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The Tangent Groups of a Lie Group and Gauge Invariance in Lagrangian Dynamics

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A tangent Lie group with elements and group operations which are tangent prolongations of those corresponding to another Lie group is examined. An action of such extended Lie group on differentiable manifold and its tangent bundle is defined by using contact transformations. It turned out that a tangent Lie symmetrical Lagrangian describes a dynamical system with first-class constraints. Geometrical aspects of the reduction procedure are considered.

Introduction

We propose to analyse dynamical systems with first-class constraints by using of the tangent Lie groups [1]. In our opinion, this approach explains the genesis of Dirac systems of this type and offers the conceptual clarity.

1 Second-order tangent group of a Lie group

Let G be an $\mathcal{R}$-dimensional Lie group. The tangent bundle TG and the second tangent bundle T^2G are also the Lie groups with elements and group operations which are tangent prolongations of those corresponding to the original Lie group G [1, 2, 3].

Let us consider the curves $\nu : I \to G$ and $\lambda : I \to G$ where I is open neighbourhood of the zero point $0 \in \mathbb{R}$. We introduce the coordinate system (U, g) in G. Henceforth Greek symbols $\nu_\alpha(a) := (g_\alpha \circ \nu)(0)$, $\lambda_\beta(b) := (g_\beta \circ \lambda)(0)$ etc. denote the local coordinates of group elements a, b etc. in manifold G. These coordinates are chosen so that $\varepsilon_\kappa(e) = 0$, $\kappa = 1, \ldots, \mathcal{R} := \overline{1, \mathcal{R}}$, for identity element e. We also use the induced coordinate systems (U^1, g^1) and (U^2, g^2) on the 1-st and 2-nd order tangent bundles TG and T^2G, respectively. Greek indices are meant to run from 1 to $\mathcal{R}$ throughout the paper; the summation convention is used for dummy indices. We denote t^1a, t^2a etc. the elements $t\nu(0)$ and $t^2\nu(0)$ of tangent Lie groups TG and T^2G, respectively. Their coordinates are $\left(\nu_\alpha, \nu_\alpha^1\right)$ and $\left(\nu_\alpha, \nu_\alpha^1, \nu_\alpha^2\right)$ where $\nu_\alpha^i = d^i(g_\alpha \circ \nu)/dt^i\big|_0$, $i = 0, 1, 2$.

Starting with a group multiplication

$$\mu : G \times G \to G, \tag{1.1}$$

we construct the multiplication law for TG [1, 2]

$$T\mu : TG \times TG \to TG$$

defined by

$$T\mu(t\lambda(0), t\nu(0)) = t(\mu(\lambda, \nu))(0).$$

We obtain $\mathcal{R}$ expressions in local coordinates

$$\eta^1_\alpha = d_T\mu_\alpha(\lambda_\beta(b), \nu_\gamma(a)) = \lambda^1_\beta \frac{\partial\mu_\alpha(b,a)}{\partial\lambda_\beta(b)} + \nu^1_\gamma \frac{\partial\mu_\alpha(b,a)}{\partial\nu_\gamma(a)}, \tag{1.2}$$

in addition to the relations

$$\eta_\alpha = \mu_\alpha(\lambda_\beta, \nu_\gamma), \tag{1.3}$$

which illustrate the law (1.1) locally. Here d_T is the Tulczyjew differential operator [4].

As it follows from eqs.(1.2), the identity element t^1e has zero-valued derivative coordinates: $\varepsilon^1_\kappa\left(t^1e\right) = 0$ for all $\kappa = \overline{1, \mathcal{R}}$.

In analogy with $T\mu$ we construct the second order prolongation $T^2\mu : T^2G \times T^2G \to T^2G$ of the multiplication law (1.1):

$$T^2\mu\left(t^2\lambda(0), t^2\nu(0)\right) = t^2(\mu(\lambda,\nu))(0),$$

where on the right side is the second tangent prolongation of the curve $\eta = \mu(\lambda,\nu) : I \to G$ taken at zero point. On the local level we have $\mathcal{R}$ relations

$$\eta^2_\alpha = d^2_T\mu_\alpha(\lambda_\beta, \nu_\gamma), \tag{1.4}$$

in addition to eqs.(1.3) and (1.2).

