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Canonical Realization of the Poincaré Algebra for a Relativistic System of Charged Particles Plus Electromagnetic Field

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The procedure of reducing of canonical field degrees of freedom for a system of charged particles plus electromagnetic field in the constraint Hamiltonian formalism is developed up to the first order in the coupling constant expansion. The canonical realization of the Poincaré algebra in the terms of physical variables is found. The relation between covariant and physical particle variables in the Hamiltonian description is studied.

1 Introduction

Usually, an interaction within a system of N charged particles is described by means of the electromagnetic field with its own degrees of freedom represented by the 4-potential $A_\mu(x)$, $x \in \mathbb{M}_4$, over the Minkowski space-time[1] [1, 2, 3].

Such a system of particles plus electromagnetic field is completely determined by the following action

$$\begin{aligned} S = &-\int \sum_{a=1}^{N} m_a \sqrt{u_a^\mu(\tau) u_{a\mu}(\tau)}\, d\tau \\ &- \int \sum_{a=1}^{N} e_a u_a^\mu(\tau) A_\mu[x_a(\tau)] d\tau - \frac{1}{16\pi} \int F_{\mu\nu}(x) F^{\mu\nu}(x) d^4x. \end{aligned} \tag{1.1}$$

Here $F_{\mu\nu}(\mathbf{x},\tau) = \partial_\mu A_\nu(\mathbf{x},\tau) - \partial_\nu A_\mu(\mathbf{x},\tau)$ is the field strength; m_a, e_a are the mass and the charge of particle a, respectively, $u_a^\mu(\tau) = dx_a^\mu(\tau)/d\tau$, and $\tau \mapsto x_a^\mu(\tau)$ gives parametric equation of the particle world line in the Minkowski space-time.

But often it is desirable to exclude the field degrees of freedom and formulate the description of the system only in the terms of particle variables. The elimination of the field variables can be performed exactly in the action (1.1). This leads to the time-symmetric Wheeler–Feynman electrodynamics [4, 5] with the Fokker action. Nonlocality of the Fokker-type actions result in serious difficulties in transition to the Hamiltonian description [6]. The same problems occur when Fokker-type action is replaced by the single-time Lagrangian depending on the infinity order derivatives of the particle coordinates [7, 8]. Although this problem can be solved within the corresponding approximation schemes [6, 9]. Here we shall consider an alternative way to

[1] The Minkowski space-time $\mathbb{M}_4$ is endowed with a metric $\|\eta_{\mu\nu}\| = \mathrm{diag}(1,-1,-1,-1)$. The Greek indices $\mu,\nu,\ldots$ run from 0 to 3; the Latin indices from the middle of alphabet, $i,j,k,\ldots$ run from 1 to 3 and both types of indices are subject of the summation convention. The Latin indices from the beginning of alphabet, a, b, label the particles and run from 1 to N. The sum over such indices is indicated explicitly.

overcome these difficulties. The main idea consists in the elimination of field degrees of freedom *after* transition to the Hamiltonian description of the particles plus field theory.

Then, we must solve the field Hamiltonian equations of motion and make the canonical transformation to the free field variables. After that the canonical free field variables will be eliminated by means of canonical constraint method. This procedure gives us the canonical realization of the Poincaré algebra in the terms of particle variables.

However, the field equations of motion are nonlinear, so we will find the solutions of these equations and other relations in the first order in the coupling constant expansion. Therefore, the Lienard–Wiechert potentials will be the expected solutions of the field equations.

The present paper is organized as follows. In Section 2, there is a canonical realization of the Poincaré algebra for the system of N point charged particles plus electromagnetic field (field theory).

In Section 3, we find solutions of the field equations of motion of first order in the coupling constant expansion, make canonical transformation to the free field variables and eliminate them with help of constraints. We obtain a canonical realization of the Poincaré generators depending on the particle coordinates and momenta. It is shown that the new generators form an algebra. There is a study of relations between new canonical coordinates and positions of particles of the reduced system.

