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Modelling System for Relaxing Media. Symmetry, Restrictions and Attractive Features of Invariant Solutions

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A model describing non-equilibrium processes in relaxing media is considered. Restrictions arising from the symmetry principles and the second law of thermodynamics are stated. System of ODE describing a set of travelling wave solutions is obtained via group theory reduction. Bifurcation analysis of this system reveals the existence of periodic invariant solutions as well as limiting to them solitary wave solutions. These families play the role of intermediate asymptotics for a wide set of Cauchy and boundary value problems

1 Introduction

Analysis of experimental studies of multi-component media subjected to shock loading [1] enables one to conclude that some internal state variations are possible at constant values of the "external" parameters (temperature T, pressure p, mass velocity u, etc.). Phenomena arising then as an afteraction of relaxing processes might be formally described by introducion of an internal variable λ, expressing deviation of the system from the state of complete thermodynamic equilibrium and formally obeying the chemical kinetics equation with unknown affinity A of the relaxing process. Connection between the "internal" and "external" variables is stated by the second law of thermodynamics, written in the Gibbs form [2]:

$$TdS = dE - p\rho^{-2}d\rho + Ad\lambda. \tag{1}$$

To describe the long nonlinear waves propagation in such media, the following system may be proposed [3]:

$$\begin{gathered}\rho\left(\frac{\partial u^i}{\partial t} + u^j\frac{\partial u^i}{\partial x^j}\right) + \frac{\partial p}{\partial x^i} = 0, \qquad \frac{\partial \rho}{\partial t} + u^i\frac{\partial \rho}{\partial x^i} + \rho\frac{\partial u^i}{\partial x^i} = 0,\\ \frac{\partial p}{\partial t} + u^i\frac{\partial p}{\partial x^i} + M\frac{\partial u^i}{\partial x^i} = N, \qquad \frac{\partial \lambda}{\partial t} + u^j\frac{\partial \lambda}{\partial x^j} = Q \equiv aA,\end{gathered} \tag{2}$$

where M, and N are functions connected with internal energy E and affinity of the relaxing processes $A = a^{-1}Q$ by means of the relations

$$M = (p - \rho^2 E_\rho)/(\rho E_p), \qquad N = -E_\lambda Q/E_p. \tag{3}$$

Here and henceforth lower indices mean partial derivatives with corresponding to subsequent variables.

The aim of this work is to show that arbitrainess in the choice of functions E and A may be reduced to the great extent if we impose restrictions arising from symmetry principles and the second law of thermodynamics. Another goal is to state the conditions leading to the invariant autowave solutions appearance as well as to study their attractive features.

2 Group theory classification of system (2)

Let us study the symmetry of system (2), that contains three unknown functions, linked together by means of equations (3). We look for infinitesimal operators (IFO), having the following form:

$$X = \xi^\alpha \frac{\partial}{\partial x^\alpha} + \eta^i \frac{\partial}{\partial u^i} + \tau \frac{\partial}{\partial p} + \theta \frac{\partial}{\partial \rho} + \gamma \frac{\partial}{\partial \lambda}, \tag{4}$$

where ξ^α, η^i, τ, θ, γ depend on x^α, u^i, p, ρ, λ, $\alpha = 0, \ldots, n$, $i = 1, \ldots, n$ (we identify variable x^0 with t). The procedure of "splitting" of a PDE system, arising from action of the first extension of the operator (4) on system (2), is very similar to that described in [4]. It is possible to select a subsystem defining coordinates ξ^μ, η^i, τ and θ:

$$\begin{gathered} \tau = p[m - (2+n)gt] + f(t), \qquad \theta = \rho(\alpha - ngt), \qquad \xi^0 = gt^2 + bt + h, \\ \xi^i = (gx^i + l^i)t + cx^i + \Sigma a^i_j x^j + q^i, \qquad \eta^i = gx^i + l^i + u^i(c - b - gt) + \Sigma a^i_j u^j, \end{gathered} \tag{5}$$

where g, b, h, l^i, m, c are arbitrary constants, $a^i_j = -a^j_i$, $\alpha = 2(b-c) + m$, $f(t)$ is an arbitrary function. Besides, it may be shown that $\gamma^j_{x^i} = \gamma^j_{u^k} = 0$. The remaining part of the system, containing the unknown functions M, N and Q, is presented in the following form:

$$\hat{Z}\begin{pmatrix} M \\ N \\ Q \end{pmatrix} \equiv \left(\theta\frac{\partial}{\partial\rho} + \tau\frac{\partial}{\partial p} + \gamma\frac{\partial}{\partial\lambda}\right)\begin{pmatrix} M \\ N \\ Q \end{pmatrix} = \begin{bmatrix} \tau_p M \\ \tau_0 + (\tau_p - \xi^0_0)N + ngM \\ \gamma_0 + \gamma_p N + \gamma_\lambda Q - \xi^0_0 Q \end{bmatrix}. \tag{6}$$