Now we consider the embedding $\iota_1 : G \to TG$, locally given by $(\nu_\alpha) \mapsto (\nu_\alpha, 0)$. The submanifold $\iota_1(G) \subset TG$ is a slice [5] of the coordinate system $\left(U^1, g^1\right)$. According to ref.[1], ι_1 is a group homomorphism and (G, ι_1) is a closed subgroup of a Lie group TG. Similarly we construct a closed subgroup $\iota_2(G) \subset T^2G$ where the inclusion map $\iota_2 : G \to T^2G$ is the embedding locally written as $(\nu_\alpha) \mapsto (\nu_\alpha, 0, 0)$.

Note that the bundle projections $\tau_G : TG \to G$ and $\tau^2_G : T^2G \to G$ are also the group homomorphisms. The projection $\tau^{2,1}_G$ is the homomorphism from group T^2G to group TG.

Therefore, an original Lie group is a Lie subgroup and a submanifold of its own first- and second-order tangent groups [1]. (More exactly, we consider the slices $\iota_1(G) \subset TG$ and $\iota_2(G) \subset T^2G$ on which all the derivative coordinates are equal to zero.) Moreover, the constants of structure of these tangent Lie groups are determined by the structure constants of G. To demonstrate it we study the involutive distribution $\mathcal{X}_L(T^2G)$ of all left invariant vector fields on T^2G and its dual space $\mathcal{X}^*_L(T^2G)$ of all left invariant one-forms.

Taking into account an exclusive role of Tulczyjew differential operator in prolongation algorithm (see eqs.(1.2) and (1.4)), we deal with $\mathcal{X}^*_L(T^2G)$. We write the local expressions for canonical left invariant one-forms [5, 6] which constitute the basis for $\mathcal{X}^*_L(T^2G)$ at a point t^2a:

$$\theta^{\gamma k} = \left[\frac{\partial}{\partial\nu^i_\alpha(t^2a)} d^k_T\mu_\gamma(b,a)\right]_{t^2b = t^2a^{-1}} d\nu^i_\alpha. \tag{1.5}$$

Small roman indices run from 0 to 2. The exterior derivatives of the $\theta^{\gamma k}$ are given by the Maurer–Cartan equation

$$d\theta^{\gamma k} = -\frac{1}{2} C^\Gamma_{\mathrm{AB}} \theta^{\alpha i} \wedge \theta^{\beta j}. \tag{1.6}$$

We use multi-index notation in structure constants $\{C^\Gamma_{\mathrm{AB}}\}$ of T^2G, where multi-indices A, B and Γ are the 2-tuples of natural numbers, e.g. $\mathrm{A} = (\alpha i)$. Particularly, for subgroup $\iota_2(G) \subset T^2G$ we have

$$d\theta^\gamma = -\frac{1}{2} c^\gamma_{\alpha\beta} \theta^\alpha \wedge \theta^\beta,$$

where $\{c^\gamma_{\alpha\beta}\}$ are structure constants of the original Lie group G (zero-valued roman indices are omitted). Left-invariant one-forms θ^α are given by eqs.(1.5) if integer k is equal to zero.

