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Nonlinear Symmetry and Unity of Spacetime and Matter*

Kazunari SHIMA

Laboratory of Physics, Saitama Institute of Technology
Okabe-machi, Saitama 369-0293, Japan

Based upon the invariances under the nonlinear global supersymmetry (NL SUSY) and the general coordinate transformation ($GL(4,R)$), a unified description of spacetime and matter is proposed. Except the graviton all elementary particles accomodated in a single irreducible representation of $N=10$ extended super-Poincaré (SP) algebra are the composites of more fundamental objects *superons* with spin 1/2, which are Nambu–Goldstone (N–G) fermions accompnying the spontaneous breakedown of the global supertranslation of spacetime. The electroweak standard model (SM) and $SU(5)$ ($SO(10)$) grand unified model (GUTs) are investigated systematically by using the superon diagrams. The stability of the proton, the suppression of the flavour changing neutral currents (FCNC), $K^0-\overline{K}^0$ and $B^0-\overline{B}^0$ mixings, CP-violation, the atmospheric ν_μ deficit, the charmless nonleptonic B decay and the absence of the electroweak lepton-flavor-mixing are understood naturally in the superon pictures of GUTs and some predictions are presented. The fundamental action of superon-graviton model (SGM) for supersymmetric spacetime and matter is obtained.

1 Introduction

As a unified gauge field theory of all particles and all forces, the strong-electroweak standard model (SM), the grand unified theories (GUT) like $SU(5)$ ($SO(10)$) and their variants with supersymmetry (SUSY) still leave many fundamental problems, for example, the lack of the explanations of the generation structure of quarks and leptons and the absence of the electroweak mixings only among the lepton generations, the stability of the proton and the missing of the gravitational interaction, ... etc. The (local) supersymmetry (SUSY) [1], although unclear yet in the low energy particle physics, is the most promising notion for explaining the rationale of beings of all elementary particles including the graviton. As shown by Gell-Mann [2], $SO(8)$ maximally extended supergravity theory (SUGRA) is too small to accommodate all observed particles as elementary fields within the framework of the local gauge field theory. However it may be interesting, even at the risk of the local gauge field theory at the moment, from the viewpoints of simplicity and beauty of nature to attempt the accomodation of all observed elementary particles in a single irreducible representation of a certain group (algebra).

In ref. [3], by extending the group theoretical arguments beyond $N=8$ we have shown that among all $SO(N)$ extended super-Poincaré (SP) symmetry, the massless irreducible representations of $SO(10)$ SP algebra gives minimally and uniqely the framework for the unification of all observed particles and forces. However the fundamental theory has left unknown.

In ref. [4], we have pointed out that the anticommutators of the supercharges of $SO(10)$ SP algebra in the light-cone frame (the massless irreducible representations) can be interpreted as canonical anticommutators of creation and annihilation operators of spin 1/2 fermions and

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that it may indicate the existence of certain fundamental objects which are the constituents of all elementary particles except the graviton. We have identified the fundamental objects with Nambu–Goldstone (N–G) fermions *superons* of the spontaneous breakdown of supertranslation of spacetime. We have proposed *superon-graviton model* (SGM) as a fundamental theory for supersymmetric structure of spacetime and matter by using Volkov–Akulov nonlinear (NL) SUSY action for N–G fermion [5] in the curved spacetime. In this article, with a concise review of ref. [3] and [4] for the selfcontained arguments we study SGM further from the viewpoints of the internal structure of the quarks, leptons and gauge bosons except the graviton. The symmetry breaking of SGM and its cosmological implications are discussed briefly.