Note that system (6) does not contain the parameters h, l^i, q^i, a^i_j . Therefore, for arbitrary functions M, N and Q system (2) admits operators $\hat{P}_0$, $\hat{P}_i$, $\hat{J}_{ab}$ and $\hat{G}_a$, i, a, $b = 1, \ldots, n$, forming the standard representation of the Galilei algebra $AG(n)$ [4]. We did not succeed in obtaining the general solution of system (6), but knowledge of particular one enables to prove that symmetry algebra is infinite-dimensional.

Theorem 1. *Let $E = p/[(\nu-1)\rho] + q\lambda$, $\nu = (n+2)/n$, $Q = \rho^{\sigma\nu-1}\psi(\omega)$, where ψ is arbitrary function of $\omega = p/\rho^\nu$. Then system (2), in addition to $AG(n)$, admits the following operators:*

$$\begin{aligned} K_1 &= t^2\frac{\partial}{\partial t} + tx^i\frac{\partial}{\partial x^i} + (x^i - tu^i)\frac{\partial}{\partial u^i} - nt\rho\frac{\partial}{\partial\rho} \\ &\quad - (2+n)tp\frac{\partial}{\partial p} + \left\{\int \frac{\rho f(\rho) - 2p}{\rho^{\sigma\nu+1}\psi(\omega)}dp + t[2p/\rho - f(\rho)]\right\}\frac{\partial}{\partial\lambda}, \\ \hat{D} &= 2t\frac{\partial}{\partial t} + x^i\frac{\partial}{\partial x^i} - u^i\frac{\partial}{\partial u^i} - (n+2)p\frac{\partial}{\partial p} - n\rho\frac{\partial}{\partial\rho} - \frac{2p}{\rho}\frac{\partial}{\partial\lambda}, \\ L &= \Gamma(\rho, \lambda + p/\rho)\frac{\partial}{\partial\lambda}, \end{aligned} \tag{7}$$

where $i, a, b = 1, \ldots, n$. Note that the latter two expressions contain arbitrary functions $f(\rho)$ and $\Gamma(\rho, \lambda + p/\rho)$.

If to decline the requirement of maximal symmetry existence, then the problem of group theory classification of system (2) may be effectively solved. The results obtained are presented in Table 1 where the following notation is used:

$$\hat{D}_1 = t\frac{\partial}{\partial t} - u^i\frac{\partial}{\partial u^i} + 2\rho\frac{\partial}{\partial\rho}, \qquad \hat{D}_2 = x_i\frac{\partial}{\partial x^i} + u^i\frac{\partial}{\partial u^i} - 2\rho\frac{\partial}{\partial\rho},$$

$$\hat{D}_3 = \rho\frac{\partial}{\partial\rho} + p\frac{\partial}{\partial p}, \qquad \hat{L}_1 = \partial/\partial\rho, \qquad \hat{L}_2 = \partial/\partial p, \qquad \hat{L}_3 = \partial/\partial\lambda.$$

It is seen from the analysis of Table 1 that symmetry extension takes place in many cases including those for which E, Q are arbitrary functions of the invariants of subsequent ISO and this gives way for effective use of qualitative methods in the relaxing media models investigations.