Tulczyjew operator d_T is the derivation of type d_* of zero degree which acts on the 0-forms as a total time derivative [4]. Having used the commutation $d\circ d_T = d_T\circ d$ and the expressions $d_T{\nu_\beta}^i = {\nu_\beta}^{i+1}$, after short calculations we establish the following relations between the "higher-order" one-forms (1.5) and the original ones:

$$\theta^{\gamma 1} = d_T\theta^\gamma, \qquad \theta^{\gamma 2} = d_T^2\theta^\gamma.$$

Thanks to commutation of Tulczyjew operator with an exterior derivative and positively signed Leibniz' rule for wedge product [3] we arrive at

$$d\theta^{\gamma 1} = -\frac{1}{2}c^\gamma_{\alpha\beta}\theta^{\alpha 1}\wedge\theta^\beta - \frac{1}{2}c^\gamma_{\alpha\beta}\theta^\alpha\wedge\theta^{\beta 1},$$

$$d\theta^{\gamma 2} = -\frac{1}{2}c^\gamma_{\alpha\beta}\theta^{\alpha 2}\wedge\theta^\beta - c^\gamma_{\alpha\beta}\theta^{\alpha 1}\wedge\theta^{\beta 1} - \frac{1}{2}c^\gamma_{\alpha\beta}\theta^\alpha\wedge\theta^{\beta 2}.$$

When comparing these expressions with the Maurer–Cartan equation (1.6) we deduce the constants of structure $\{C^\Gamma_{\mathrm{AB}}\}$. It is convenient to write them as the following block matrices:

$$\hat{C}^{(\gamma 0)} = \begin{bmatrix} \hat{c}^\gamma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \hat{C}^{(\gamma 1)} = \begin{bmatrix} 0 & \hat{c}^\gamma & 0 \\ \hat{c}^\gamma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \hat{C}^{(\gamma 2)} = \begin{bmatrix} 0 & 0 & \hat{c}^\gamma \\ 0 & 2\hat{c}^\gamma & 0 \\ \hat{c}^\gamma & 0 & 0 \end{bmatrix}.$$

Here symbol $\hat{c}^\gamma$ denotes the skew-symmetric matrix $\|c^\gamma_{\alpha\beta}\|$ with fixed integer γ.

Similarly we obtain the structure constants of a Lie group TG:

$$\hat{C}^{(\gamma 0)} = \begin{bmatrix} \hat{c}^\gamma & 0 \\ 0 & 0 \end{bmatrix}, \qquad \hat{C}^{(\gamma 1)} = \begin{bmatrix} 0 & \hat{c}^\gamma \\ \hat{c}^\gamma & 0 \end{bmatrix}.$$

The basis for $\mathcal{X}_L(T^2G)$ consists of the left invariant vector fields [2, 5, 6], say $X^{(2)}_B$, locally given by

$$X^{(2)}_{(\beta 0)} = L^\alpha_\beta\frac{\partial}{\partial g_\alpha} + d_T(L^\alpha_\beta)\frac{\partial}{\partial {g_\alpha}^1} + d_T^2(L^\alpha_\beta)\frac{\partial}{\partial {g_\alpha}^2},$$

$$X^{(2)}_{(\beta 1)} = L^\alpha_\beta\frac{\partial}{\partial {g_\alpha}^1} + 2d_T(L^\alpha_\beta)\frac{\partial}{\partial {g_\alpha}^2},$$

$$X^{(2)}_{(\beta 2)} = L^\alpha_\beta\frac{\partial}{\partial {g_\alpha}^2}.$$

Here $L^\alpha_\beta(g)$ are the components of the left invariant vector fields X_β which form the basis for $\mathcal{X}_L(G)$.

Vector field $X^{(2)}_{(\beta 0)}$ is the 2-lift of corresponding one X_β to tangent bundle T^2G, i.e. $X^{(2)}_{(\beta 0)} = X^{(2,2)}_\beta$ (see refs. [1, 9]). The former belongs to the basis of the sub distribution $\mathcal{X}_L(\iota_2(G)) \subset \mathcal{X}_L(T^2G)$. The others $X^{(2)}_{(\beta 1)}$ and $X^{(2)}_{(\beta 2)}$ are intimately connected with the 1-st and 0-th lifts [1, 9] of X_β to T^2G, respectively. Namely, we have $X^{(2)}_{(\beta 1)} = J_1X^{(2,2)}_\beta$ and $X^{(2)}_{(\beta 2)} = (1/2)(J_1)^2X^{(2,2)}_\beta$, where J_1 is the canonical almost tangent structure [10] of order 2 on T^2G.