The conclusions in Section 4 contain some final remarks and the outline of future research.

2 Field theory Poincaré generators

Action (1.1) for the system of field and particles is manifestly Poincaré-invariant. Its invariance leads to the conservation of the symmetric energy-momentum tensor [1, 3]

$$\begin{aligned}\theta^{\mu\nu}(z) &= \sum_{a=1}^{N} m_a \int \frac{u_a^\mu(\tau)u_a^\nu(\tau)}{\sqrt{u_a^2(\tau)}}\delta^4(x_a(\tau)-z)d\tau \\ &+ \frac{1}{4\pi}\left(-F^{\mu\lambda}(z)F^{\nu}{}_{\lambda}(z)+\frac{1}{4}F_{\lambda\sigma}(z)F^{\lambda\sigma}(z)\eta^{\mu\nu}\right).\end{aligned} \tag{2.1}$$

For transition to the Hamiltonian description we use $3+1$ splitting of the Minkowski space-time corresponding to the instant form of dynamics [10, 11]. In geometric approach the instant form of dynamics is determined by foliation of the Minkowski space-time by the hyperplanes $x^0=\tau$, $\tau\in\mathbb{R}$.

In this case the Lagrangian of the system is

$$\begin{aligned}L = &-\sum_{a=1}^{N} m_a\sqrt{1-\mathbf{u}_a^2(\tau)}d\tau - \sum_{a=1}^{N} e_a\left(u_a^i(\tau)A_i[\mathbf{x}_a(\tau),\tau]+A_0[\mathbf{x}_a(\tau),\tau]\right) \\ &-\frac{1}{16\pi}\int F_{\mu\nu}(\mathbf{x},\tau)F^{\mu\nu}(\mathbf{x},\tau)d^3x d\tau,\end{aligned}$$

where $\mathbf{x}_a=(x_a^i)$, $\mathbf{u}_a=(u_a^i)$, $\mathbf{A}(\mathbf{x},\tau)=(A^i(\mathbf{x},\tau))$.

The canonical momenta are given by

$$p_{ai}(\tau) = -\frac{\partial L}{\partial u_a^i} = \frac{m_a u_{ai}(\tau)}{\sqrt{1-\mathbf{u}_a^2(\tau)}} + e_a A_i[\mathbf{x}_a(\tau),\tau],$$

$$E^i(\mathbf{x},\tau) = \frac{\delta L}{\delta \dot{A}_i(\mathbf{x},\tau)} = \frac{1}{4\pi}F^{i0}(\mathbf{x},\tau), \qquad E^0(\mathbf{x},\tau) = \frac{\delta L}{\delta \dot{A}_0(\mathbf{x},\tau)} = 0.$$

The canonical and Dirac Hamiltonians are

$$H = \sum_{a=1}^{N} \left[\sqrt{m_a^2 + [\mathbf{p}_a - e_a \mathbf{A}(\mathbf{x}_a)]^2} + e_a A_0(\mathbf{x}_a) \right] + \int \left(\frac{1}{16\pi} F_{ij} F_{ij} + 2\pi E^i E^i - A_0 \partial_i E^i \right) d^3x,$$

$$H_D = H + \int \lambda E^0 d^3x,$$

where λ is the Dirac multiplier.

The basic Poisson brackets are

$$\{x_a^i(\tau), p_{bj}(\tau)\} = -\delta_{ab}\delta_j^i, \qquad \{A^\mu(\mathbf{x},\tau), E^\nu(\mathbf{y},\tau)\} = \eta^{\mu\nu}\delta^3(\mathbf{x}-\mathbf{y}). \tag{2.2}$$