Table 1

E, Q	IFO
$E = p\rho^{-1}f(\omega)$, $Q = \lambda g(\omega)$, $\omega = p\lambda^{\kappa}$	$\hat{Z}_1 = \lambda\hat{L}_3 - \kappa\hat{D}_3$, $\hat{Z}_2 = \hat{D}_2$
$E = \rho^{-1}[f(\omega) - p]$, $Q = e^{-p}g(\omega)$, $\omega = \lambda - \sigma\ln\rho$,	$\hat{Z}_1 = \hat{D}_1 + \hat{D}_2 + \hat{L}_2$, $\hat{Z}_2 = \hat{D}_2 - 2\sigma\hat{L}_3$
$E = \rho^{-1}[f(\omega) - p]$, $Q = \rho^{\nu}e^{p}g(\omega)$, $\omega = \lambda/\rho^{\sigma}$	$\hat{Z}_1 = \hat{D}_1 + \hat{D}_2 - \hat{L}_2$, $\hat{Z}_2 = \hat{D}_2 + 2(\sigma - \nu)\hat{L}_2 + 2\sigma\lambda\hat{L}_3$
$E = p\rho^{-1}f(\omega)$, $\omega = \lambda - \tau\ln\rho$, $Q = p^{\mu}\rho^{\nu}g(\omega)$,	$\hat{Z}_1 = \nu\hat{D}_1 + (\nu + 1/2)\hat{D}_2 - \tau\hat{L}_3$, $\hat{Z}_2 = (\mu + \nu)\hat{D}_2 + 2\nu\hat{D}_3 - 2\mu\tau L_3$
$E = \rho^{\xi-1}F(\omega_1, \omega_2)$, $Q = \rho^{\sigma-\beta}G(\omega_1, \omega_2)$ $\omega_1 = p/\rho^{\xi}$, $\omega_2 = \lambda/\rho^{\sigma}$	$\hat{Z} = 2\beta\hat{D}_1 + (2\beta + \xi - 1)\hat{D}_2 + 2\xi\hat{D}_3 + 2\sigma\lambda\hat{L}_3$
$E = \rho^{-1}[F(\omega_1, \omega_2) - \tau\ln\rho]$, $Q = \rho^{\sigma-\beta}G(\omega_1, \omega_2)$, $\omega_1 = \rho^{\tau}e^{-p}$, $\omega_2 = \lambda\rho^{-\sigma}$,	$\hat{Z} = 2\beta\hat{D}_1 + 2\sigma\lambda\hat{L}_3 + (2\beta - 1)\hat{D}_2 + 2\tau\hat{L}_2$
$E = \rho^{\xi-1}F(\omega_1, \omega_2)$, $Q = \rho^{-\beta(1+\xi)}G(\omega_1, \omega_2)$, $\omega_1 = p/\rho^{\xi}$, $\omega_2 = \lambda - \delta\ln\rho$	$\hat{Z} = 2\beta(1 + \xi)\hat{D}_1 + 2\xi\hat{D}_3 + 2\delta\hat{L}_3 +$ $+[2\beta(1 + \xi) + (\xi - 1)]\hat{D}_2$
$E = pF(\rho, \omega)$, $Q = p^{-\beta}G(\rho, \omega)$, $\omega = \lambda - \tau\ln p$	$\hat{Z} = \beta\hat{D}_1 + (\beta + 1/2)\hat{D}_2 + \hat{D}_3 + \tau\hat{L}_3$
$E = F(\rho, \omega) - p/\rho$, $Q = e^{-p}G(\rho, \omega)$, $\omega = \lambda - \nu p$	$\hat{Z} = \hat{D}_1 + \hat{D}_2 + \hat{L}_2 + \nu\hat{L}_3$
$E = F(\rho, \omega) - p/\rho$, $Q = G(\rho, \omega)$, $\omega = \lambda - \nu p$	$\hat{Z} = \hat{L}_2 + \nu\hat{L}_3$
$E = \rho^{\tau-1}F(\lambda, \omega)$, $Q = p^{-\beta}G(\lambda, \omega)$, $\omega = p/\rho^{\tau}$,	$\hat{Z} = 2\tau\beta(\hat{D}_1 + \hat{D}_2) - (1 - \tau)\hat{D}_2 + 2\tau\hat{D}_3$
$E = \rho^{\tau-1}F(\lambda, \omega)$, $Q = G(\lambda, \omega)$, $\omega = p/\rho^{\tau}$,	$\hat{Z} = (1 - \tau)\hat{D}_2 - 2\tau\hat{D}_3$
$E = \rho^{-1}[F(\lambda, \omega) - \tau(1 + \ln\rho)]$, $Q = e^{-p}G(\lambda, \omega)$, $\omega = \rho^{\tau}e^{-p}$	$\hat{Z} = 2\tau(\hat{D}_1 + \hat{L}_2) + (2\tau - 1)\hat{D}_2$
$E = pF(\rho, \lambda)$, $Q = p^{-\beta}G(\rho, \lambda)$	$Z = \hat{D}_3 + \beta\hat{D}_1 + (\beta + 1/2)\hat{D}_2$
$E = \rho^{-1}F(p, \lambda)$, $Q = \rho^{\tau}G(p, \lambda)$	$\hat{Z} = 2\tau\hat{D}_1 + (2\tau + 1)\hat{D}_2$