2 $SO(10)$ super-Poincaré algebra

In ref. [3] and [4], by noting that 10 generators Q^N ($N = 1, 2, \ldots, 10$) of $SO(10)$ SP algebra are the fundamental represemtations of $SO(10)$ internal symmetry and that $SO(10) \supset SU(5) \supset SU(3) \times SU(2) \times U(1)$ we have decomposed 10 generators Q^N of $SO(10)$ SP algebra as follows with respect to $SU(5)$

$$\begin{aligned} \underline{10} &= \underline{5} + \underline{5}^* \\ &= \left\{ \left(\underline{3}, \underline{1}; -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3} \right) + (\underline{1}, \underline{2}; 1, 0) \right\} + \left\{ \left(\underline{3}^*, \underline{1}; \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) + (\underline{1}, \underline{2}^*; -1, 0) \right\}, \end{aligned} \tag{1}$$

where we have specified ($\underline{SU(3)}$, $\underline{SU(2)}$; electric charges). This assignment is the extention of that of $SO(8)$ SUGRA of Gell-Mann and interestingly coincides with $\underline{5}$ of $SU(5)$ GUT of Georgi and Glashow [6]. To obtain a smaller single irreducible representation we have studied the massless representation. For massless case the little algebra for the supercharges in the light-like frame $P_\mu = \epsilon(1, 0, 0, 1)$ becomes after a suitable rescaling

$$\left\{ Q^M_\alpha, Q^N_\beta \right\} = \left\{ \bar{Q}^M_{\dot{\alpha}}, \bar{Q}^N_{\dot{\beta}} \right\} = 0, \qquad \left\{ Q^M_\alpha, \bar{Q}^N_{\dot{\beta}} \right\} = \delta_{\alpha 1} \delta_{\dot{\beta} \dot{1}} \delta^{MN}, \tag{2}$$

where $\alpha, \beta = 1, 2$ and $M, N = 1, 2, \ldots, 5$. Note that the spinor charges Q^M_1, $\bar{Q}^M_{\dot{1}}$ satisfy the algebra of annihilation and creation operators respectively and can be used to construct a finite-dimensional supersymmetric Fock space with positive metric. It is natural to identify the graviton with the Clifford vacuum $\mid \Omega(\lambda)\rangle$ ($SO(10)$ singlet but not necessarily the lowest energy state) satisfying $Q^M_\alpha \mid \Omega(\lambda)\rangle = 0$, for the adjoint representation with helicity ± 1 appears automatically. We obtain $2 \cdot 2^{10}$ dimensional irreducible representation of the little algebra (2) of SO(10) SP algebra as follows: $\left[\underline{1}(+2), \underline{10}\left(+\frac{3}{2}\right), \underline{45}(+1), \underline{120}\left(+\frac{1}{2}\right), \underline{210}(0), \underline{252}\left(-\frac{1}{2}\right), \underline{210}(-1), \underline{120}\left(-\frac{3}{2}\right), \underline{45}(-2), \underline{10}\left(-\frac{5}{2}\right), \underline{1}(-3)\right]$ + [CPT-conjugate], where $\underline{d}(\lambda)$ represent $SO(10)$ dimension $\underline{d}$ and the helicity λ.

3 Superon quintet model (SQM) for matter

3.1 Particles in SQM

By noting that the helicities of these states are automatically determined by $SO(10)$ SP algebra and that Q^M_1 and $\bar{Q}^M_{\dot{1}}$ satisfy the algebra of the annihilation and the creation operators for the spin $\frac{1}{2}$ particle, we speculate boldly that these states spanned upon the mathematical (not the physical true vacuum with the lowest energy) Clifford vacuum $\mid \Omega(\pm 2)\rangle$ are the *relativistic* (gravitationally induced) massless composite eigenstates made of the fundamental massless object Q^N *superon* with spin $\frac{1}{2}$. Therefore we regard (1) as *a superon-quintet* and *an antisuperon-quintet*. The unfamiliar identification of the generators of $SO(10)$ SP algebra with the fundamental objects is discussed later. Now we envisage the Planck scale physics as follows.

Nature (spacetime and matter) have the symmetric structure described by $SO(10)$ SP algebra at (above) the Planck energy scale, where the gravity dominates and induces the spontaneous breakdown of the supertranslation of spacetime acompanying the pair production of N–G formions (the superon-quintet and the antisuperon-quintet) from the vacuum in such a way as all the possible nontrivial multiplicative combinations of superons span the massless irreducible representations (i.e. eigenstates) of $SO(10)$ SP algebra. As shown later, the interaction of superons is highly nonlinear.