Let $X^{(1)}_{(\beta i)}$, $i = 0, 1$, be the canonical left-invariant vector fields on $\mathcal{X}_L(TG)$. If $\tau^2_1 : T^2G \to TG$ is the canonical projection, then $X^{(2)}_{(\beta i)}$ and $X^{(1)}_{(\beta i)}$ are τ^2_1-related, i.e. $T\tau^2_1(X^{(2)}_{(\beta i)}) = X^{(1)}_{(\beta i)}$. Each of homomorphism of groups, mentioned in this Section, corresponds the Lie algebra homomorphism which describes its effect on left invariant vector fields, as well as the mapping which relates the dual algebras. The former are then nothing but the differential of originating group homomorphism and the latter is precisely the transpose of this differential [5].

2 An action of the tangent Lie group on a smooth manifold

When considering the group parameters as constants, an action $r : Q \times G \to Q$ of a Lie group G on an $\mathcal{N}$-dimensional smooth manifold Q lifts to an action $r^1 : TQ \times G \to TQ$ of G on the tangent bundle TQ as follows [6, 11, 9]: $\left(r^1\right)_a : TQ \to TQ$, where $\left(r^1\right)_a = Tr_a$ for any fixed $a \in G$. Treatment of group parameters via the time-dependent variables makes the notion of lift of a group action quite different from mentioned above. The desired map is $Tr : TQ \times TG \to TQ$ [1]. It defines the transformation $(Tr)_{t^1 a} : TQ \to TQ$ for any fixed $t^1 a \in TG$.

We introduce the coordinate system (V, q) in Q and we also use the induced charts $\left(V^1, q^1\right)$ in TQ. The action of TG on TQ induces a Lie algebra homomorphism of $\mathrm{Lie}\,(TG) := T_{t^1 e}(TG)$ into vector space $\mathcal{X}(TQ)$. To each vector field $\xi^{(1)}_{(\alpha i)} := X^{(1)}_{\alpha i}\left(t^1 e\right)$, $i = 0, 1$, we assign the vector field $Y^{(1)}_{(\alpha i)}$ on TQ:

$$Y^{(1)}_{(\alpha 0)} = Y^b_\alpha \frac{\partial}{\partial q_b} + d_T(Y^b_\alpha)\frac{\partial}{\partial {q_b}^1}, \qquad Y^{(1)}_{(\alpha 1)} = Y^b_\alpha \frac{\partial}{\partial {q_b}^1}. \tag{2.1}$$

Symbol $Y^b_\alpha(q_a)$, $a, b = \overline{1, \mathcal{N}}$, denotes the component of the fundamental vector field Y_α corresponding to $\xi_\alpha \in \mathrm{Lie}\,(G)$. Actually $\left\{Y^{(1)}_{(\alpha i)} | \alpha = \overline{1, \mathcal{R}}; i = 0, 1\right\}$ is a Lie subalgebra of the set $\mathcal{X}(TQ)$ of all vector fields on TQ.

Let us compare these results with standard situation where coordinates of the group elements are meant to be constants. In such a case the transformations of TQ are generated by fundamental vector fields which are complete lifts of their prototypes, acting on Q [9]. Since $\dim TG = 2 \dim G$, we have double number of infinitesimal generators, namely $Y^{(1)}_{(\alpha 0)}$ and $Y^{(1)}_{(\alpha 1)}$, which are then nothing but the complete and vertical lifts [7] of the original one Y_α. Therefore, it is reasonable to say that we deal with the *total* 1-st lift of an action of G on Q.