3 Restrictions imposed by the second law of thermodynamics

Let us consider the governing functions, defining in the following form, widely used in applications [5]:

$$E = \frac{p}{(\sigma - 1)\rho} - h(\lambda), \qquad Q \equiv aA = a\,g(\lambda)\phi(p, \rho). \tag{8}$$

Employing the consequences of the second law of thermodynamics (1), we may obtain some restrictions on functions $g(\lambda)$, $h(\lambda)$ and $\phi(p, \rho)$.

Equating partial derivative of the entropy function S with the corresponding terms standing at the RHS of the formula (1), we obtain:

$$(S_p)_{V,\lambda} = T^{-1}\,(E_p)_{V,\lambda}\,, \tag{9}$$

$$(S_V)_{p,\lambda} = T^{-1}[(E_V)_{p,\lambda} + p], \tag{10}$$

$$(S_\lambda)_{V,p} = T^{-1}[(E_\lambda)_{V,p} + A], \tag{11}$$

where $V = \rho^{-1}$. Comparison of the mixed partial derivatives $S_{p,V}$, $S_{p,\lambda}$ and $S_{V,\lambda}$, calculated from (9)–(11), gives the following expressions for T and S:

$$T = V^{-1/\Gamma}\Phi(\Omega), \qquad \Omega = p\,V^{\sigma}, \tag{12}$$

$$S = S_1(\lambda) + \Gamma \int \frac{d\Omega}{\Phi(\Omega,\lambda)}, \tag{13}$$

where $\Gamma = (\sigma - 1)^{-1}$. Functions $\Phi(\Omega, \lambda)$ and $S_1(\lambda)$ are connected with functions defining E and Q by means of the equation

$$V^{-1/\Gamma}\Phi(\Omega,\lambda)\left(\dot{S}_1(\lambda) - \Gamma \int \frac{d\Omega}{\Phi(\Omega,\lambda)}\Phi_\lambda\right) = g(\lambda)\phi(p,\rho) - \dot{h}(\lambda). \tag{14}$$

Assuming that $g(\lambda) = \dot{h}(\lambda)/m$, $\phi(p,\rho) = m + \rho^{1/\Gamma}\theta(\Omega)$ and $\Phi(\Omega,\lambda) = f(\lambda)R(\Omega)$ we obtain the solution

$$A = g(\lambda)\left\{m + \rho^{1/\Gamma}R(\Omega)\left(1 + \Gamma\int\frac{d\Omega}{R(\Omega)}\right)\right\}, \qquad E = \Gamma\rho^{1/\Gamma}\Omega - h(\lambda), \tag{15}$$

where $f = C\exp[g(\lambda)]$, $g(\lambda) = \dot{h}(\lambda)/m$. Note that at $R = C_1\exp(\Omega/r)$, $r = \mathrm{const}$ function A describes kinetics of the Ahrrenius type [5].

Assuming that $R(\Omega) = \kappa^{-1}\Omega$, $h(\lambda) = -q\,(\lambda - \lambda_0)$ we obtain the governing equations

$$A = q\kappa^{-1}\left(\frac{p}{\rho} - \kappa\right), \tag{16}$$

$$E = \frac{p}{\rho(\sigma - 1)} + q\,(\lambda - \lambda_0), \tag{17}$$

at which system (2) admits scaling symmetry group, generated by the operator $\hat{D}_3$. So the symmetry requirements together with the restrictions arising from the second law of thermodynamics completely remove the arbitrariness in the choice of the functions E and A.

If the processes under consideration are not far from the state of complete thermodynamic equilibrium, which is the case, e.g., when the long nonlinear wave propagation is studied, then we may substitute the energy balance equation for the the finite-differenc equation

$$p - p_0 = +\sigma\kappa V_0^{-2}(V - V_0) + q(\sigma - 1)V_0^{-1}(\lambda - \lambda_0) = 0, \tag{18}$$

where p_0, V_0 and λ_0 denote the values of the subsequent parameters in the state of complete thermodynamic equilibrium. Expressing λ from this equation, differentiating (18) with respect to temporal variable and next employing (16) with RHS expanded near the equilibrium, it is possible to express the governing equation merely in terms of “external” variables. Restricting our consideration to one-dimensional case and going to the Lagrangian representation