Now from the viewpoints of the superon-quintet model (SQM) for matter we can study more concretely the physical meaning of the results obtained in ref. [3] and [4].

Hereafter we use the following symbols for superons Q^N $(N = 1, 2, \ldots.10)$.

For the superon-quintet $\underline{5}$: $\left[\left(\underline{3}, \underline{1}; -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right), (\underline{1}, \underline{2}; 1, 0)\right]$, we use

$$[Q_a, Q_m;\ a = 1, 2, 3; m = 4, 5] \tag{3}$$

and for the antisuperon-quintet $\underline{5}^*$: $\left[\left(\underline{3}^*, \underline{1}; +\frac{1}{3}, +\frac{1}{3}, +\frac{1}{3}\right), (\underline{1}, \underline{2}^*; -1, 0)\right]$, we use

$$[Q_a^*, Q_m^*;\ a = 1, 2, 3; m = 4, 5]\,, \tag{4}$$

i.e. Q_a and Q_m represent color- and electroweak-components of superon quintets respectively. Accordingly all the states are specified explicitly with respect to ($SU(3)$, $SU(2)$; electric charges). In order to see the low energy particle contents, suppose that through the symmetry breaking:

$$[SO(10)\text{ SP symmetry}] \longrightarrow [\cdots] \longrightarrow [SU(3) \times SU(2) \times U(1)] \longrightarrow [SU(3) \times U(1)]$$

which is discussed later, the states with higher helicities $\left(\pm 3, \pm\frac{5}{2}, \pm 2, \pm\frac{3}{2}, \pm 1\right)$ of $SO(10)$ SP algebra absorb the lower helicity states$\left(\pm 2, \pm\frac{3}{2}, \pm 1, \pm\frac{1}{2}, 0\right)$ in $SU(3) \times SU(2) \times U(1)$ invariant way and become massive as many as possible in so far as the SM with three generations of quarks and leptons survive in the residual massless states. We have carried out the recombinations among $2 \cdot 2^{10}$ helicity states and found surprisingly just three generations of quarks and leptons of the SM appear in the surviving massless states of the fermions. At least one new spin $\frac{3}{2}$ lepton-like (gravitino) electroweak doublet (ν_Γ, Γ^-) with the mass of the electroweak scale is predicted [3]. ((ν_Γ, Γ^-) may be included in $\underline{5}$ of $\underline{1}0 = \underline{5} + \underline{5}^*$ of helicity $\pm\frac{3}{2}$ state.)

Towards the construction of the fundamental theory of SQM and for surveying the physical (phenomenological) implications of the superons for the unified gauge models (SM and GUTs) it is very important to understand all the gauge and the Yukawa couplings of the unified gauge models in terms of the superon pictures. For simplicity we neglect the mixing between superons and take the following left-right symmetric assignment for quarks and leptons by using the conjugate representations naively , i.e. $(\nu_l, l^-)_R = (\bar{\nu}_l, l^+)_L$, etc. [3].

For three generations of leptons $[(\nu_e, e), (\nu_\mu, \mu), (\nu_\tau, \tau)]$, we take

$$[(Q_m \varepsilon_{ln} Q_l^* Q_n^*), (Q_a Q_a^* Q_m^*), (Q_a Q_a^* Q_b Q_b^* Q_m^*)] \tag{5}$$

and the conjugate states respectively.

For three generations of quarks $[(u, d), (c, s), (t, b)]$, we have *uniquely*

$$[(\varepsilon_{abc} Q_b^* Q_c^* Q_m^*), (\varepsilon_{abc} Q_b^* Q_c^* Q_l \varepsilon_{mn} Q_m^* Q_n^*), (\varepsilon_{abc} Q_a^* Q_b^* Q_c^* Q_d Q_m^*)] \tag{6}$$

and the conjugate states respectively.

