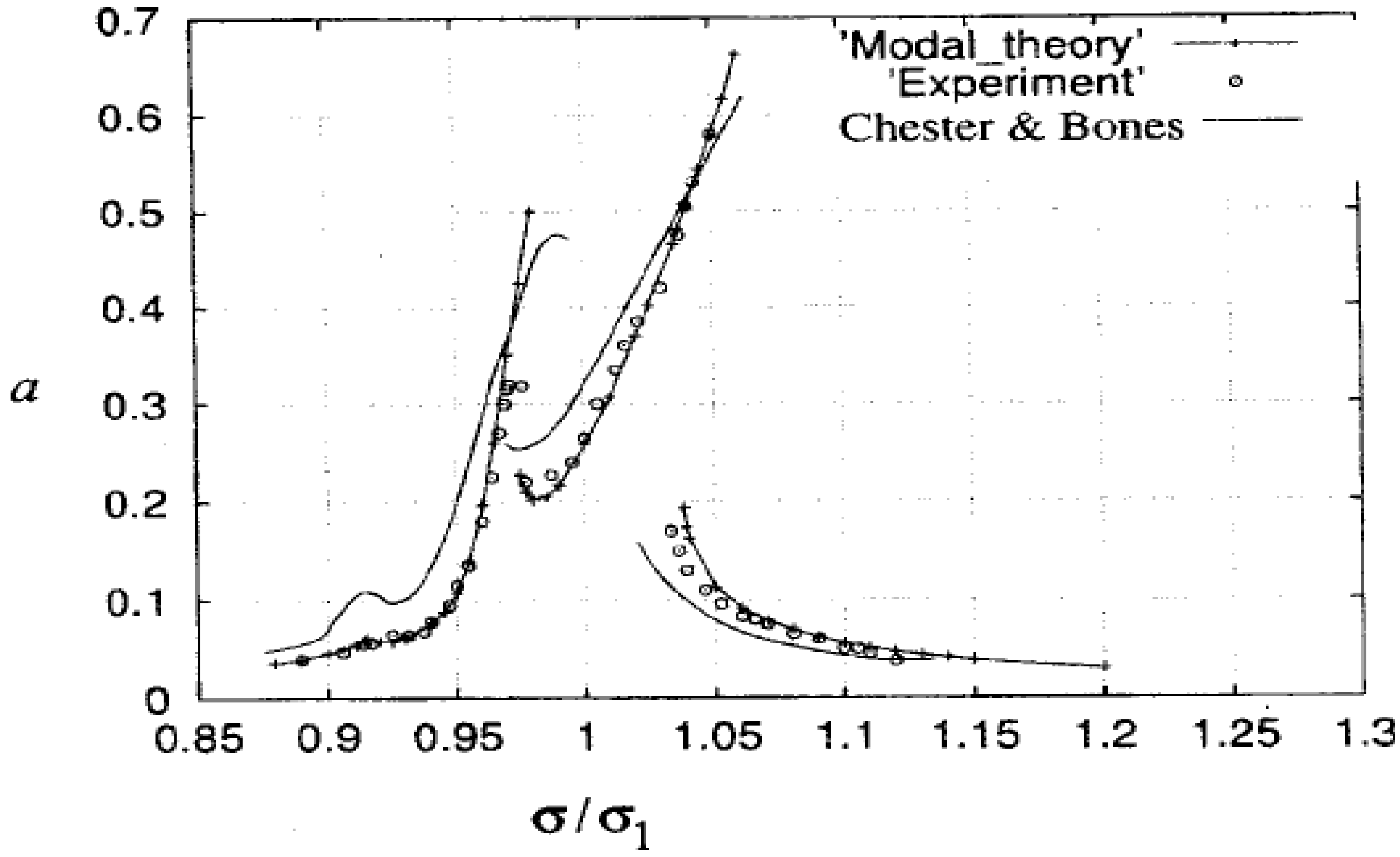
Wave profiles for intermediate depth



Wave profiles for nearly-shallow depth. $A/L=0.028, T/T_1=1.09, h/L=0.173$



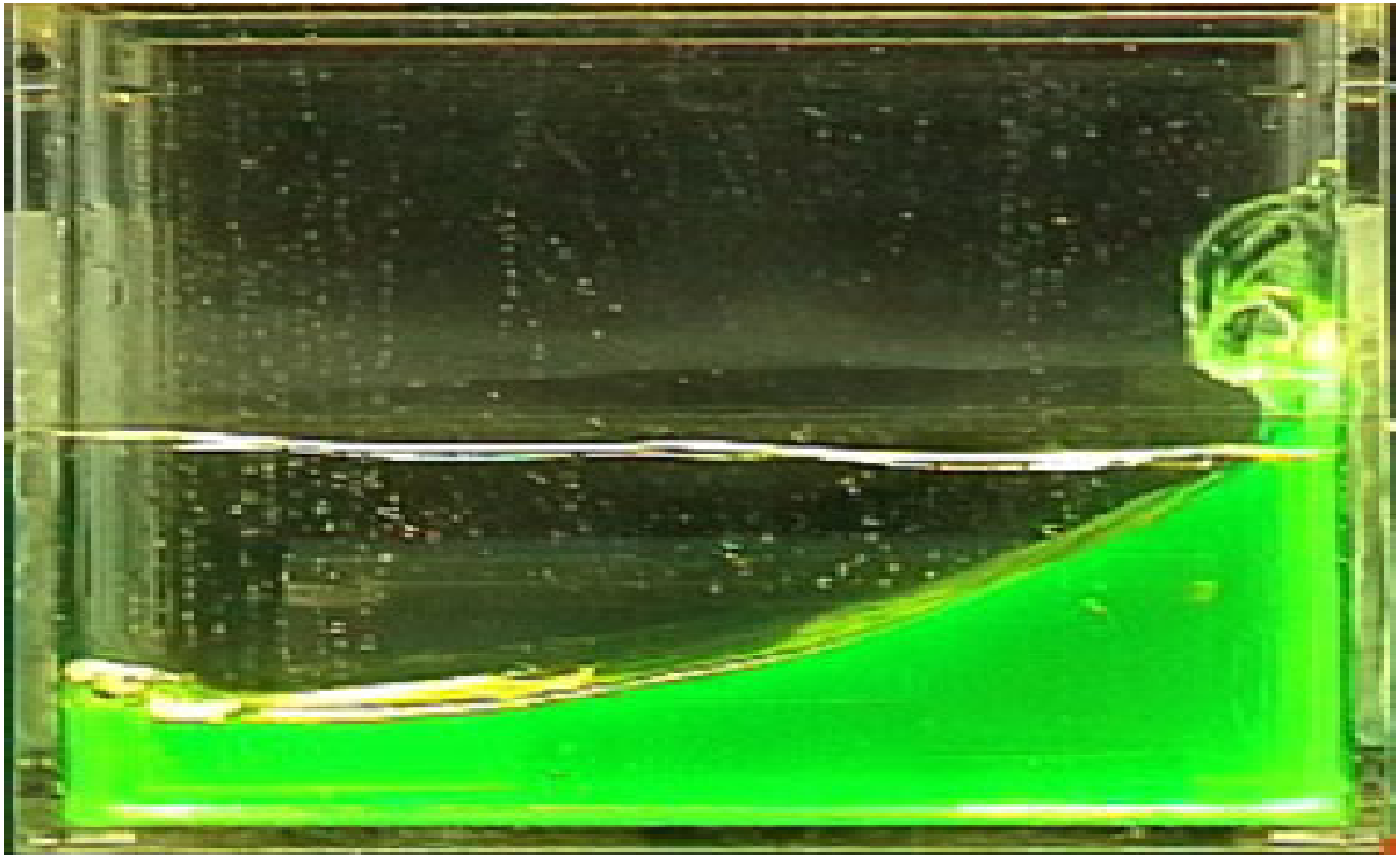
Comparison with experiments for shallow sloshing