$$t_l = t, \qquad x_l = \int \rho\, dx \tag{19}$$

we obtain a closed system which is more simple than (2):

$$\frac{\partial u}{\partial t_l} + \frac{\partial p}{\partial x_l} = \Im, \qquad \frac{\partial V}{\partial t_l} - \frac{\partial u}{\partial x_l} = 0, \qquad \tau\left[\frac{\partial p}{\partial t_l} + \frac{\chi}{\tau V^2}\frac{\partial u}{\partial x_l}\right] = \frac{\kappa}{V} - p, \tag{20}$$

where $\Im$ is an external force, $V \equiv \rho^{-1}$ is the specific volume,

$$\tau^{-1} = -a\,(A_\lambda)_V = a(\sigma-1)q^2/\kappa, \qquad \chi/\tau = (\partial p/\partial\rho)_\lambda = \kappa\sigma, \qquad \kappa = (\partial p/\partial\rho)_{A=0}\,.$$

(we will drop index "l" in the forthcoming formulae). So, instead of unknown functions, system (20) contains three parameters that completely define dynamical features of relaxing media near the equilibrium [3].

4 On attractive features of invariant solutions of system (20)

A wide employment of the group theory methods in non-linear mathematical physics is justified to the great extent by the fact that invariant solutions of evolution systems very often play the role of intermediate asymptotics for sufficiently large class of Cauchy and boundary value problems. It is shown below that attractive features are inherent to periodic invariant solutions of system (22) as well as reducible to them solitary wave solutions.

When $\Im = \gamma = \text{const}$ then ansatz

$$u = U(\omega), \qquad V = \frac{R(\omega)}{x_0 - x}, \qquad p = (x_0 - x)\,\Pi(\omega), \qquad \omega = t\xi + \ln\frac{x_0}{x_0 - x} \tag{21}$$

leads to an ODE system. Substituting (21) into the second equation of systems (20) we find that $\dot U = \xi\dot R$. Variables R, Π satisfy the following system of ODE:

$$\begin{aligned} &\xi\left[\tau(\xi R)^2 - \chi\right]\dot R = -R\left[(1+\tau\xi)R\Pi - \kappa + \tau\gamma\xi R\right],\\ &\xi\left[\tau(\xi R)^2 - \chi\right]\dot \Pi = \xi\left[\xi R(R\Pi - \kappa) + \xi(\Pi + \gamma)\right]. \end{aligned} \tag{22}$$

It is easy to see that the singular point $\mathbf{A}(R_1, \Pi_1)$, where $\Pi_1 = -\gamma > 0$, $R_1 = \kappa/\Pi_1$, corresponds to the invariant stationary solution of system (20), belonging to the set (21):

$$u_0 = 0, \qquad p_0 = \gamma(x - x_0), \qquad V_0 = \kappa/[\gamma(x - x_0)]. \tag{23}$$

For this special case we are able to express transition to the Eulerian co-ordinate x_e in explicit form:

$$x_e = (\kappa/\gamma)\ln\left[(x_0 - x_l)/x_0\right]. \tag{24}$$

So, according to the formula (24), the Lagrangian co-ordinate $x_l = x_0$ corresponds to the point on infinity in the Eulerian reference frame.

We are going to formulate conditions assuring existence of periodic solutions in vicinity of the singular point $\mathbf{A}(R_1, \Pi_1)$. For this purpose we rewrite the linear part of system (22) in co-ordinates $x = R - R_1$, $y = \Pi - \Pi_1$:

$$\xi\Delta\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{bmatrix} -\kappa, & -R_1^2\sigma \\ \kappa\xi^2, & (\xi R_1)^2 + \chi\xi \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + O(|x|, |y|). \tag{25}$$

Periodic solutions appearance would take place when the eigenvalues of matrix $\hat{\mathbf{M}}$ standing at the RHS of equation (25) intersect imaginary axis [6], and this is so when the following relations hold:

$$\xi = \xi_{cr} = -\left(\chi + \sqrt{\chi^2 + 4\kappa R_1^2}\right)/(2R_1^2), \qquad 0 < R_1 < \sqrt{\chi/(\tau\xi^2)}. \tag{26}$$