For $[SU(2) \times U(1)$ gauge bosons], we have from $\underline{2} \times \underline{2}^*$

$$\left[-Q_4 Q_5^*, \frac{1}{\sqrt{2}}(Q_4 Q_4^* - Q_5 Q_5^*), Q_5 Q_4^*; \frac{1}{\sqrt{2}}(Q_4 Q_4^* + Q_5 Q_5^*)\right].$$

For $[SU(3)$ gluons], we have

$$\left[Q_1 Q_3^*, Q_2 Q_3^*, Q_1 Q_2^*, \frac{1}{\sqrt{2}}(Q_1 Q_1^* - Q_2 Q_2^*), Q_2 Q_1^*, \frac{-1}{\sqrt{6}}(2Q_3 Q_3^* - Q_2 Q_2^* - Q_1 Q_1^*), Q_3 Q_2^*, Q_3 Q_1^*\right].$$

For $[SU(2)$ Higgs Boson], we have $[\varepsilon_{abc} Q_a Q_b Q_c Q_m]$ and the conjugate state.

For $[(X, Y)$ leptoquark bosons] in GUTs, we have $[Q_a^* Q_m]$ and the conjugate state.

For [a color- and $SU(2)$-singlet neutral gauge boson] from $\underline{3} \times \underline{3}^*$ (which we call simply S boson to represent the singlet) we have $Q_a Q_a^*$.

The specification of (X, Y) gauge boson is important for the proton decay in $SU(5)$ GUT. The specification of S boson may be interesting as an additional $U(1)$ of the gauge structure $SU(3) \times SU(2) \times U(1) \times U(1)$ of SUSY SM. As shown later S boson plays crucial roles in the process concerning the third generation of quarks and leptons. We have considered only two-superons states $\underline{45}$ of the adjoint representation of $SO(10)$ SP algebra for the vector gauge bosons.

3.2 Superon diagrams

Now in order to see the physical implications of SQM for SM (GUTs) for matter we try to interpret the Feynman diagrams of SM (GUTs) in terms of the Feynman diagrams of SQM. The superon-line Feynman diagrm of SQM is obtained by replacing the single line in the Feynman diagram of the unified gauge models by the corresponding multiple superon lines. To translate the vertex of the Feynman diagram of the unified gauge models into that of SQM, we assume that the superon-antisuperon pair creation and pair annihilation within a single state for a quark, a lepton and a (gauge) boson (i.e. within a single $SO(10)$ SP eigenstate) are rigorously forbidden. This rule seems natural because every state is an irreducible representation of $SO(10)$ SP algebra and is prohibitted from the decay without any remnants, i.e. without the interaction between the superons contained in another state. As discussed later this means the absence of the excited states of quarks, leptons, gauge bosons despite their compositeness. Here we just mention that all the states necessary to the SM and GUTs with three genarations of quarks and leptons appear up to five-superons states (i.e. one half of the full occupation ten-superons). As mentioned later this observation may be crucial for the spontaneous symmetry breaking with a large mass hierarchy. At the moment we naievely assume that all exotic states besides higher spin states composed of more than five superons have large masses in the low energy.

Now the translation is unique and straightforward. We see that in the Yukawa coupling of SQM the observed quark (lepton) interacts with the Higgs boson and a new quark(lepton) which is exotic with respect to $SU(2)$ and/or spin. Then the Yukawa coupling of SM (GUTs) can be reproduced effectively only in the higher orders of the Yukawa couplings of SQM, which gives potentially the Yukawa coupling of SM (GUTs) a small factor of the order of the inverse of the large mass of the exotic quark and lepton. This mechanism may be the origin of CKM mixing matrix for the quark sector but may be dangerous so far for the lepton sector because of the disastrous violations of the lepton family quantum numbers by the lepton mixings. However we find that at every gauge coupling vertex there is a stringent selection rule for generations which is characteristic to SQM, for each generation is identical only with respect to $SU(3) \times SU(2) \times U(1)$ quantum numbers but has another superon content corresponding to the flavor quantum number. This selection rule is the matching of the superons, i.e. the superon number conservation, at the gauge coupling vertex. For the quark sector, surprisingly, the selection rule respects the CKM mixing of the Yukawa coupling sector and maintains the successful gauge current structure of SM. While for the lepton sector, remarkably the selection rule prohibits basically the lepton flavor changing electroweak currents between lepton generations at the tree level and reproduces the success of SM. We regard that SQM can explain the absence of the lepton-flavor-mixing in the *electroweak* gauge interaction.