We may lift an action of TG on Q to the action $T^2 G$ on TQ in similar circumstances. We introduce a smooth map

$$\begin{aligned} r^{(0,1)} : \quad & Q \times TG \to Q, \\ & (y(0), t\nu(0)) \mapsto x(0), \end{aligned} \tag{2.2}$$

which is an action of a Lie group TG on manifold Q on the right [6]. The bracketed and separated by comma integers $(0, 1)$ up to letter r are associated with the orders of tangent bundles over Q and G, respectively. The curve $y : \mathbb{R} \to Q$ runs across a point $y(0) \in V$ with coordinates $\left\{y_a | a = \overline{1, \mathcal{N}}\right\}$ and the curve $x : \mathbb{R} \to Q$ passes through a point $x(0) \in V$ with coordinates $\left\{x_a | a = \overline{1, \mathcal{N}}\right\}$. In local coordinates (2.2) is written as

$$x_a = f_a(y_b, \nu^i_\alpha(t^1 a)).$$

An action of TG on Q induces a Lie algebra homomorphism of the Lie algebra $\mathrm{Lie}\,(TG)$ into vector space $\mathcal{X}(Q)$ [6]. To each vector field $\xi^{(1)}_{(\alpha i)} \in \mathrm{Lie}\,(TG)$, $i = 0, 1$, we assign the following fundamental vector field on Q:

$$Y^{(0,1)}_{(\alpha i)} = \left.\frac{\partial f_a(q_b, t^1 a)}{\partial {\nu_\alpha}^i}\right|_{t^1 a = t^1 e} \frac{\partial}{\partial q_a}. \tag{2.3}$$

Each of them is the infinitesimal generator of an 1-parameter group of transformations of Q.

The map (2.2) lifts to the right action $r^{(1,2)} : TQ \times T^2G \to TQ$ of group T^2G on tangent bundle TQ by composition of the tangent mapping $Tr^{(1,0)} : T(Q\times TG) \to TQ$ with the canonical embedding

$$\begin{aligned} i_{1,0} : \quad & TQ \times T^2G \to T(Q \times TG), \\ & (ty(0), t^2\nu(0)) \mapsto t(y, t\nu)(0). \end{aligned}$$

The tangent prolongation $ty(0)$ is represented in TV by $(y_b, {y_b}^1)$, where $y_b = (q_b \circ y)(0)$, and ${y_b}^1 = d(q_b \circ y)/dt|_0$. In local coordinates we obtain the following transformational law for first-order derivative coordinates: ${x_a}^1 = d_T f_a$, where ${x_a}^1 = d(q_a \circ x)/dt|_0$.

The fundamental vector fields which correspond to $\xi^{(2)}_{(\alpha i)} \in \mathrm{Lie}\,(T^2G)$ may be expressed in terms of both complete and vertical lifts [7] of vector fields (2.3):

$$Y^{(1,2)}_{(\alpha 0)} = \left(Y^{(0,1)}_{(\alpha 0)}\right)^c, \qquad Y^{(1,2)}_{(\alpha 1)} = \left(Y^{(0,1)}_{(\alpha 1)}\right)^c + \left(Y^{(0,1)}_{(\alpha 0)}\right)^v, \qquad Y^{(1,2)}_{(\alpha 2)} = \left(Y^{(0,1)}_{(\alpha 1)}\right)^v. \tag{2.4}$$

They constitute an involutive distribution [2] on TQ.

3 Dynamical system with first-class constraints

In ref. [8] the degenerate Lagrangian system was examined in which the action integral is invariant with respect to the so-called gauge transformation. (By this is meant that the coordinate transformation of the configuration manifold is specified by some time-dependent parameters.) The theorem was proved that a necessary and sufficient condition of such invariance is the existence of relations linking the expressions for Euler–Lagrange equations together. It was shown that a Hamiltonian system with first-class constraints is derived from this degenerate Lagrangian. Conversely, a Dirac system with first-class constraints admits the symmetry of this type.