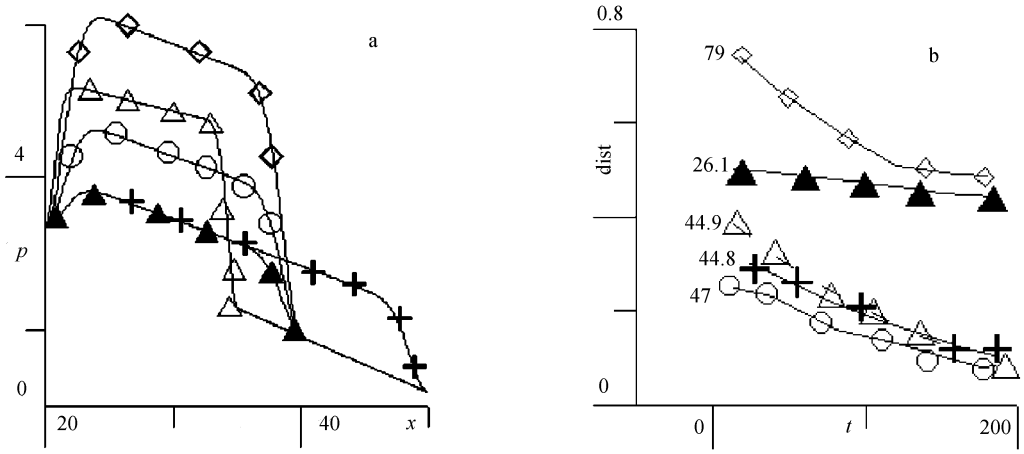

Fig. 1. Perturbations used in numerical experiments (a) and temporal dependence of distances between the wave packs and the solitary wave invariant solution (b). Numbers near the graphs show the energies of the initial perturbations with the same marks.

Numerical analysis of system (22) gives such changes of regimes in a vicinity of the singular point $\mathbf{A}\,(R_1, \Pi_1)$. When value of the parameter ξ is a little less than ξ_{cr}, then the singular point is a stable focus. Above this value a stable limiting cycle appears in a soft manner. Its radius grows with further increase of the parameter ξ until the homoclinic bifurcation takes place. After that the singular point becomes unstable focus.

We performed numerical simulation of system (20) based on the Godunov numerical scheme [7]. The values of the parameters were choosen in accordance with the requirements posed by (26). In numerical experiments we observed that solutions of the Cauchy problems evolved in self-similar modes when invariant periodic solutions belonging to set (21) as well as limiting to them homoclinic solutions were taken as Cauchy data.

Numerical simulations have also shown that wave packs created by sufficiently large class of perturbations of the initial inhomogeneous state (23) tend to the solution associated with the homoclinic loop. Whether or not the wave pack would tend to the homoclinic solution depends on the energy of initial perturbation, more preciesly, on that part of the total energy that is travelled with the pack moving "downward" i.e. towards the domain with decreasing p.

Using the equation (18), we can express the energy of perturbation in the following form:

$$E^{\rm tot} = \int_0^x \left\{ \frac{u^2}{2} - p_0\,(V - V_0) + \int_0^{x'} \left[\frac{\partial p}{\partial x} - \gamma \right] V\,dx'' \right\} dx'.$$

Numerical simulation shows that energy estimation of the initial perturbation well enough characterizes convergency to the invariant soliton-like solution. For $\chi = 1.5$, $\tau = 0.07$, $\kappa = 10$ convergency is observed when $E^{\rm tot}$ is close to 45. Fig. 1a shows variety of perturbation used whereas Fig. 1b – temporal dependence of minimal distances between the wave packs created by perturbations and the family of solutions associated with the homoclinic loop. On the left side of Fig. 2 initial perturbations are shown together with invariant homoclinic solution marked by the dotted line, whereas on the right side the homoclinic trajectories and the wave packs created by the subsequent perturbations are shown at large distances from the origin. Case b corresponds to the initial perturbations having the energy close to 45, cases a and c – to the initial perturbations having sufficiently different energies.

We also interested in attracting features of periodic invariant solutions. Our experiments showed that it is impossible to obtain convergency to a periodic invariant solution when Cauchy data are chosen among monotonic functions. But if we solve the boundary value problem with periodically initiated impulses then convergency may be attained. Here again the energy criterion works, besides, perturbations should be separated by proper temporal intervals.We solved numerically a piston problem taking again as Cauchy data stationary invariant solutions (23), associated with the critical point $A\,(R_1, \Pi_1)$. It was observed that the convergency takes place when the energy of a single perturbation created by the piston that works in a pulse regime is close to 17 and the temporal intervals between the pulses lies near 22. Fig. 3 shows typical patterns obtained. An invariant periodic solution envelopes succession of wave perturbations, that are essentialy different from the autowave mode near the origin (Fig. 3a), but approaches it in the long run (Fig. 3b).

