SYNOPSIS

of the book

"VARIATIONAL METHODS IN NONLINEAR PROBLEMS OF THE DYNAMICS OF A LIMITED LIQUID VOLUME"[1]

by

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CONTENTS

[1] Kiev:Institute of Mathematics, 1995.– 400pp. (in Russian)

SUMMARY

The book develops variational approaches to nonlinear free boundary problems associated with dynamics of a limited liquid volume in a container (sloshing). After announcing basic physical models used to describe nonlinear wave motions of a liquid in a rigid tank *Chapter 1* focuses on formulations of the governing equations and boundary conditions of evolutional boundary problems associated with dynamics of a limited volume of a perfect liquid with the free surface in physical fields of various nature (gravity, surface tension, inertia, acoustics and so on). Chapter 1 introduces also main mathematical definitions and denoting as well as delivers the standard terminology accepted in this branch of hydrodynamics. Furthermore, the main book's text dials with a series of variational approaches to three classes of the nonlinear free boundary problems arising in spacecraft and microgravity technology. These are

1. The free boundary problem on nonlinear sloshing of an incompressible liquid with irrotational flows in a moving tank (coupled "rigid body–liquid" motions) and its Lagrangian variational formulation; the projective methods with natural modal basis and numerical schemes for calculation of hydrodynamic force (moment) on the tank (Chapter 2 and some examples in Chapter 7).

2. The nonlinear boundary problem on compressible interfacial potential "gas-liquid" flows due to triad of the gravitation, the surface tension

and a steady acoustic field created in the gas domain and its variational Lagrange-type formulations; the asymptotic averaging reduction in the original and variational problems (Chapters 3 and 4).

3. Sloshing of a compressible perfect liquid in a rigid tank exposed to high-frequency translatory vibration, variational Lagrange-type formulation; the asymptotic averaging reduction in the original and variational problems (Chapters 5 and 6).

Introduction presents a detailed (32p.) survey on existing theoretical and experimental studies and *Chapter 7* gives a set successful implementation of the theoretical results from Chapters 2-6 to some applied problems.

The main text in the theoretical chapters 2-6 is as follows:

- *Chapter 2* is devoted to the Bateman-Luke variational principle and its generalisations onto the liquid sloshing in a rigid tank. Following this principle the chapter derives the nonlinear free boundary problem on sloshing a perfect fluid in a moving tank. Effect of the surface tension is ignored. It is shown that, in contrast to the classical Lagrangian variational principle, this variational approach makes it possible to derive the full set of governing equations and boundary conditions (including so named kinematic and dynamic free boundary conditions on free (unknown) surface) from the necessary extrema condition. This point yields an attractive idea to develop variational projective (modal) methods based on this variational formulation. Authors realize this idea in by several steps. First, they consider the nonlinear free boundary problem on sloshing of a heavy liquid in an unmoving tank (nonlinear gravity waves in a closed basin) and derive corresponding infinite-dimensional modal system. Later on, the modal method is expanded onto the problem on sloshing in a tank performing known spatial motion (inertial waves). The explicit form of the infinite-dimensional systems of ordinary differential equations (modal systems) coupling generalised coordinates implying perturbation of the natural standing waves are derived. Finally, the method is generalized for the case of coupled "rigid body-liquid" motions. This modal system has six additional generalized coordinates implying spatial motions of the carrying body.

- *Chapter 3* is devoted to the theory of compressible interfacial potential "liquid-gas" flows under acoustic field, created by a vibrator posed in the gas domain over the interface (the problem on acoustic positioning). Authors formulate corresponding nonlinear initial-boundary problem and find up small parameters in normalised formulation of this problem. These small parameters appear near both the inhomogeneous terms and local pseudo-differential operators in the boundary conditions and governing equations. These quantities imply potential forces (gravity and surface

tension quantities). Physically, the same redistribution of small parameters in dynamic systems is established in the governing ordinary differential equations describing the pendulum dynamics due to external high-frequency loading. The pendulum vibrational problem was earlier analyzed by Kapitsa. He has shown that this dynamic system allows a multi-scale time technique in order to separate quick (high-frequency) pulsations and slow motions. After applying similar asymptotic procedure to the normalized interfacial problem authors derive a series of nonlinear and linear boundary value problems. When averaging acoustic pulsation in solutions of these problems they reduce asymptotically the original interfacial problem to the problem on the slow-time liquid sloshing under acoustic radiant pressure (asymptotic reduction). Mathematically, this problem is similar to the problem on sloshing in an unmoving basin considered in the previous chapter. However, the dynamic condition on the free surface includes an additional local nonlinear pseudo-differential operator implying the acoustic radiant loading. The acoustic pressure depends parametrically on the instantaneous shape of the interface and solution of the Neumann boundary value problem with respect to wave function in the instantaneous gas domain. The main part of the chapter contains description of the mathematical theory for this free boundary "sloshing" problem (with the nonlinear operator in the dynamic condition). First of all, authors introduce and analyse the concept of capillary-acoustic equilibria, namely, they consider static solutions of the derived nonlinear evolutional free boundary problem. It was shown, that these solutions (averaged interface shapes) are usually different from the capillary shapes. Authors present explicit analytical examples and, in addition, analyse their bifurcation in order to show, that corresponding static free boundary problem (capillary-acoustic equilibria) may have non-unique solution. Further, the spectral theory of linear (natural) sloshing relative to the capillary-acoustic equilibrium shapes is constructed. The spectral theorems establish the real pointer spectrum with only a finite number of negative eigenvalues. Whereas treating the stability of the hydrodynamic system by signs of the spectrum, this implies that breakdown of the equilibria may occur only due to a finite set of linearly independent interfacial perturbations. A number of analytical examples, given in the chapter showed that acoustic field can either stabilise or destabilize the interface. The result depends on the excitation frequency and the energy of the standing acoustic field in the gas. Finally, the last section gives the generalisation of the earlier mathematical results onto the case, when the tank moves in the space and inertial forces are of concern.

- The results in *Chapter 4* are associated with variational approaches to the free boundary problems of the previous chapter. The analysis starts from generalisation of the variational approaches by Hamilton-Ostrogradsky, Bateman-Luke and Luke-Berdichevsky to the compressible potential flows with a free surface. First of all, the equivalence between the original inter-

facial problem of the chapter 3 and corresponding variational formulations is proved. Further, the multi-scale time separation technique (asymptotic reduction) is generalised onto variational statement and a potential implying quasi-potential energy is derived. Authors give the explicit form for both the first and second Frechet derivatives of this nonlinear static functional. The extremal points of this functional are reached on solutions of the static free boundary problems on the capillary-acoustic equilibrium shapes. It is shown that the spectrum of the Jacobi operator (derived from the second derivative) is positive, if and only if, the spectral problem of the previous chapter (on natural sloshing relative to capillary-acoustic equilibrium shape) is positive. Since functional of the quasi-potential energy has strong minima on stable capillary-acoustic shapes, this point makes it possible to introduce the concept of generalised solution for this static free boundary problem. In addition, the variational extremal formulation can be used for projective schemes. Authors give successful examples of the realisation of appropriate projective schemes for the case of circular cylindrical tank.

- Theoretical results of the chapters 3 and 4 are generalised onto the case of high-frequency vibration of a tank filled partially by a compressible fluid with the free surface. Corresponding theory of the vibrocapillary equilibria, the spectral theorems on natural sloshing and the variational formulations are given in *Chapters 5 and 6*.