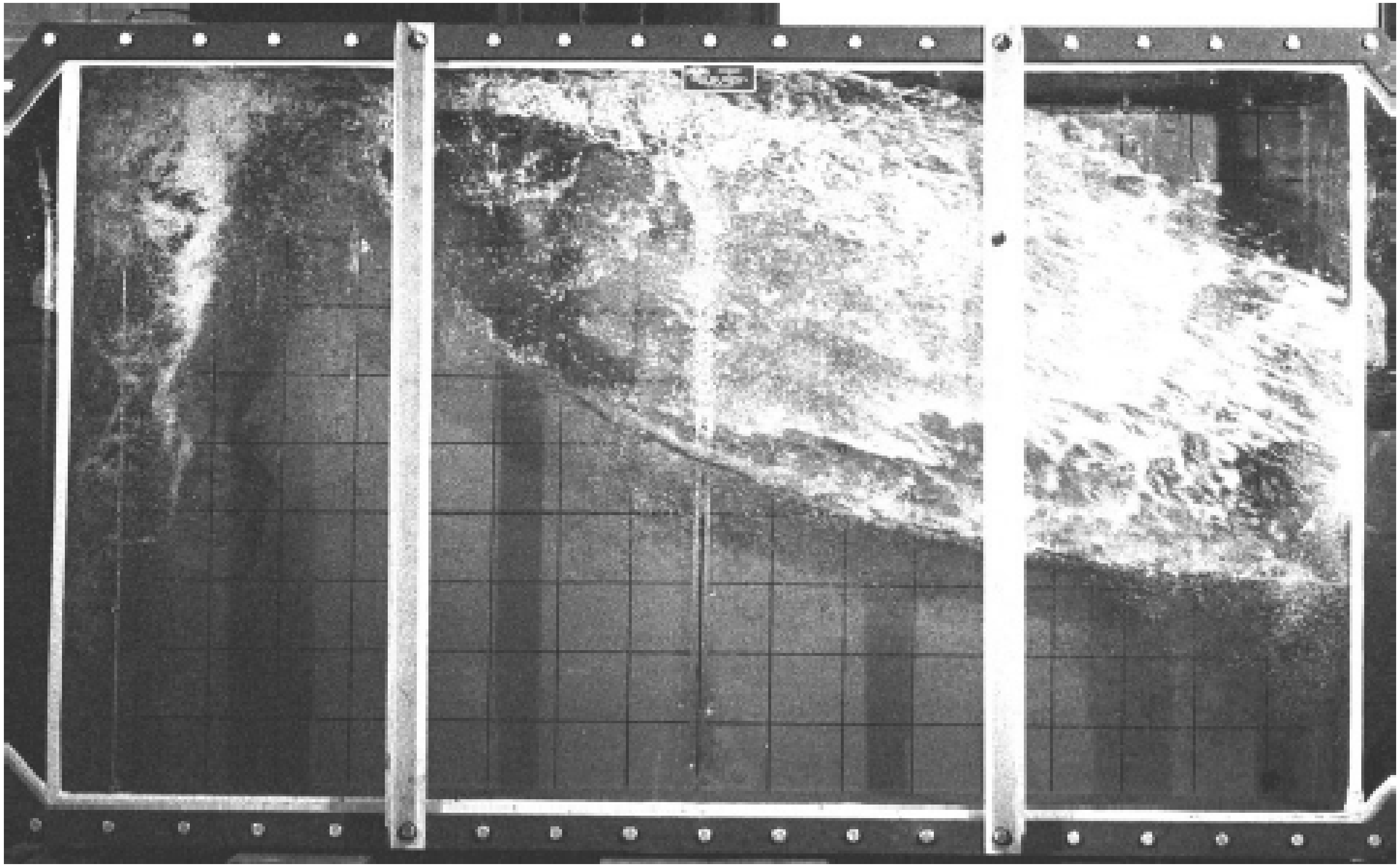
Dissipation

- Increased importance with decreasing depth
- Very long transient effects for finite depth and a smooth tank with no roof impact
- Short transient period for intermediate and shallow depth
- Due to run up along walls and subsequent overturning of free surface and impact on underlying fluid
- Due to breaking waves in the middle of the tank
- Viscous boundary layer effects are not important

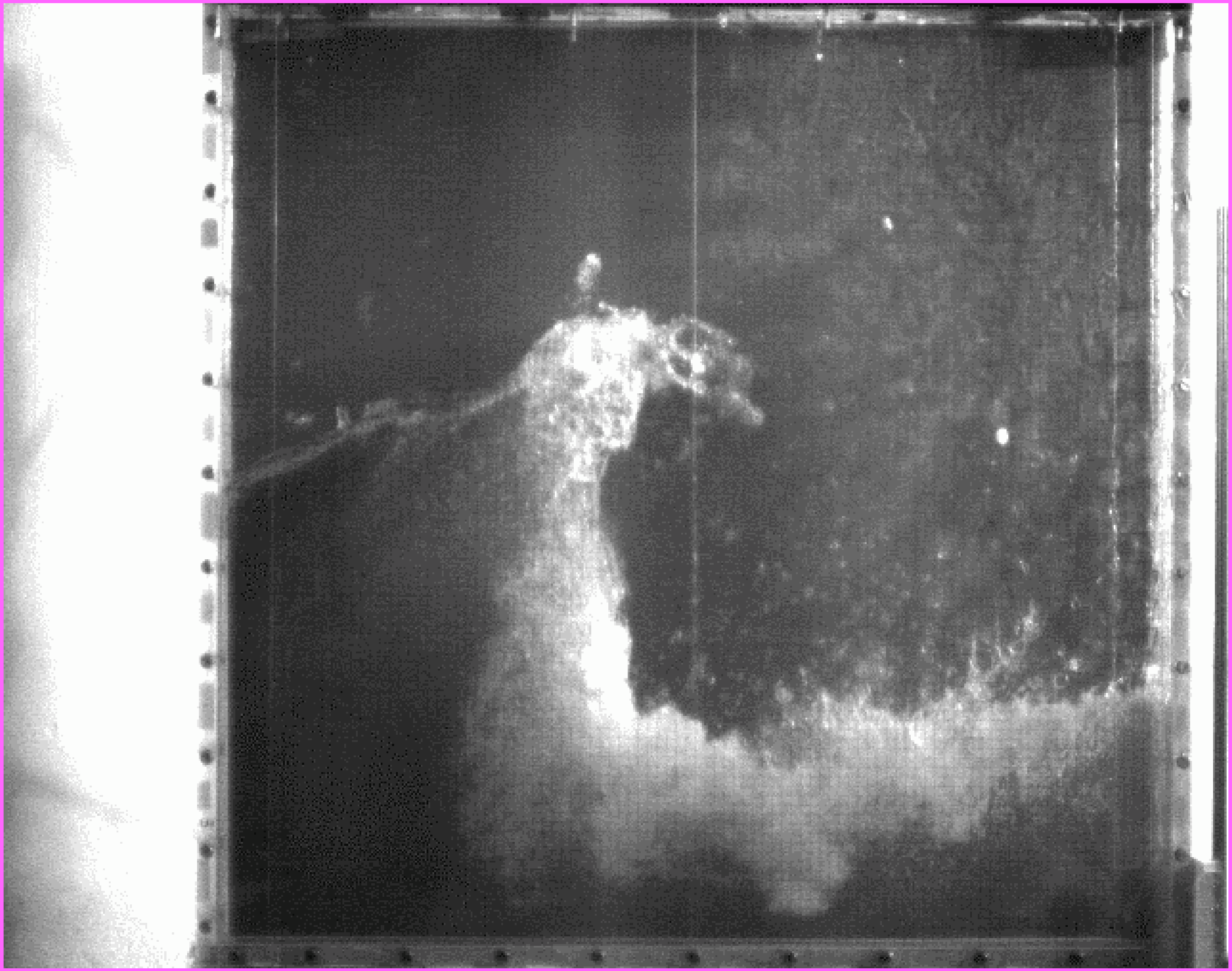
Dissipation: importance of near-wall phenomena