The constraint $E^0(\mathbf{x},\tau) \approx 0$ ($\approx$ means "weak equality" in the sense of Dirac) reflects the gauge invariance of S; its time constancy produces the only secondary constraint, $\partial_i E^i(\mathbf{x},\tau) - \rho(\mathbf{x},\tau) \approx 0$, where $\rho(\mathbf{x},\tau) = \sum_{a=1}^{N} e_a \delta^3(\mathbf{x} - \mathbf{x}_a(\tau))$. The two constraints $E^0(\mathbf{x},\tau) \approx 0, \partial_i E^i(\mathbf{x},\tau) - \rho(\mathbf{x},\tau) \approx 0$ are first class with vanishing Poisson brackets. Therefore, the corresponding conjugate variables $A_0(\mathbf{x},\tau)$, $\int \Delta^{-1}(\mathbf{x}-\mathbf{y})\partial_i A_i(\mathbf{y},\tau) d^3y$, ($\Delta^{-1}(\mathbf{x}) = -1/(4\pi|\mathbf{x}|)$) are arbitrary functions.

Conservation of the energy-momentum tensor (2.1) leads to ten conserved Poincaré generators:

$$P^\mu = \int \theta^{\mu 0}(\mathbf{x},\tau) d^3x, \qquad M^{\mu\nu} = \int \left(x^\mu \theta^{\nu 0}(\mathbf{x},\tau) - x^\nu \theta^{\mu 0}(\mathbf{x},\tau) \right) d^3x.$$

They can be rewritten in terms of canonical variables as

$$P^0 = \sum_{a=1}^{N} \sqrt{m_a^2 + [\mathbf{p}_a - e_a \mathbf{A}(\mathbf{x}_a)]^2} + \int \left(\frac{1}{16\pi} F_{ij} F_{ij} + 2\pi E^i E^i \right) d^3x,$$

$$P^k = \sum_{a=1}^{N} \left[p_a^k - e_a A^k(\mathbf{x}_a) \right] + \int E^l F^{lk} d^3x,$$

$$M^{k0} = \sum_{a=1}^{N} x_a^k \sqrt{m_a^2 + [\mathbf{p}_a - e_a \mathbf{A}(\mathbf{x}_a)]^2} + \int \left(\frac{1}{16\pi} F_{ij} F_{ij} + 2\pi E^i E^i \right) x^k d^3x - \tau P^k,$$

$$M^{ik} = \sum_{a=1}^{N} (x_a^i p_a^k - x_a^k p_a^i) + \int \left(x^k E^l \partial^i A^l - x^i E^l \partial^k A^l \right) d^3x + \int \left(A^i E^k - A^k E^i \right) d^3x,$$

where $\mathbf{p}_a = (p_a^i)$. They satisfy the commutation relations of the Poincaré algebra,

$$\{P^\mu, P^\nu\} = 0, \qquad \{P^\mu, M^{\nu\lambda}\} = \eta^{\mu\nu} P^\lambda - \eta^{\mu\lambda} P^\nu,$$

$$\{M^{\mu\nu}, M^{\lambda\sigma}\} = -\eta^{\mu\lambda} M^{\nu\sigma} + \eta^{\nu\lambda} M^{\mu\sigma} - \eta^{\nu\sigma} M^{\mu\lambda} + \eta^{\mu\sigma} M^{\nu\lambda},$$

in terms of the Poisson brackets (2.2).

3 Reduction of field degrees of freedom

The equations of motion in first order in the coupling constant expansion are

$$\dot{x}_a^i = \frac{p_a^i}{\sqrt{m_a^2+\mathbf{p}_a^2}} + \frac{e_a \Pi_a^{ij}}{\sqrt{m_a^2+\mathbf{p}_a^2}} A_j(\mathbf{x}_a), \qquad \dot{p}_{ai} = \partial_{ai} A_j(\mathbf{x}_a) \frac{e_a p_a^j}{\sqrt{m_a^2+\mathbf{p}_a^2}} + e_a \partial_{ai} A_0(\mathbf{x}_a),$$