As a few examples of the gauge interactions and the selection rule at the gauge coupling vertex we have discussed the following typical processes [4], i.e. (i) β decay: $n \longrightarrow p + e^- + \bar{\nu}_e$, (ii) $\pi^0 \longrightarrow 2\gamma$, (iii) the proton decay: $p \longrightarrow e^+ + \pi^0$, (iv) a flavor changing neutral current process (FCNC): $K^+ \longrightarrow \pi^+ + \nu_e + \bar{\nu}_e$ and (v) an advocated typical process of the (non-gauged) compositeness: $\mu \longrightarrow e + \gamma$.

Now the translation is unique and straightforward. For the processes (i) and (ii) we can draw the corresponding similar tree-like superon line diagrams easily, where the triangle-like superon diagram does not appear. For the process (iii) we examine the Feynman diagrams for the proton decay of GUTs and find that the corresponding superon line diagrams do not exist due to the selection rule, i.e. the mismatch of the superons contained in the quarks(u and d) and the gauge bosons(X and Y) at the gauge coupling vertices. This means that irrespective of the massses of the gauge bosons the proton is stable at the tree level against $p \longrightarrow e^+ + \pi^0$. For FCNC process (iv) the penguin-type and the box-type superon line diagrams are to be studied corresponding to the penguin- and box-Feynmann diagrams for $K^+ \longrightarrow \pi^+ + \nu_e + \bar{\nu}_e$ of GUTs. Remarkablely the superon line diagrams which have only the u and c quarks for the internal quark line exist due to the selection rule and GIM mechanism of the SM is reproduced. The third generation t quark for the internal line is decoupled due to the selection rule. This is the indication of the strong suppression of the FCNC process, $K^+ \longrightarrow \pi^+ + \nu_e + \bar{\nu}_e$. This simple mechanism may hold in general for FCNC processes. For the process (v) the corresponding tree-like superon line diagram does not exist due to the selection rule at the gauge coupling vertex, i.e. $\mu \longrightarrow e + \gamma$ decay mode is absent at the tree-level in the superon (composite) model. The process $\tau \longrightarrow e(\mu) + \gamma$ is suppressed similarly.

As for the CP-violation, the mixing K^0-$\overline{K^0}$ is natural in SQM, for remarkably K^0 and $\overline{K^0}$ have *the same superon contents* (i.e. indistinguishable and superposing at the superon level) but have the different superon combinations distinguished by the interactions which lead to mass differences. GIM mechanism works for the superon picture of K^0-$\overline{K^0}$ mixing box diagram of SM but remarkably t quark (the third generation) decouples due to the selection rule at the gauge coupling vertices. However in SQM there is another higher order box (ladder-like) diagram contributing to K^0-$\overline{K^0}$ mixing amplitude, where S gauge boson emitted by the transition $(u, d) \leftrightarrow (t, b)$ and t quark play crucial (dominant) roles besides W boson. The relative phase of these two amplitudes may be an origin of CP-violation in the neutral K-meson decay. This mechanism of CP-violation without requiring complex gauge coupling constants seem natural from the viewpoint of the unification of all forces including gravity (which is a singlet, neutral and universal force) in a (semi)simple gauge group with one universal gauge coupling constant. It is interesting that t quark (the third generation of quarks) which appears automatically in SQM is needed for CP-violation in SQM context. The mixings B^0-$\overline{B^0}$ and D^0-$\overline{D^0}$ are natural in the same reason but the preliminary analyses suggest the similar new mechanisms for mixing and CP-violation characteristic of the SQM. SQM explains qualitatively the Weinberg angle (i.e. the mixing of the neutral electroweak gauge bosons) and predicts the mixing of a gluon and S boson by the same reason. The low energy $SU(3)$ color symmetry may be a residual gauge symmetry like $U(1)$ electromagnetic gauge symmetry in SM. As for the charmless nonleotonic B decay [7] in SQM the transition $(t, b) \leftrightarrow (c, s)$ occurrs not at the tree level of the weak charged current but at the higher orders of the gauge couplings due to the selection rule for the quark sector, where the transition $(t, b) \leftrightarrow (c, s)$ is achieved by the emissions of S boson and W boson and may give an explanation of the excess of the charmless (or the suppression of the charm mode) nonleotonic B decay. Furthermore for the lepton sector amazingly S gauge boson induces the transition *only* $\nu_\mu \leftrightarrow \nu_\tau$ (i.e. between the second and the third generation) at the tree level due to the selection rule, which may solve simply and naturally the ν_μ deficit problem of the atomospheric neutrino [8].