Dissipation: importance of roof impact



Dissipation: importance of breaking waves



Dissipative phenomena by modal modelling

- Linear damping terms in each modal system (pendulum analogy) $\ddot{\beta}_i + 2\gamma_i\dot{\beta}_i + \sigma_i^2\beta_i = 0 \quad (i \geqslant 1).$
- Damping terms amount viscous etc. dissipation
- For steady-state sloshing – constant values accounting for energy less per period

$$\gamma_i = \gamma_i^{tank'surface} + \gamma_i^{bulk} + \gamma_i^{others}.$$

$$\gamma_i^{bulk} = \frac{4}{3}\nu\frac{\varkappa_i^4(h^*)^2}{1+(h^*)^2\frac{2}{3}\varkappa_i^2},$$

$$\gamma_i^{tanksurface} = \frac{1}{2}\sqrt{\frac{\nu\sigma_i}{2}}\frac{1}{B^*}\left[1+\tfrac{1}{2}B^* + B^*\frac{1-h^*}{2h^*(1+\frac{2}{3}(h^*)^2\varkappa_i^2)}\right]$$

Extension of multimodal method to 3D sloshing in square base tank

- Possible steady-state wave motions are
  - planar (Stokes standing) waves (2D)
  - square-like waves
  - swirling

- No stable steady wave motions may exist

Free surface elevation for square based tank with finite depth

- $\zeta = \sum\sum \beta_{ik}(t) \cos(\pi i(x+0.5L))\cos(\pi k(y+0.5L))$

- $\beta_{10} = O(\varepsilon^{1/3})$ $\beta_{01} = O(\varepsilon^{1/3})$

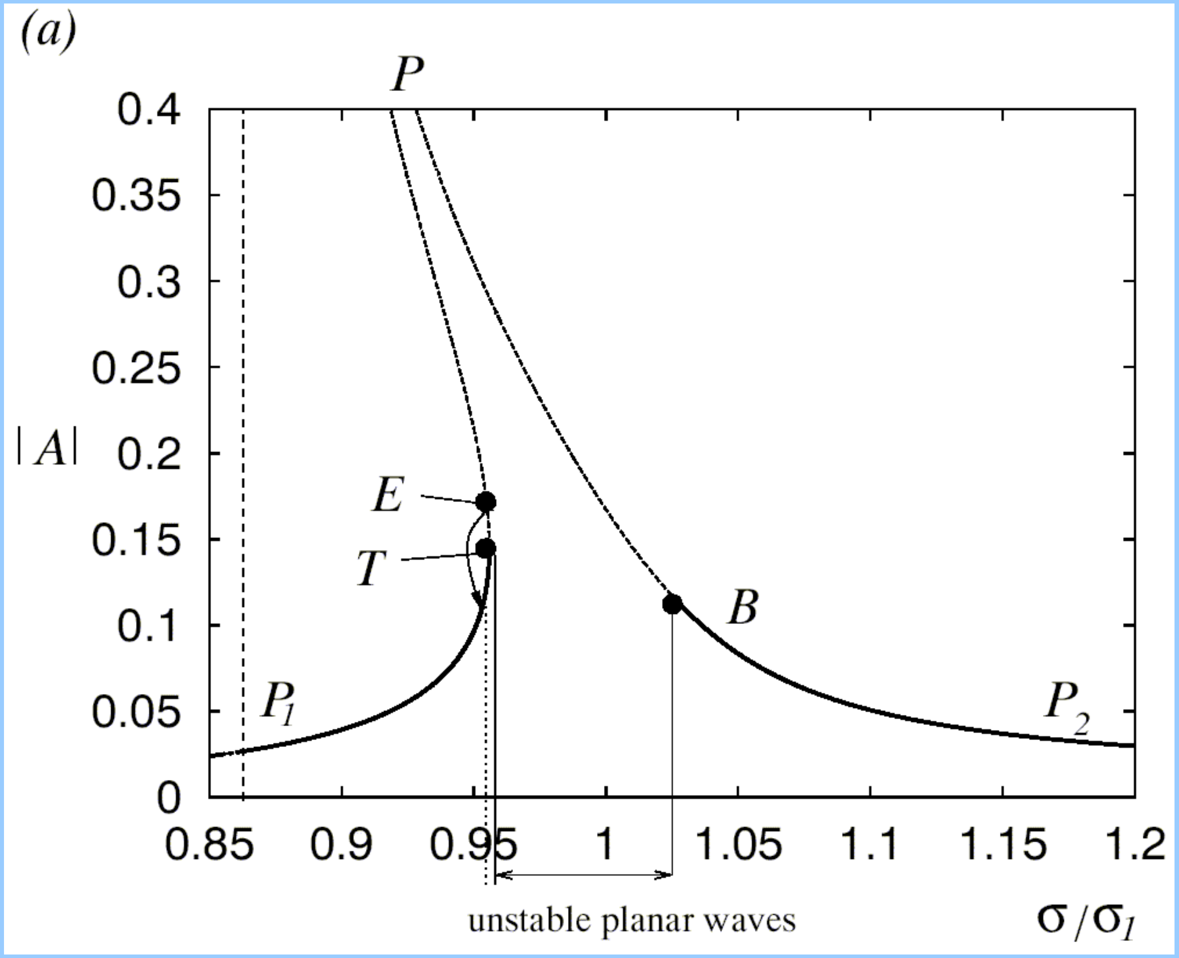
- $\beta_{20} = O(\varepsilon^{2/3})$ $\beta_{11} = O(\varepsilon^{2/3})$ $\beta_{02} = O(\varepsilon^{2/3})$

- $\beta_{30} = O(\varepsilon)$ $\beta_{21} = O(\varepsilon)$ $\beta_{12} = O(\varepsilon)$ $\beta_{03} = O(\varepsilon)$

Longitudinal harmonic excitation of square-based tank (experiments made by NTNU, Norway)

- Fluid depth h=0.508L (in experiments)
- Excitation amplitude 0.0078L (in experiments)
- Steady-state wave response
- Stable and unstable theoretical results