In geometrical approach [12, 13] a first-class constraint set C is the co-isotropic submanifold of phase space P. The symplectic polar $T^\P C \subset TC$ is the integrable distribution on C which is called the *characteristic distribution of* C. This distribution induces the *characteristic foliation* of C. The necessary and sufficient condition for the Dirac system to possess the solutions is that the Hamiltonian $H : C \to \mathbb{R}$ takes a constant value on leaves of the characteristic foliation of C (see refs. [12, 13, Theorem 1]). This theorem can be coordinated with the results obtained in [8] as follows.

Let $P = T^*Q$ and let Q admits the foliation caused by an integrable distribution E. A vector field $Y \in E$ induces the transformation $Fl^Y_t : Q \to Q$ of the configuration manifold. This flow should be identified with the gauge transformation introduced in [8]. The parameters of gauge transformation distinguish the points of an individual leaf of foliation. We interpret this transformation as the invertible contact transformation [14]:

$$\begin{aligned} y'_a &= f_a(y_b, \nu_\alpha, {\nu_\alpha}^1), \\ \nu'_\alpha &= \nu_\alpha\,, \qquad \alpha = \overline{1,\mathcal{R}}, \end{aligned} \tag{3.1}$$

which leaves a Lagrangian $L : TQ \to \mathbb{R}$ invariant. Since the transformed Lagrangian does not depend on the variables ν_α and their time derivatives ${\nu_\alpha}^1$, corresponding Jacobi–Ostrogradski momenta $\hat{\eta}_{\alpha,0}$ and $\hat{\eta}_{\alpha,1}$ [10]

$$\hat{\eta}_{\alpha,1} = \frac{\partial(L\circ f^1)}{\partial \nu_\alpha^2} = \left(\frac{\partial L}{\partial \dot{y}'_a}\circ f^1\right)\frac{\partial f_a}{\partial \nu_\alpha^1},$$

$$\hat{\eta}_{\alpha,0} = \frac{\partial(L\circ f^1)}{\partial \nu_\alpha^1} - d_T\hat{\eta}_{\alpha,1} = \left(\frac{\partial L}{\partial \dot{y}'_a}\circ f^1\right)\frac{\partial f_a}{\partial \nu_\alpha},$$

are equal to zero. Symbol f^1 denotes the first holonomic prolongation [14] of the map (3.1). Taking the $\nu_\alpha \to 0$ limits we obtain the constraint manifold

$$C = \left\{(y_c,\pi_c)\in T^*Q;\ \pi_b\, Y^b_{\alpha k}(y_c) = 0\right\}, \tag{3.2}$$

where the index $k = 1, 0$ enumerates steps of iterative constraint algorithm [13]. So, the initial constraint set is

$$C^0 = \left\{(y_c,\pi_c)\in T^*Q;\ \pi_b\, Y^b_{\alpha 1}(y_c) = 0\right\}.$$

Note that $Y^b_{\alpha k}(y_c)$ are the components of vector fields $Y_{\alpha k}\in E$ which generate the gauge transformations (3.1).

The characteristic distribution $T^{\P}C$ of the constraint manifold (3.2) is spanned by the vector fields

$$Y^*_{\alpha k} = Y^b_{\alpha k}(y_c)\frac{\partial}{\partial y_b} - \pi_b\frac{\partial Y^b_{\alpha k}}{\partial y_c}\frac{\partial}{\partial \pi_c},$$

which are complete lifts of vector fields from the distribution E to the phase manifold T^*Q [9]. The tangent manifold TQ admits the foliation caused by the distribution E^c involving both the complete, Y^c, and the vertical, Y^v, lifts [7] of the original vector fields $Y \in E$:

$$\begin{aligned}
\dot{Y}_{\alpha 0} &= Y^b_{\alpha 0}(y_c)\frac{\partial}{\partial y_b} + d_T\left(Y^b_{\alpha 0}(y_c)\right)\frac{\partial}{\partial {y_b}^1} = Y^c_{\alpha 0},\\
\dot{Y}_{\alpha 1} &= Y^b_{\alpha 1}(y_c)\frac{\partial}{\partial y_b} + \left(Y^b_{\alpha 0}(y_c) + d_T\left(Y^b_{\alpha 1}(y_c)\right)\right)\frac{\partial}{\partial {y_b}^1} = Y^c_{\alpha 1} + Y^v_{\alpha 0},\\
\dot{Y}_{\alpha 2} &= Y^b_{\alpha 1}(y_c)\frac{\partial}{\partial {y_b}^1} = Y^v_{\alpha 1},
\end{aligned} \tag{3.3}$$