Next we just mention the excited states of quarks, leptons and gauge bosons. As stated before these particles (i.e. the massless eigenstates of $SO(10)$ SP symmetry) do not have the low energy excited states in SQM, because each particle is a single (massless) eigenstates of $SO(10)$ SP symmetry composed of superons and transits to *another eigenstate* through the interaction, i.e. through the absorption or the emission of superons (i.e. eigenstates).

4 Superon-graviton model (SGM) for spacetime and matter

4.1 Fundamental action for SGM

Finally we consider the fundamental theory of superon-graviton model (SGM) for supersymmetric spacetime and matter. In carrying through the canonical quantization of the elementary N–G spinor field $\psi(x)$ of two dimensional Volkov–Akulov model [4] of the NL SUSY, we have shown that the supercharges Q given by the supercurrents

$$J^\mu(x) = \frac{1}{i}\sigma^\mu \psi(x) - \kappa \; \{\text{the higher orders of } \kappa,\ \psi(x) \text{ and } \ \partial\psi(x)\} \tag{7}$$

obtained by the ordinary Noether procedures can satisfy the super-Poincaré algebra at the cnonically quantized level [9], where κ is a fundamental volume of the superspace of the NL SUSY with the mass dimension -2 (for the two dimensional case). Remarkably (7) means the field-current identity between the fundamental Nambu–Goldstone spinor $\psi(x)$ field and the supercurrent, which justify our basic assumption that the generator(supercharge) Q^N $(N = 1, 2, \ldots, 10)$ of $SO(10)$ SP algebra for the massless case represents the fundamental object *superon with spin* $\frac{1}{2}$. And our qualitative arguments are valid in the leading order for the small κ and/or in the low energy (momentum) as seen from (7). Therefore we speculate that the fundamental theory of SQM for matter is $SO(10)$ NL SUSY and that the fundamental theory of SGM for spacetime and matter at (above) the Planck scale is $SO(10)$ NL SUSY in the curved spacetime which corresponds to the Clifford vacuum $\mid \Omega(\pm 2)\rangle$. We regard that all the helicity-states of $SO(10)$ SP algebra including the observed quarks, leptons and gauge bosons except the graviton are the relativistic (gravitational) composite massless states of N–G fermion superons. SGM may show that the relativistic version of the composite (quark) model [11] of matter is realized as eigenstates of $SO(10)$ SP algebra at the superon level.

We propose the following Lagrangian as the fudamental theory of SGM of spacetime and matter.

$$L_{SGM} = -\frac{c^3}{16\pi G} e(R + \Lambda)|W|, \tag{8}$$

$$|W| = \det W^\nu_\mu = \det\left(\delta^\nu_\mu + \kappa T^\nu_\mu\right), \qquad T^\nu_\mu = \frac{1}{2i}\sum_{i,j=1}^{10}\left(\bar{s}^i O_{ij}\gamma_\mu D^\nu s^j - D^\nu \bar{s}^i \gamma_\mu O_{ij} s^j\right), \tag{9}$$