Planar wave amplitude A as a function of excitation frequency σ



’Square’- like (diagonal) steady state wave elevation

$\zeta = [$**A(cos(π(x+0.5L))- cos(π(y+0.5L)))+**

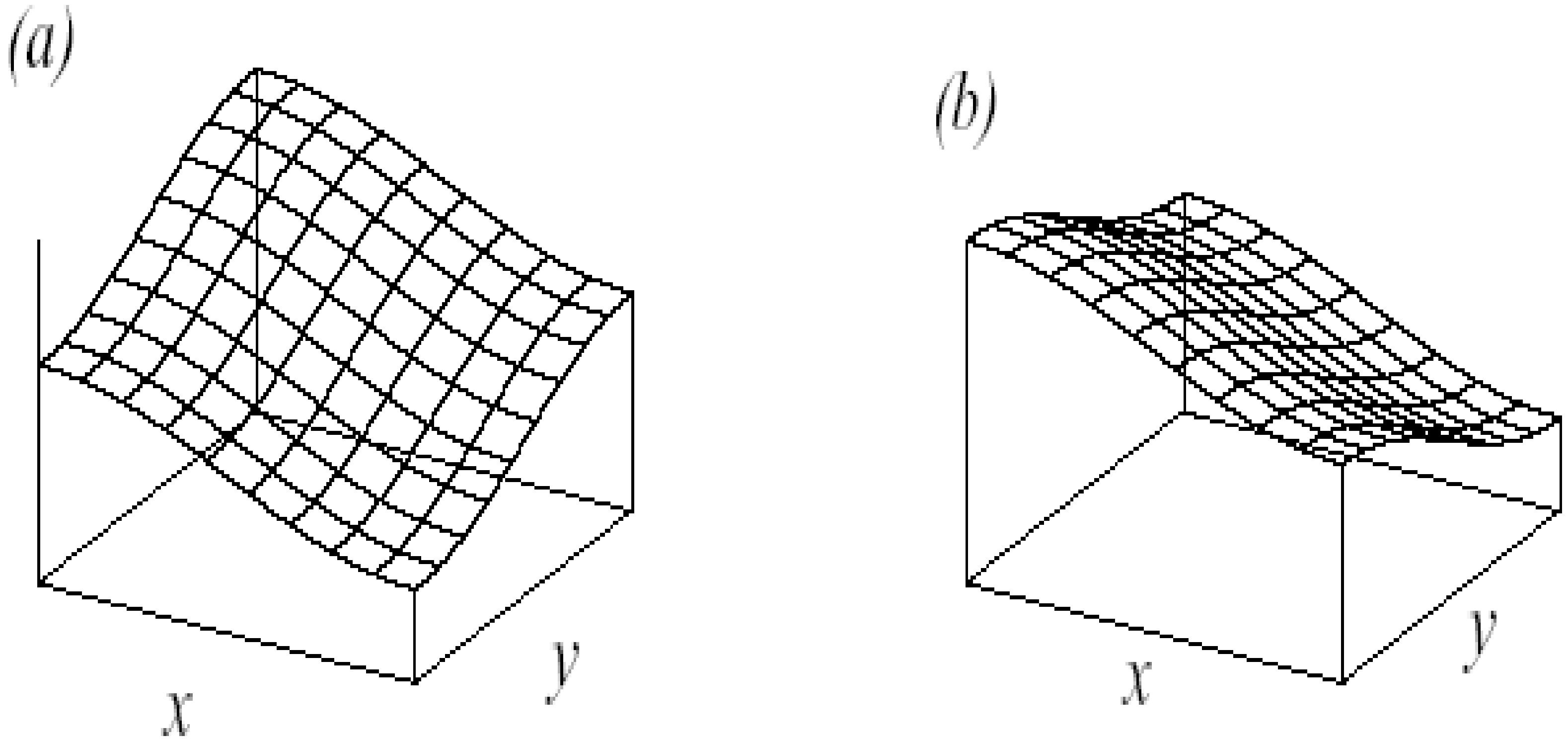
B**(cos(π(x+0.5L))+ cos(π(y+0.5L)))] cos(ωt)**

+o($\varepsilon^{1/3}$)

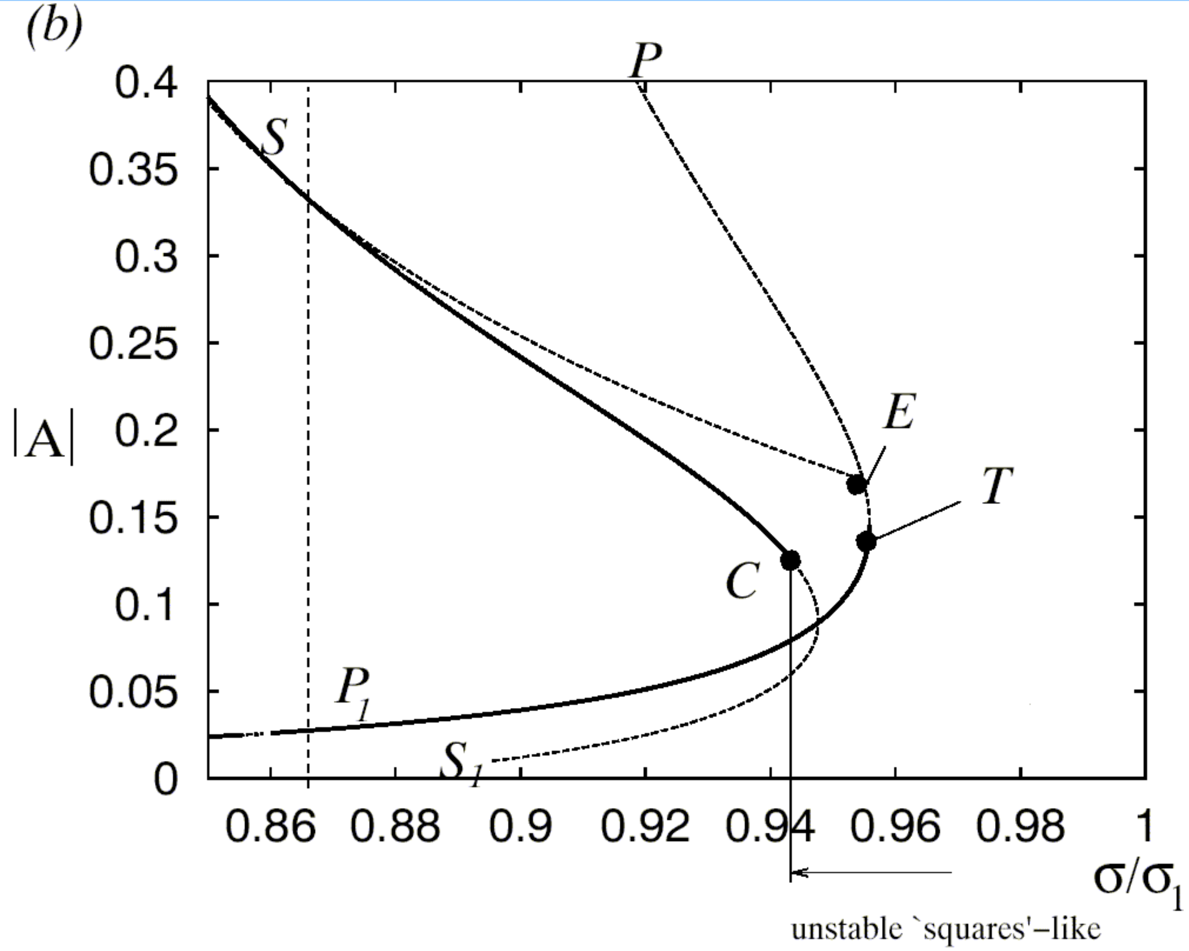
Square mode :**cos(π(x+0.5L))- cos(π(y+0.5L))**