$$\dot{A}_i = -4\pi E_i + \partial_i A_0, \qquad \dot{E}^i = -j^i - \frac{\Delta}{4\pi} A^i - \frac{1}{4\pi}\partial^i \left(\partial_j A^j\right), \tag{3.1}$$

$$\dot{E}^0 = \partial_i E^i - \rho \approx 0, \qquad \dot{A}_0 = \lambda,$$

where $\Pi_a^{ij} \equiv \delta^{ij} - p_a^i p_a^j/(m_a^2+\mathbf{p}_a^2)$, $j^i = \sum\limits_{a=1}^{N} \left(e_a p_a^i/\sqrt{m_a^2+\mathbf{p}_a^2}\right) \delta^3(\mathbf{x}-\mathbf{x}_a(\tau))$ is current density, and λ is an arbitrary function of the evolution parameter τ. They are generated by the Dirac Hamiltonian

$$H_D = \sum_{a=1}^{N} \left[\sqrt{m_a^2+\mathbf{p}_a^2} + \frac{e_a p_{ai}}{\sqrt{m_a^2+\mathbf{p}_a^2}} A_i(\mathbf{x}_a) + e_a A_0(\mathbf{x}_a)\right]$$
$$+ \int \left(\frac{1}{16\pi} F_{ij}F_{ij} + 2\pi E^i E^i - A_0 \partial_i E^i\right) d^3x + \int \lambda E^0 d^3x.$$

From Eqs.(3.1) one gets

$$\ddot{A}_k - \Delta A_k - \partial_k \left(\dot{A}_0 - \partial_l A_l\right) = 4\pi j_k.$$

If we require that $\dot{A}_0 - \partial_l A_l = 0$ (the Lorentz gauge), then by using the constraint $\partial_i E^i - \rho \approx 0$ we obtain wave equations for the potentials

$$\ddot{A}_k - \Delta A_k = 4\pi j_k, \qquad \ddot{A}_0 - \Delta A_0 = 4\pi\rho. \tag{3.2}$$

The general solutions of the inhomogeneous Eqs.(3.2) can be presented in the form

$$A_\mu = A_\mu^{\mathrm{rad}} + A_\mu^1,$$

where A_μ^{rad} is the general solution of the corresponding homogeneous equation and

$$\begin{aligned} A_k^1(\mathbf{x},\tau) &= 4\pi \sum_{a=1}^{N} \int D(\tau-\tau'|\mathbf{x}-\mathbf{x}_a(\tau')) \frac{e_a p_{ak}(\tau')}{\sqrt{m_a^2+\mathbf{p}_a^2(\tau')}} d\tau', \\ A_0^1(\mathbf{x},\tau) &= 4\pi \sum_{a=1}^{N} e_a \int D(\tau-\tau'|\mathbf{x}-\mathbf{x}_a(\tau')) d\tau', \end{aligned} \tag{3.3}$$

with the real Green function D which satisfies the equation $(\partial_\tau^2 - \Delta) D(\tau|\mathbf{x}) = \delta(\tau)\delta^3(\mathbf{x})$.

In a given approximation, the expressions (3.3) do not depend on the concrete choice of the Green function (retarded or advanced) and after integration and using free-particle equations we obtain

$$A_k^1(\mathbf{x},\tau) = \sum_{a=1}^{N} \frac{e_a u_{ak}}{\sqrt{[\mathbf{u}_a(\mathbf{x}-\mathbf{x}_a(\tau))]^2 + (1-\mathbf{u}_a^2)(\mathbf{x}-\mathbf{x}_a(\tau))^2}},$$

$$A_0^1(\mathbf{x},\tau) = \sum_{a=1}^{N} \frac{e_a}{\sqrt{[\mathbf{u}_a(\mathbf{x}-\mathbf{x}_a(\tau))]^2 + (1-\mathbf{u}_a^2)(\mathbf{x}-\mathbf{x}_a(\tau))^2}}, \tag{3.4}$$

where $u_a^k = p_a^k/\sqrt{m_a^2+\mathbf{p}_a^2}$ is the free-particle velocity.