where κ is a fundamental volume of the superspace of the NL SUSY with the mass dimension -4, $e = \det e^a_\mu$, $D_\mu = \partial_\mu + \frac{1}{2}\omega^{ab}_\mu \sigma_{ab}$ and R and Λ are the scalar curvature and the cosmological constant, respectively. O_{ij} is a 10×10 unitary matrix representing the mixing among the superons, which may be probable but unpleasant from the elementary nature of the superon. The multiplication of the Einstein-Hilbert action by SQM action $|W|$ in (8) is essential and unique for the fundamental theory if we require that (i) it should be reduced to $SO(10)$ NL SUSY *a la* Volkov–Akulov in the flat spacetime by taking only $R \to 0$, (ii) also to the Einstein–Hilbert action (i.e. Clifford vacuum action) by taking the superonless limit $s^i \to 0$, (iii) except the graviton all fields participating in the superHiggs(recombination) mechanism should be the composites of superons and (iv) the action (8) should be invariant under the global $SO(10)$ NL SUSY,

$$\delta s^M = \frac{\epsilon^M}{\sqrt{\kappa}} - 2i\sqrt{\kappa}\left(\bar{\epsilon}^L \gamma^\mu s^L\right) D_\mu s^M, \tag{10}$$

$$\delta e^a{}_\mu = i\sqrt{\kappa}\left(\bar{\epsilon}^L \gamma^\rho s^L\right) \mathcal{D}_\rho e^a{}_\mu, \tag{11}$$

where ϵ^M $(M = 1, 2, \ldots, 10)$ is a constant spinor parameter with spin 1/2. (8) is manifestly invariant at least under the general coordinate transformation and global $SO(10)$. Furthermore the all order invariance of (8) under the global $SO(10)$ NL SUSY (10) and (11) in the similar sense of ref. [12] and [13] can be anticipated, which may be included in the scope of ref. [12] and [13]. The states with helicity ± 3, $\pm\frac{5}{2}$ and ± 2 (except the graviton) made of 10-, 9- and 8-superons appear afetr specifying the contorsion in the spin conection $\omega^\mu_{ab}(e^\mu_a, s^i)$ [10]. The fundamental Lagrangian (8) can be rewritten in the following simple form $L_{SGM} = -\frac{c^3}{16\pi G} n(R + \Lambda)$, where $n = \det n^a_\mu = \det(e^a_\nu W^\nu_\mu)$.

4.2 Symmetry breaking of SGM

As for the abovementioned spontaneous symmetry breaking it is urgent to study the structure of the true vacuum of (8). To see clearly the (low energy) mass spectrum of the particles spanned upon the true vacuum, we should convert the highly nonlinear SGM Lagrangian (8) into the equivalent linearized broken SUSY $SO(10)$ (or SM) Lagrangian. The orders of the mass scales of spontaneous SUSY and $SO(10)$ breaking are given by κ and Λ. The low-energy structure of the linearized broken SUSY Lagrangian should involve GUTs, at least the SM with three generations. For carrying through the complicated scenario it is encouraging that the linearlization of such a nonlinear fermionic system was already carried out explicitly [12, 13]. They investigated in detail the conversions between $N = 1$ NL SUSY (Volkov–Akulov) model and the equivalent linear (broken) $N = 1$ SUSY Lagrangian in the flat spacetime. The extension of the generic and the systematic arguments by using the superspace [13] may be useful for the linerization of SGM. From the mathematical viewpoint an equivalent linear theory would exist. It is a challenge to pursue the scenario. We expect that by taking *non-perturbatively* the true vaccum of (8) the conversions into the linear representation is achieved, where SUSY is broken spontaneously at the tree level and the bosonic and the fermionic high-spin massless states turn out to be massive states. This may be only the possible way to circumvent the no-go theorem [14] and to accomodate successfully high spin (massless) states in the local field theoretical GUTs. The massless tensor fields (states) in the adjoint representation $\underline{45}$ of $SO(10)$ may play important roles in the early spontaneous symmetry breakings: $[SO(10)\ \mathrm{SP}] \longrightarrow [\cdots] \longrightarrow [SU(3) \times SU(2) \times U(1)] \longrightarrow [SU(3) \times U(1)]$.