Square mode :**cos(π(x+0.5L))+ cos(π(y+0.5L))**

Square(diagonal) wave patterns



Square-like wave amplitude as a function of excitation frequency σ



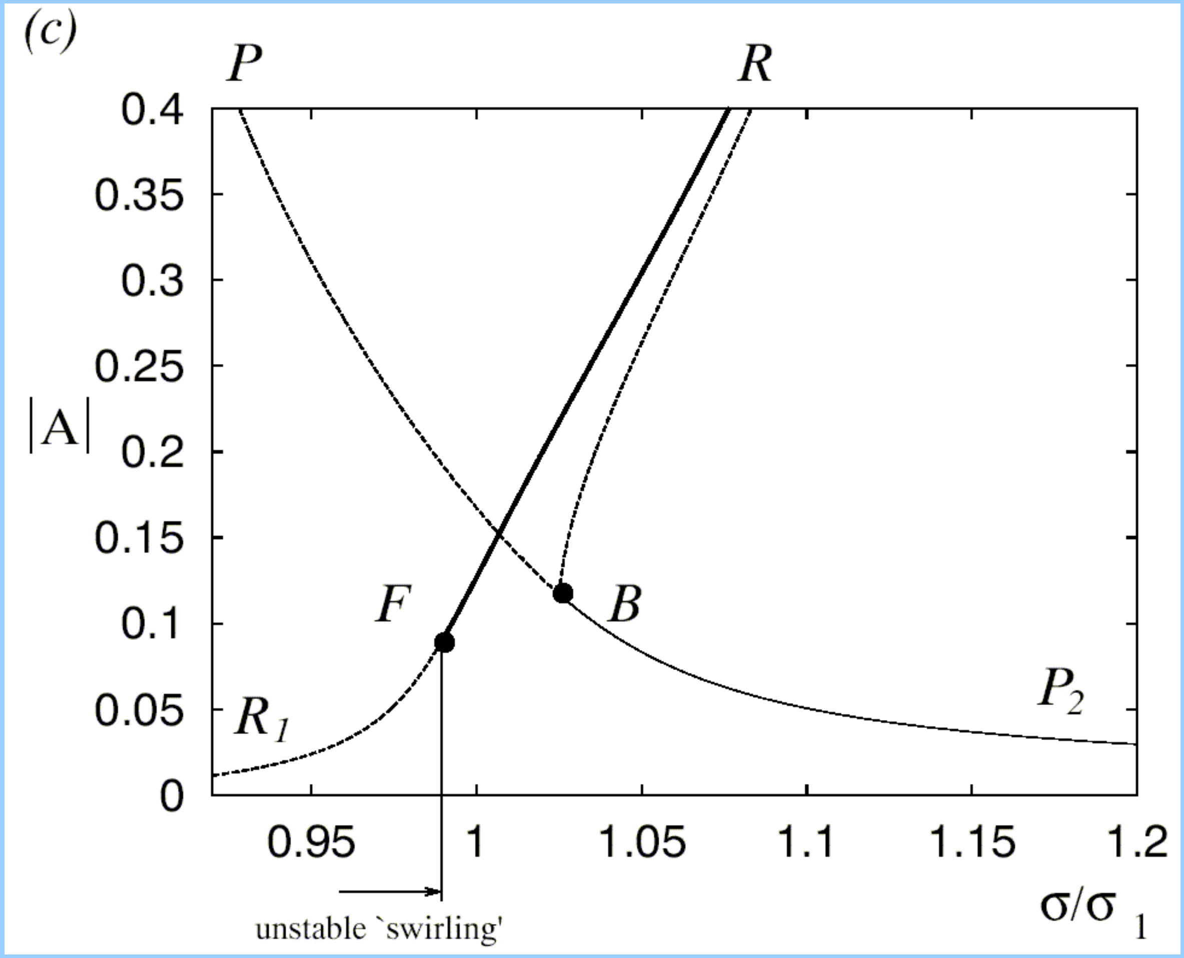
Swirling. Steady-state wave elevation

$$\zeta = \mathbf{A}\ \mathbf{cos}(\pi(\mathbf{x+0.5L}))\mathbf{cos}(\omega \mathbf{t}) + B\ \mathbf{cos}(\pi(\mathbf{y+0.5L}))\ \mathbf{sin}(\omega \mathbf{t}) + \mathbf{o}(\varepsilon^{1/3})$$

Swirling – rotational wave motion

- Special feature of three-dimensional flow in square base tank, vertical circular tank or spherical tank

Swirling wave amplitude A as a function of excitation frequency σ



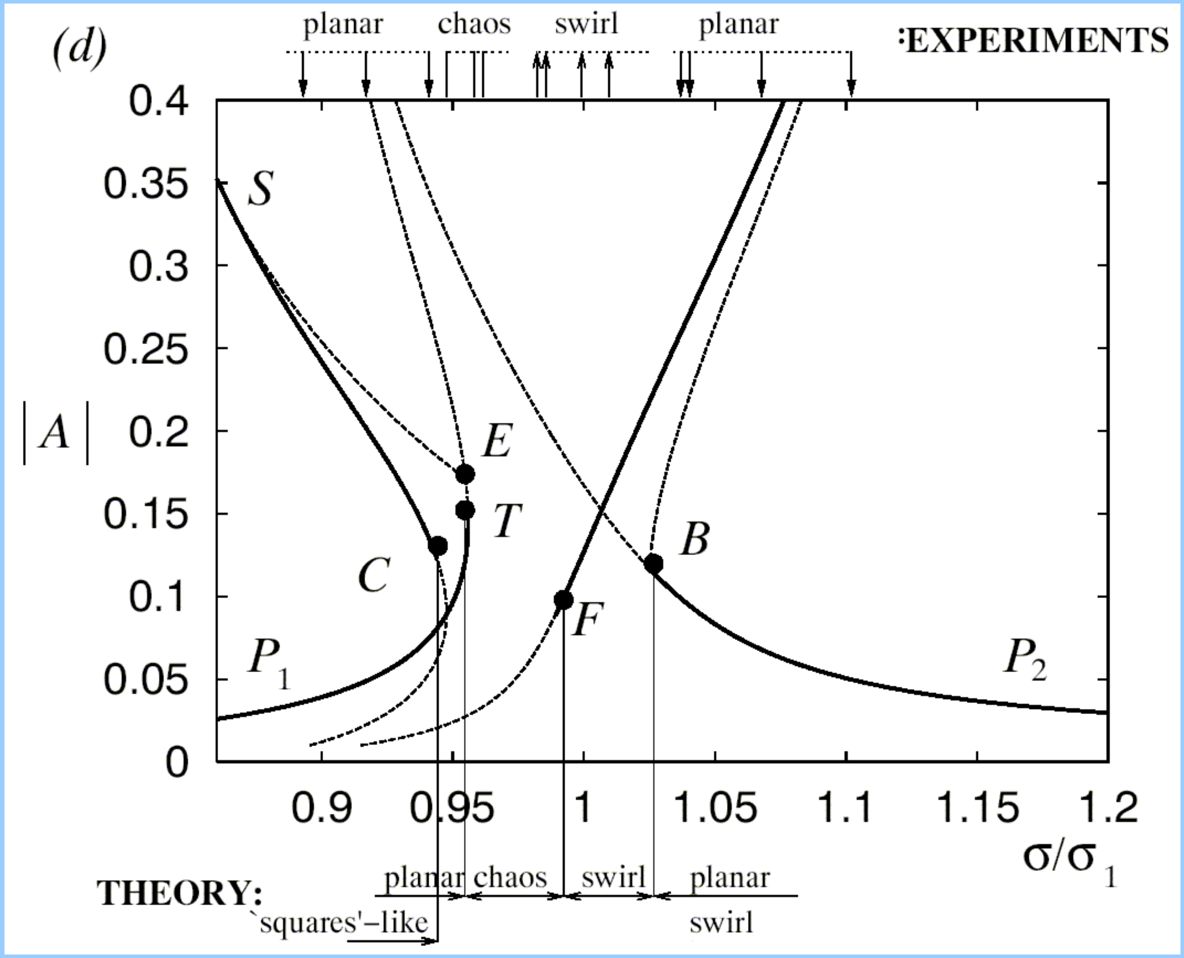
Stability of steady state solutions

- $\beta_{10}=(A_1+\alpha(\tau))\cos(\omega t)+(A_2+\delta(\tau))\sin(\omega t)+o(\varepsilon^{1/3})$
- $\beta_{01}=(A_3+\gamma(\tau))\cos(\omega t)+(A_4+\kappa(\tau))\sin(\omega t)+o(\varepsilon^{1/3})$
- A_i are steady state values
- $\tau=O(\varepsilon^{2/3})$
- $\omega-\sigma_1= O(\varepsilon^{2/3})$