(cf. eqs.(2.4)). From the theorem about the local structure of foliation [2] we see that an invariant Lagrangian (Hamiltonian) depends on the coordinates of points of plaque and their derivatives (conjugates) only. The momentum canonically conjugated to leaf's coordinate variable is equal to zero. All the momenta of this type constitute the first-class constraint set, i.e. the co-isotropic submanifold of T^*Q. Thus, the distinguished chart [2] for the distribution (3.3) is the key to the reduction procedure here.

The *primary* first-class constraints only hold the independent degrees of freedom [8]. Whence the reduction procedure leads to the constrained Hamiltonian system which does not involve the secondary ones. This Dirac system is derived from a degenerate Lagrangian which does not depend explicitly on some variables.

Concluding remarks

The requirement of invariance of a Lagrangian function under the action of a tangent Lie group leads to degeneracy of this Lagrangian. Fundamental vector fields corresponding to such group constitute the involutive distributions which are important particular cases of the integrable distributions associated with the characteristic distribution of constraint manifold. Distinguished charts for the foliations induced by these involutive distributions may help to explain the geometrical essence of gauge theories.

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References

[1] Yano K. and Ishihara S., Tangent and Cotangent Bundles, M. Dekker Inc., New York, 1973.

[2] Kolář I., Michor P.W. and Slovák J., Natural Operations in Differential Geometry, Springer-Verlag, Berlin Heidelberg, 1993.

[3] Saunders D.J., The Geometry of Jet Bundles, London Math. Soc., Lecture Notes Series, V.142, Cambridge Univ. Press, 1989.

[4] Tulczyjew W.M., Sur la différentiellee de Lagrange, *C.R. Acad.Sci. Paris, Sér. A*, 1975, V.280, 1295–1298.

[5] Warner F.W., Foundations of Differentiable Manifolds and Lie Groups, Graduate Texts in Mathematics, V.94, Springer-Verlag, 1983.

[6] Abraham R. and Marsden J., Foundations of Mechanics, 2nd ed, Benjamin, New York, 1978.

[7] de León M. and Rodrigues P.R., Methods of Differential Geometry in Analytical Mechanics, North-Holland Math. Studies, Ser. 158, Amsterdam, 1989.

[8] Gitman D.M. and Tyutin I.V., Canonical Quantization of Fields with Constraints, Nauka, Moscow, 1986 (in Russian).

[9] de León M. and de Diego D.M., Symmetries and constants of the motion for higher-order Lagrangian systems, *J. Math. Phys.*, 1995, V.36, N 8, 4138–4161.

[10] de León M. and Rodrigues P.R., Generalized Classical Mechanics and Field Theory, North-Holland Math. Studies, Ser. 112, Amsterdam, 1985.

[11] de León M., Pitanga P. and Rodrigues P.R., Symplectic reduction of higher-order Lagrangian systems with symmetry, *J. Math. Phys.*, 1994, V.35, N 12, 6546–6556.

[12] Mendella G., Marmo G. and Tulczyjew W.M., Integrability of implicit differential equations, *J. Phys. A: Math. Gen.*, 1995, V.28, 149–163.

[13] Marmo G., Mendella G. and Tulczyjew W.M., Constrained Hamiltonian systems as implicit differential equations, *J. Phys. A: Math. Gen.*, 1997, V. 30, 1997, 277–293.

[14] Yaremko Y., Contact transformations in Wheeler–Feynman electrodynamics, *Ann. Inst. H. Poincaré*, 1997, V.66, N 3, 293–322.