By generalizing the idea of the strong gravity [15] all tensor fields of the adjoint representation can have $U(M) \times U(N) \times \cdots$ invariant masses by the spontaneous symmetry breaking induced by the Higgs potential analogue gauge invariant self-interactions, provided these tensor fields are the gauge fields of the nonlinear realization of $SL(2M,C) \times SL(2N,C) \times \cdots$ with $45 = M^2 + N^2 + \cdots$. $GL(4,R)$ does not break spontaneously. $SL(12,C) \times SL(6,C)$ and $[SL(6,C) \times SL(4,C) \times SL(2,C)]^3$ which allow $U(6) \times U(3)$ and $[U(3) \times U(2) \times U(1)]^3$ invariant masses respectively are interesting from simplicity and may be relevant to SGM scenario. Especially this mechanism of the spontaneous symmetry breaking is worthwhile to be studied in detail to see whether it generates large masses spontaneously to all the states composed of more than five superons that are irrelevant to the (low energy) GUTs as mentioned before. It is very interesting if we can regard the yet hypothetical SGM (8) may be for the unified gauge models (SM and GUTs) what the BCS (electron-phonon) theory is for the Landau–Ginzburg theory of the superconductivity. The boundary condition (the global structure) of spacetime(universe) may be crucial.

Alternatively, disregarding the linearlization it is interesting from the purely phenomenological viewpoint to fit all the decay data of leptons and low lying hadrons in terms of the quark model [16] analogue $SO(10)$ superon current algebra including the higher order terms of (7), which potentially gives all the transition matrix elements in terms of superon pictures and may describe the nonlinear superon dynamics at the short ditance of the spacetime and may give a qualitative test of SQM [4]. Also it is worth studying other assignment for quarks and leptons than $R = L^*$ symmetric SQM ((5) and (6)). The left-right assymmetric assignment for quarks and leptons is also possible from only group theoretical investigations.

The cosmological implications of SGM (8) is also worth studying. Because SGM (8) describes a pre-history of quark-lepton era, i.e. N–G superons are created (i.e. pre-big bang is ignited) by the spontaneous breakdown of the supertranslation of spacetime and $SO(10)$ SP invariant massless superon composite states (quark-lepton era) are spanned, which lead to the big bang of the universe inducing the spontaneous breakedown of $SO(10)$ SUSY by the interactions among the massless composite states.

Finally we just mention that in SGM the singularities of the gravitational collapse may be prohibitted by the phase transition to the N-G pahase achieved *gravitationally*. It is a challenge to test these conjecture quantitatively by starting from the SGM action (8), where the higher order terms of κ and momentum (derivatives) become dominant.

5 Conclusion

We have shown by the qualitative arguments that the unified gauge models (SM and GUTs) are strengthened or revived by taking account of the topology of the superon diagram of SGM, while drawing the superon diagram (i.e. extracting the low energy physical implications) of SGM is guided by the Feynman diagram of SM (GUTs). We regard that these beautiful complimentality between the gauge unified models (SM and GUTs) and SGM may be an evidence of $SO(10)$ SP symmetric structure of spacetime and matter behind the gauge models, i.e. an evidence of the superon-quintet hypothesis for matter (SQM) and superon-graviton model (SGM) (8) for spacetime and matter. The experimental searches for a predicted new spin $\frac{3}{2}$ lepton-type (gravitino) doublet (ν_Γ, Γ^-) with the mass of the electroweak scale [3] and a new gauge boson S are important. Also SQM predicts two doubly-charged, electroweak- and color-singlet (unconfined) particles E^{2+} and M^{2+} with spin $\frac{1}{2}$ [3]. Their masses are left unknown within this study. From the present experimental data for τ^- decay S boson mass seems much larger than the W boson mass. The clear signals of (ν_Γ, Γ^-) may be similar to the top-quark pair production event without jets production, i.e. $e + \bar{e} \longrightarrow l + \bar{l}$ + *missing large* P_T *(energy)* [3]. The evidence of S boson may be seen already and will become clear in the high energy B meson experiment.

Besides those interesting aspects of SGM (8), much more open questions are left.

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