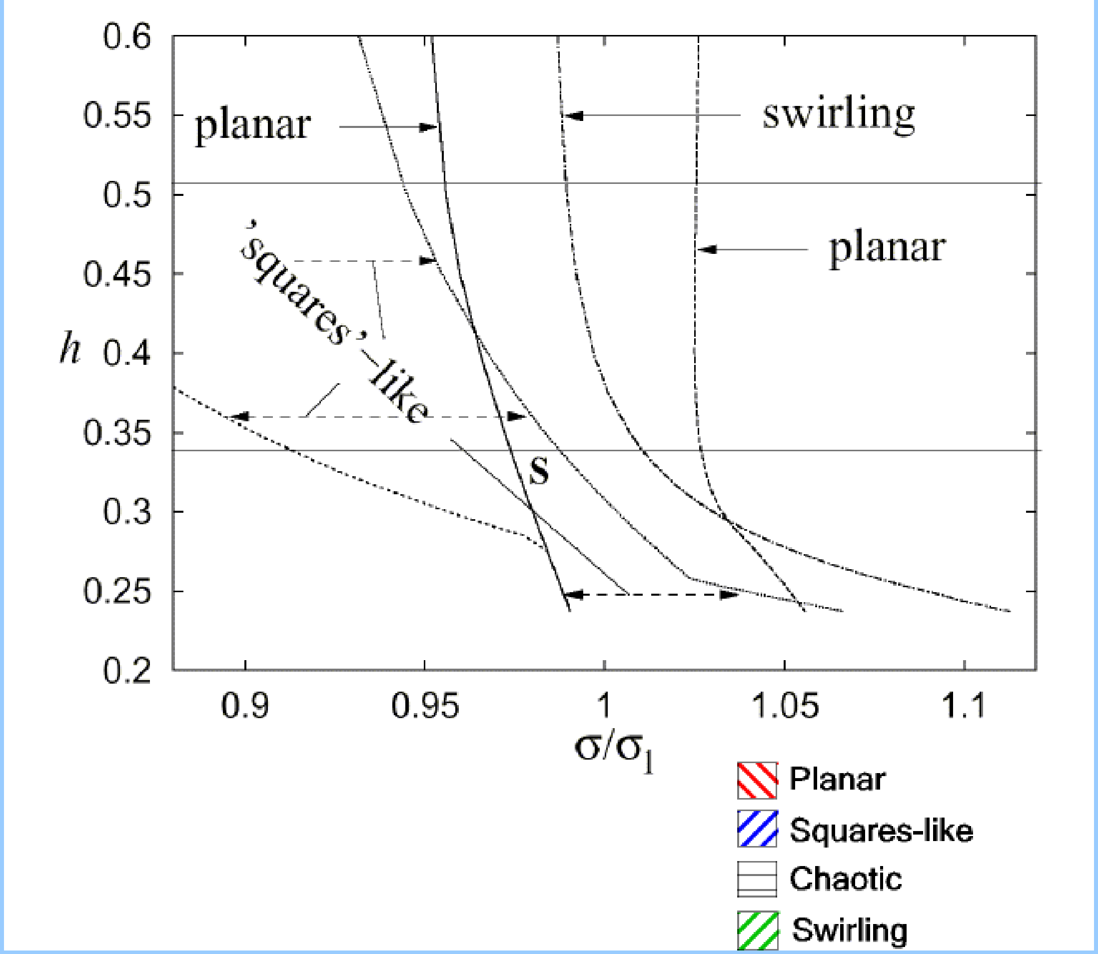
Comparison between theory and experiments

- Theory predicts more than one stable steady-state wave type for certain frequencies
- Wave type with smallest amplitude is then most likely

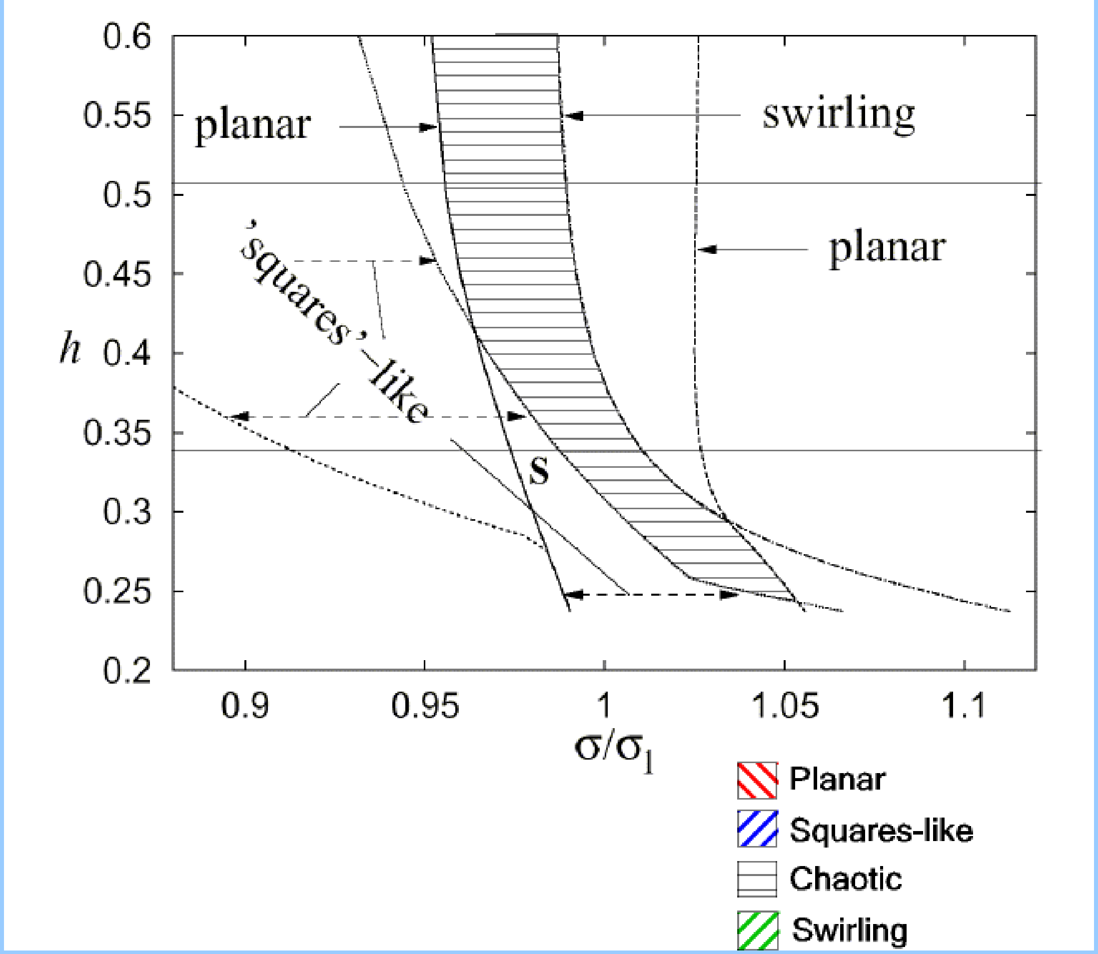
Wave amplitudes A as a function of excitation frequency σ



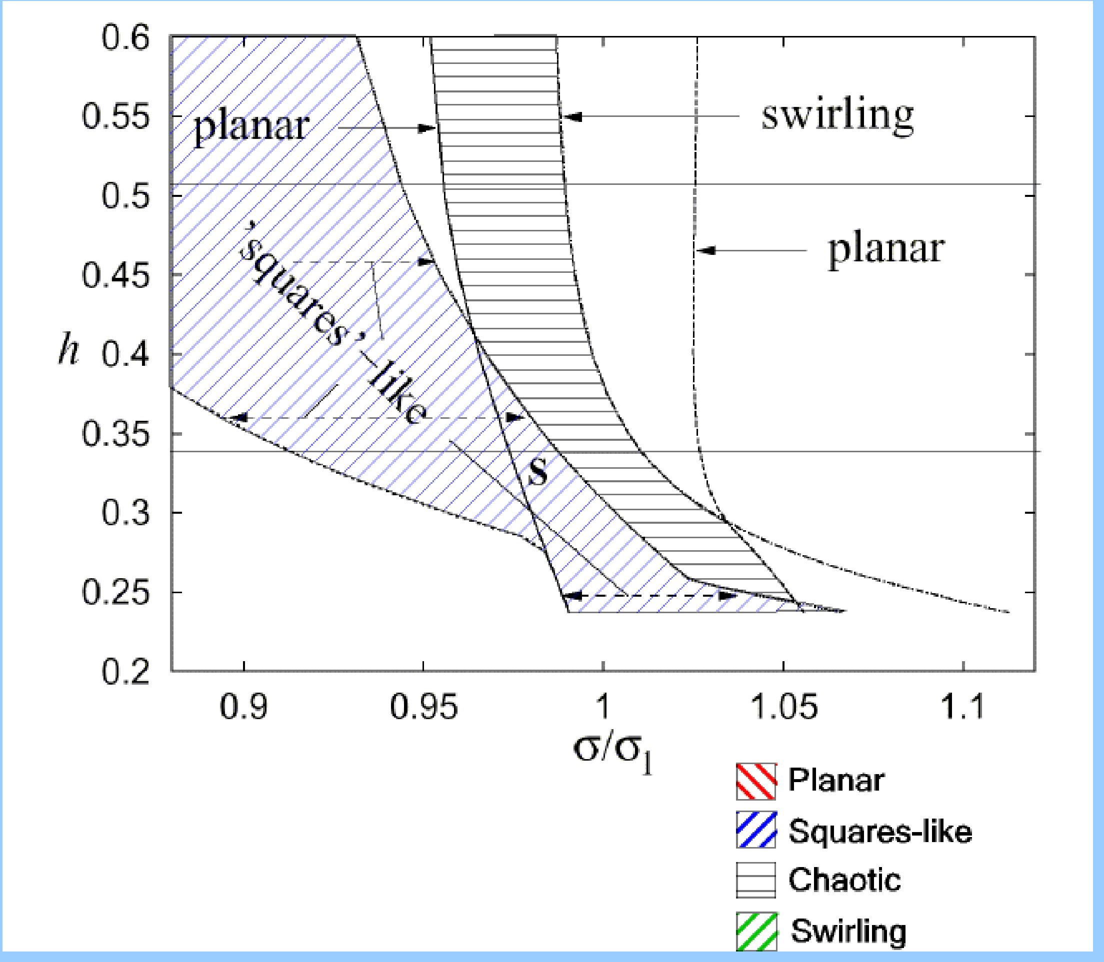
Flow types in square based tank with longitudinal excitation.Effect of fluid depth