Let us perform the canonical transformation to the new field variables:

$$\phi_\mu(\mathbf{x},\tau) = A_\mu(\mathbf{x},\tau) - A_\mu^1(\mathbf{x},\tau),\ \ \chi^k(\mathbf{x},\tau) = E^k(\mathbf{x},\tau) - E_1^k(\mathbf{x},\tau), \tag{3.5}$$

where $E_1^k(\mathbf{x},\tau)$ is

$$E_1^k(\mathbf{x},\tau) = -\frac{1}{4\pi}\left(\dot{A}_1^k(\mathbf{x},\tau) - \partial^k A_0^1(\mathbf{x},\tau)\right)$$
$$= -\frac{1}{4\pi}\sum_{a=1}^{N} \frac{e_a(1-\mathbf{u}_a^2)(x^k - x_a^k(\tau))}{\sqrt{[\mathbf{u}_a(\mathbf{x}-\mathbf{x}_a(\tau))]^2 + (1-\mathbf{u}_a^2)(\mathbf{x}-\mathbf{x}_a(\tau))^2}}.$$

This transformation changes the particle variables: $(x_a^i, p_{ai}) \mapsto (q_a^i, k_{ai})$, where

$$x_a^i = q_a^i + \int \left[\left(\phi_k + \frac{1}{2}A_k^1\right)\frac{\partial E_1^k}{\partial k_{ai}} - \left(\chi^k + \frac{1}{2}E_1^k\right)\frac{\partial A_k^1}{\partial k_{ai}} - E^0\frac{\partial A_0^1}{\partial k_{ai}}\right] d^3x, \tag{3.6}$$
$$p_{ai} = k_{ai} - \int \left[\left(\phi_k + \frac{1}{2}A_k^1\right)\frac{\partial E_1^k}{\partial q_a^i} - \left(\chi^k + \frac{1}{2}E_1^k\right)\frac{\partial A_k^1}{\partial q_a^i} - E^0\frac{\partial A_0^1}{\partial q_a^i}\right] d^3x.$$

In the considered approximation the equalities (3.5) may be put into the form

$$A_\mu = \phi_\mu + A_\mu^1(\mathbf{q}_a,\mathbf{k}_a) = \phi_\mu + A_\mu^1(\mathbf{x}_a,\mathbf{p}_a),$$
$$E^k = \chi^k + E_1^k(\mathbf{q}_a,\mathbf{k}_a) = \chi^k + E_1^k(\mathbf{x}_a,\mathbf{p}_a).$$

Let us note some useful transformation properties of A_k^1, E_1^k, A_0^1

$$\{A_1^l(\mathbf{x},\tau), x^ip^k - x^kp^i - m^{ik}\} = \delta^{il}A_1^k(\mathbf{x},\tau) - \delta^{kl}A_1^i(\mathbf{x},\tau),$$
$$\{A_1^l(\mathbf{x},\tau), x^kp^0 - m^{k0}\} = \delta^{kl}A_0^1(\mathbf{x},\tau),$$
$$\{E_1^l(\mathbf{x},\tau), x^ip^k - x^kp^i - m^{ik}\} = \delta^{il}E_1^k(\mathbf{x},\tau) - \delta^{kl}E_1^i(\mathbf{x},\tau),$$
$$\{E_1^l(\mathbf{x},\tau), x^kp^0 - m^{k0}\} = \frac{1}{4\pi}\left(\partial^l A_1^k(\mathbf{x},\tau) - \partial^k A_1^l(\mathbf{x},\tau)\right),$$
$$\{A_0^1(\mathbf{x},\tau), x^ip^k - x^kp^i - m^{ik}\} = 0,$$
$$\{A_0^1(\mathbf{x},\tau), x^kp^0 - m^{k0}\} = A_1^k(\mathbf{x},\tau),$$

here $p^0 = \sum_{a=1}^{N}\sqrt{m_a^2+\mathbf{k}_a^2}$, $p^i = \sum_{a=1}^{N} k_a^i$, $m^{k0} = \sum_{a=1}^{N} q_a^k\sqrt{m_a^2+\mathbf{k}_a^2}$, $m^{ik} = \sum_{a=1}^{N}(q_a^ik_a^k - q_a^kk_a^i)$.