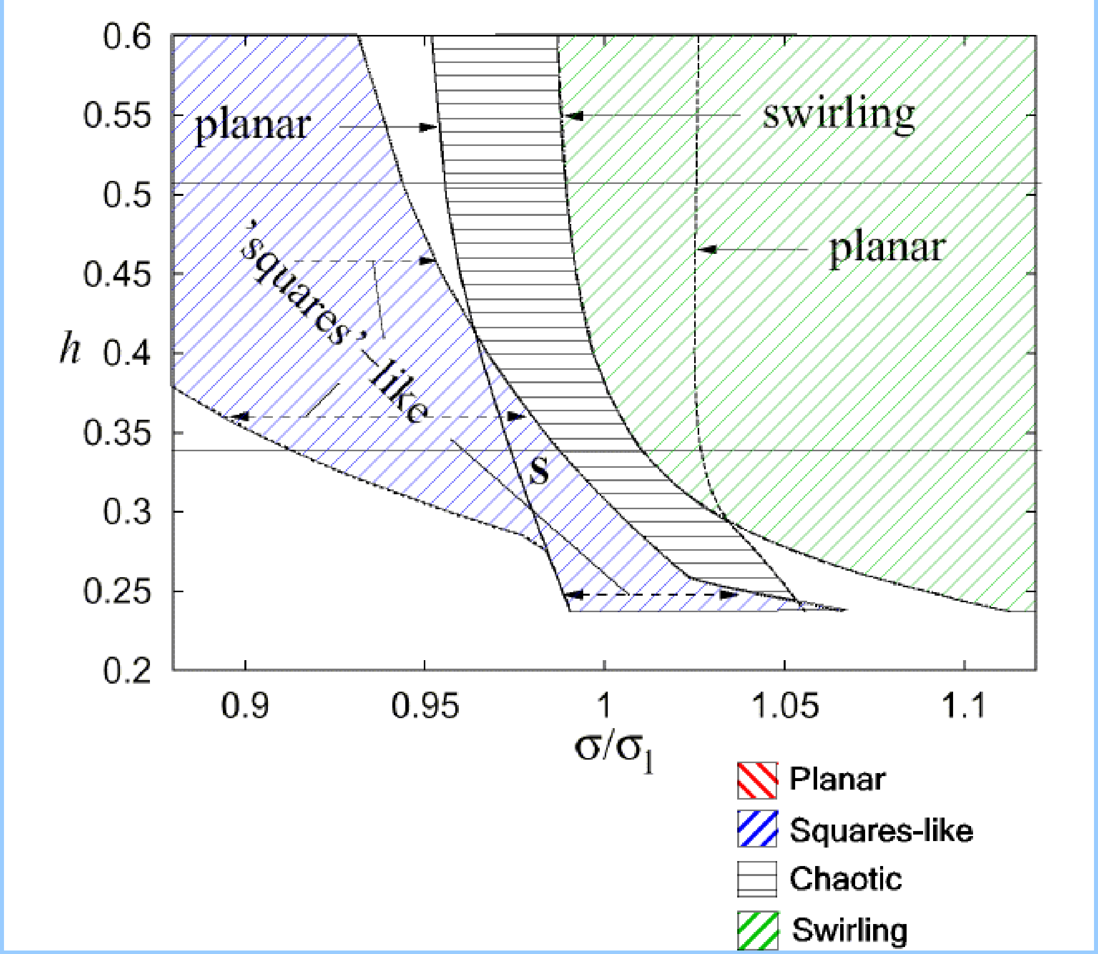
Flow types (classified) in square based tank with longitudinal excitation. Effect of fluid depth



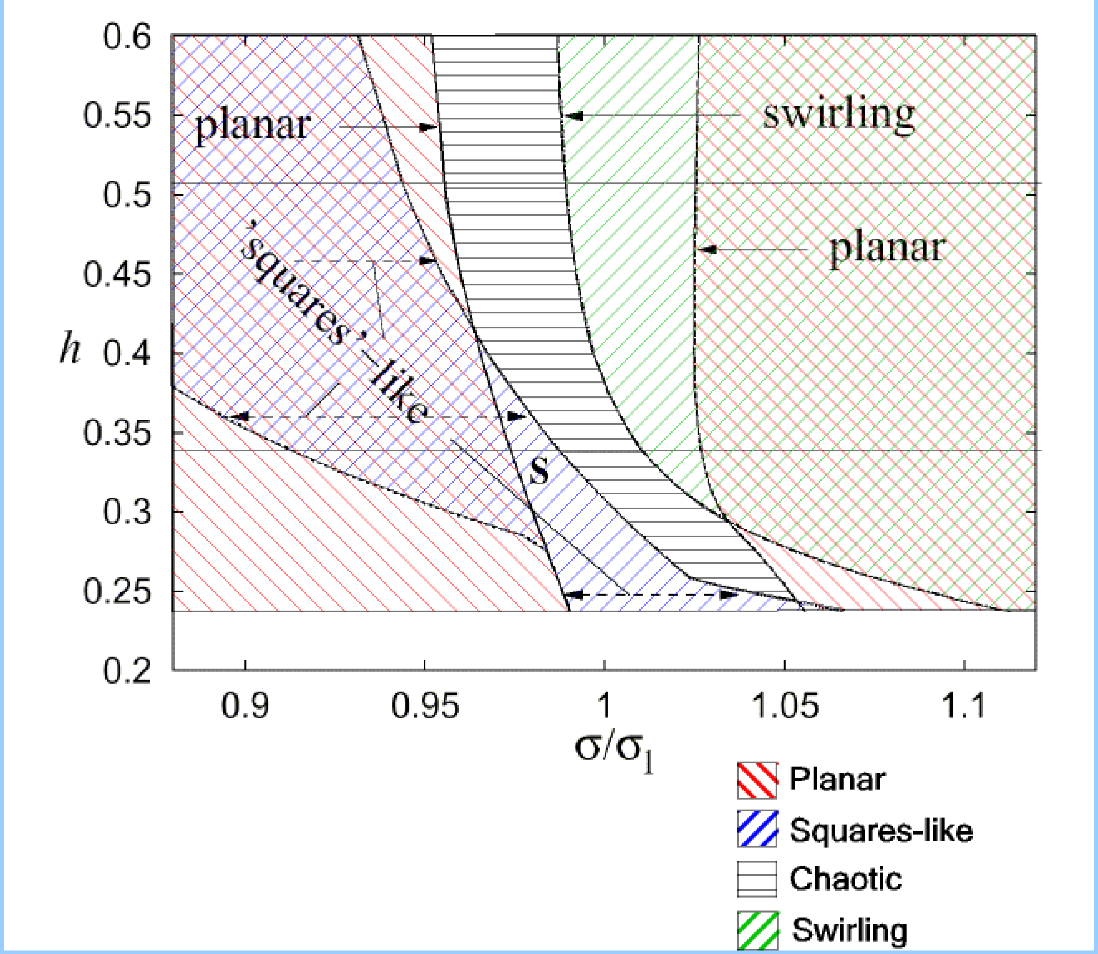
Flow types (classified) in square based tank with longitudinal excitation. Effect of fluid depth