The conserved quantities after the canonical transformation may be rewritten as

$$P^0 = \sum_{a=1}^{N}\sqrt{m_a^2+\mathbf{k}_a^2} + \frac{1}{2}\sum_{a=1}^{N}\left[\frac{e_ak_a^i}{\sqrt{m_a^2+\mathbf{k}_a}}A_i^1(\mathbf{q}_a) + A_0(\mathbf{q}_a)\right]$$
$$+ \int\left[\frac{1}{16\pi}\Phi_{ij}\Phi_{ij} + 2\pi\chi^i\chi^i\right] d^3x,$$

$$P^i = \sum_{a=1}^{N} k_a^i + \int \chi^k\Phi^{ki} d^3x,$$

$$M^{k0} = \sum_{a=1}^{N} q_a^k \sqrt{m_a^2 + \mathbf{k}_a^2} + \frac{1}{2}\sum_{a=1}^{N} q_a^k \left[\frac{e_a k_a^i}{\sqrt{m_a^2 + \mathbf{k}_a}} A_i^1(\mathbf{q}_a) + A_0(\mathbf{q}_a) \right]$$
$$+ \int \left[\frac{1}{16\pi}\Phi_{ij}\Phi_{ij} + 2\pi\chi^i\chi^i \right] x^k d^3x - \tau P^k,$$
$$M^{ik} = \sum_{a=1}^{N}(q_a^i k_a^k - q_a^k k_a^i) + \int \left(x^k\chi^l\partial^i\phi^l - x^i\chi^l\partial^k\phi^l \right) d^3x + \int \left(\phi^i\chi^k - \phi^k\chi^i \right) d^3x,$$

here $\Phi_{ij} = \partial_i\phi_j - \partial_j\phi_i$.

We reduce field degrees of freedom using the following set of constraints

$$(\Psi_\alpha) = (\phi_k, \chi^k, \phi_0, E^0) \approx 0. \tag{3.7}$$

The constraints depending on gauge A_k, A_0 potentials already contain gauge-fixing constraints. Indeed, the equations of motion lead to the conclusion that A_0 is an arbitrary function. However, the additional constraint $\dot{A}_0 - \partial_l A_l \approx 0$ together with the pure secondary constraint $\partial_i E^i - \rho \approx 0$ defines A_0 as a function of particle variables (see Eq.(3.4)). In this case, $\partial_l A_l$ can be found from the additional constraint in the terms of the coordinates and the momenta of particles too. Using Hodge decomposition for A_k

$$A_k(\mathbf{x},\tau) = A_k^{\perp}(\mathbf{x},\tau) + \partial_k \int \Delta^{-1}(\mathbf{x}-\mathbf{y})\partial_l A_l(\mathbf{y},\tau) d^3y$$
$$\approx A_k^{\perp}(\mathbf{x},\tau) + \partial_k \int \Delta^{-1}(\mathbf{x}-\mathbf{y})\partial_l A_l^1(\mathbf{y},\tau) d^3y,$$

we see that the constraint $A_k - A_k^1 \approx 0$ or $\phi_k \approx 0$ analogously determines $\partial_l A_l$ as $\dot{A}_0 - \partial_l A_l \approx 0$. This means that the gauge-fixing constraints and the constraints Eqs.(3.7) does not need to be separated.