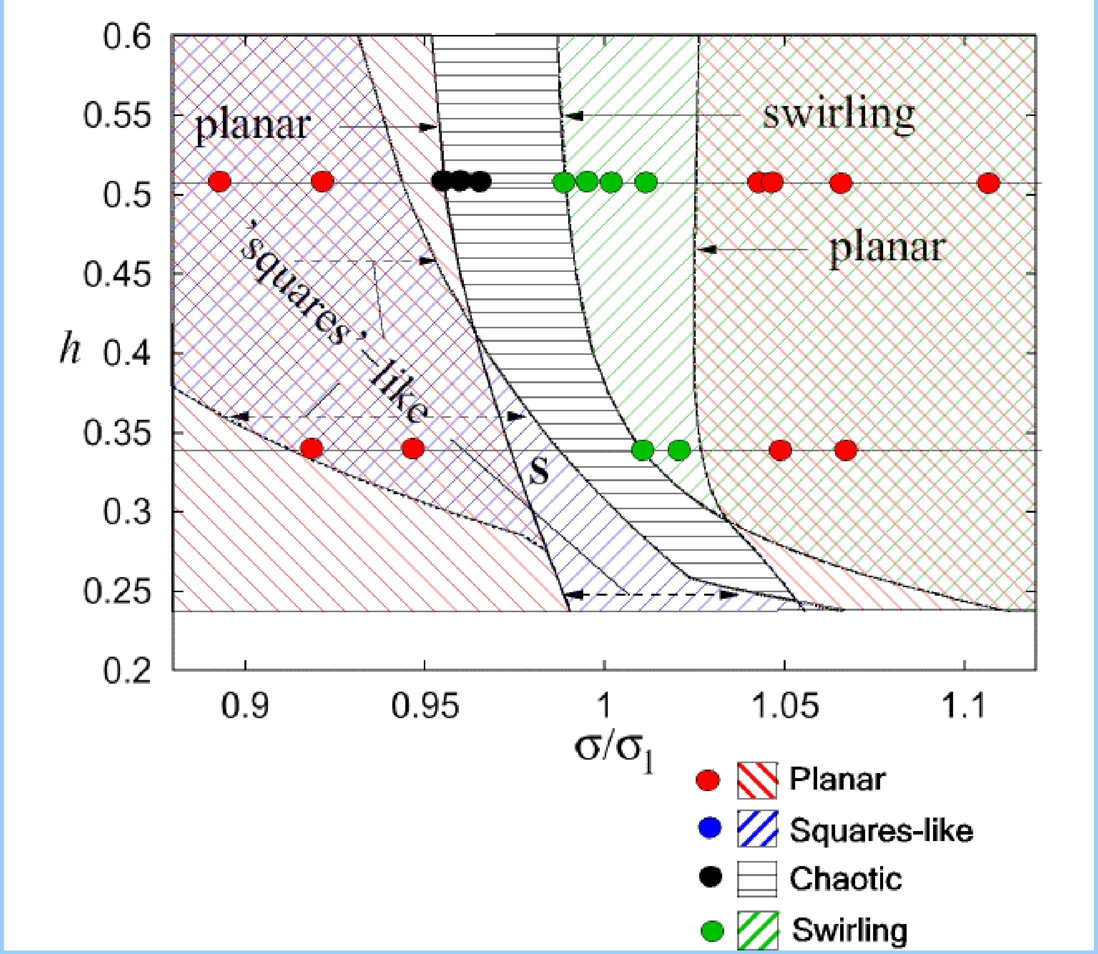
Flow types (classified) in square based tank with longitudinal excitation. Effect of fluid depth



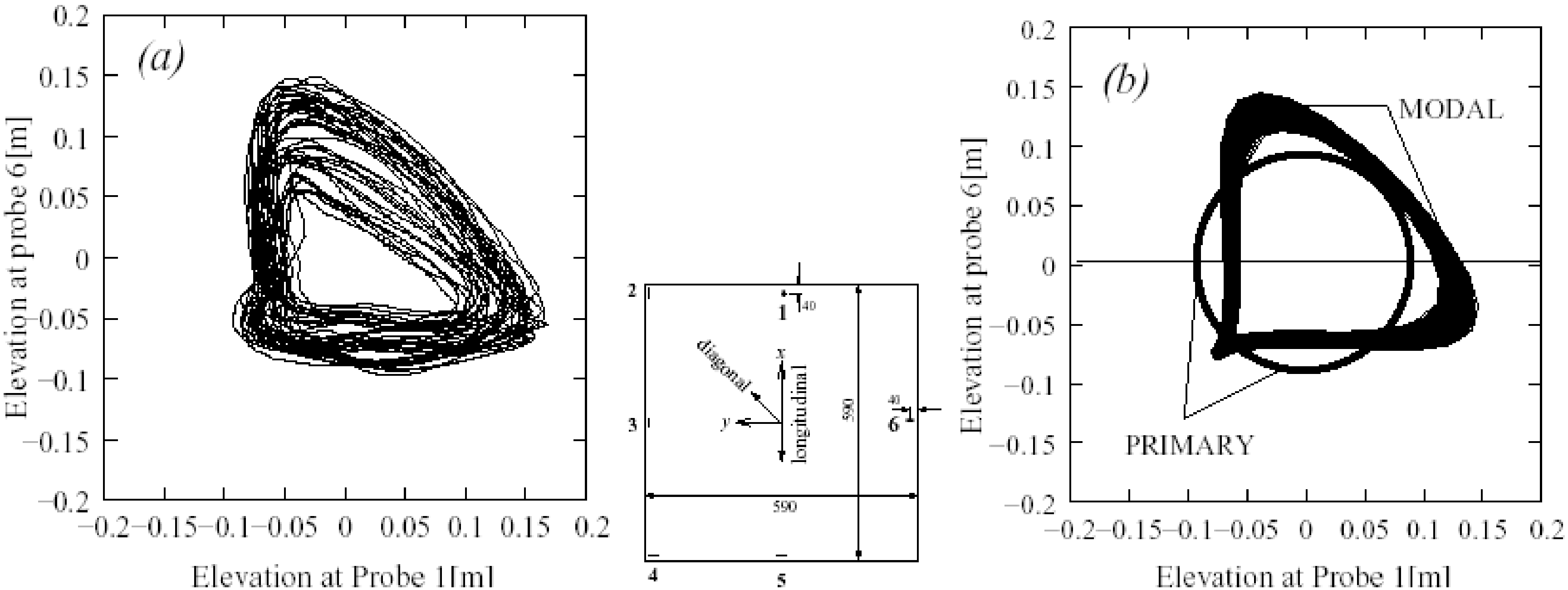
Flow types (classified) in square based tank with longitudinal excitation. Effect of fluid depth



Flow types (classified) in square based tank with longitudinal excitation. Effect of fluid depth



Swirling.Experiments and theory



CPU time for multimodal method

A typical calculation of 100 forced motion oscillation periods takes 1 to 20 seconds on a standard PC.

This means that the calculations are very efficient compared to other CFD methods.

Future perspectives

- Must we base our future developments to account for two phase flow?
- Local phenomena with CFD methods and patching with modal systems?
- Complex geometry of the tanks (there are results for conical tanks)?
- TLD applications, coupling with building dynamics?
- Dissipation?

„Analysis of Strongly Nonlinear Sloshing Phenomena in Moving Tanks of Real Industry Applications“

Thank you for your attention

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