The constraints Eqs.(3.7) are second class, so we can eliminate them by means of use of the Dirac brackets:

$$\{F,G\}_D = \{F,G\} - \int \{F, \Psi_\alpha(\mathbf{x},\tau)\} C^{-1}_{\alpha\beta}(\mathbf{x}-\mathbf{y}) \{\Psi_\beta(\mathbf{y},\tau), G\}\, d^3x d^3y$$
$$= \sum_{a=1}^{N} \left(\frac{\partial F}{\partial q_a^i}\frac{\partial G}{\partial k_a^i} - \frac{\partial G}{\partial q_a^i}\frac{\partial F}{\partial k_a^i} \right),$$

where $\|C^{-1}_{\alpha\beta}(\mathbf{x}-\mathbf{y})\|$ is the inverse matrix to $\|\{\Psi_\alpha(\mathbf{x},\tau), \Psi_\beta(\mathbf{y},\tau)\}\|$.

Thus we obtain the Poincaré generators of the reduced system

$$P^0 = \sum_{a=1}^{N} \sqrt{m_a^2 + \mathbf{k}_a^2} + \frac{1}{2}\sum_{a=1}^{N} \left[\frac{e_a k_a^i}{\sqrt{m_a^2 + \mathbf{k}_a}} A_i^1(\mathbf{q}_a) + A_0(\mathbf{q}_a) \right], \qquad P^i = \sum_{a=1}^{N} k_a^i,$$
$$M^{k0} = \sum_{a=1}^{N} q_a^k \sqrt{m_a^2 + \mathbf{k}_a^2} + \frac{1}{2}\sum_{a=1}^{N} q_a^k \left[\frac{e_a k_a^i}{\sqrt{m_a^2 + \mathbf{k}_a}} A_i^1(\mathbf{q}_a) + A_0(\mathbf{q}_a) \right] - \tau P^k,$$
$$M^{ik} = \sum_{a=1}^{N}(q_a^i k_a^k - q_a^k k_a^i),$$

which act on the particle phase space $T^*\mathbb{R}^{3N}$. They satisfy the commutation relations of the Poincaré algebra in a given approximation.

According to the Eq.(3.6) the covariant particle positions x^i_a are connected with the canonical variables as

$$x^i_a = q^i_a + \frac{1}{2}\int \left[A^1_k \frac{\partial E^k_1}{\partial k_{ai}} - E^k_1 \frac{\partial A^1_k}{\partial k_{ai}} \right] d^3x. \tag{3.8}$$

These relations cannot be complemented to the canonical transformation to the reduced phase space $T^*\mathbb{R}^{3N}$ in full accordance with the famous no-interaction theorem [12]. It can be verified directly that in a given approximation the expression (3.8) satisfies the world line condition

$$\{x^i_a, M^{k0}\}_D = \{x^i_a, P^0\} x^k_a - \tau\delta^{ik}.$$

The Poisson brackets between particle positions are

$$\{x^i_a, x^j_b\}_D = \int \left(\frac{\partial A^1_k}{\partial k_{bj}} \frac{\partial E^k_1}{\partial k_{ai}} - \frac{\partial E^k_1}{\partial k_{bj}} \frac{\partial A^1_k}{\partial k_{ai}} \right) d^3x \not\equiv 0.$$

4 Conclusions

In this paper a method of reduction the field degrees of freedom by means of canonical constraints has been developed for a system of N charged particles plus electromagnetic field. In the first order in the coupling constant expansion it is shown that the properties of the Poincaré algebra are preserved after field reduction.

We found the solutions of the inhomogeneous field equations of motion as the sum of the Lienard–Wiechert potentials and the free fields. By means of the canonical transformation to the free field variables we got new form for the Poincaré generators. It is turned out that the Poincaré generators may be presented as the sum of free field and particle terms. In our approximation we eliminate the radiation phenomenon connected with the free electromagnetic fields. The commutation and transformation properties of particle positions are studied.

The obtained description may be used for the statistical description of the system of charged particles interacting without field. The elaborated procedure of reduction can be realized for the gravity in near future.

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