So both autowave invariant solutions and solitary waves play roles of intermediate asymptotics for sufficiently large classes of solutions of system (20).

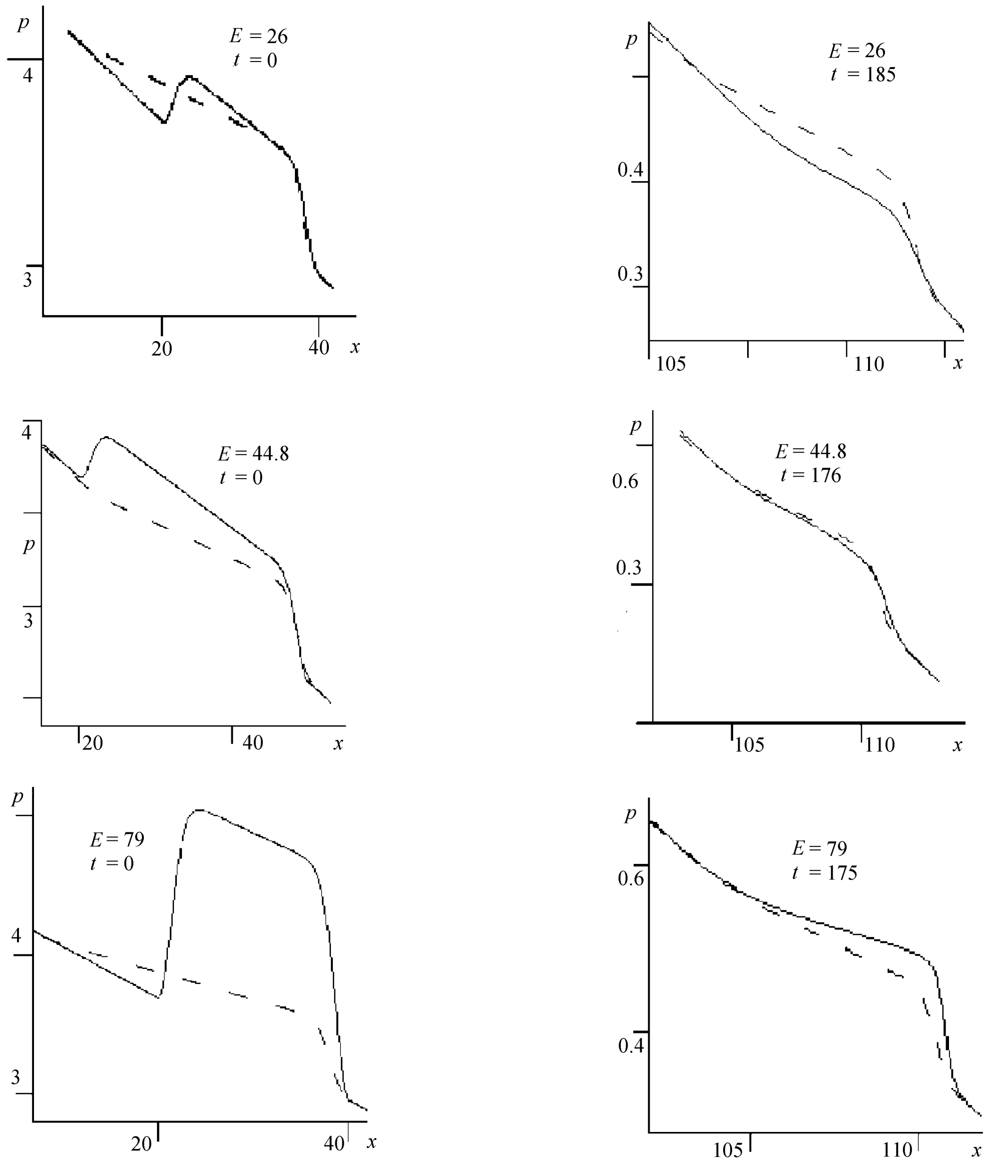

Fig. 2. Perturbations of the stationary inhomogeneous solution (23) (left) and wave packs created by these perturbations (right) on the background of the invariant solitary wave solution indicated by dotted lines.

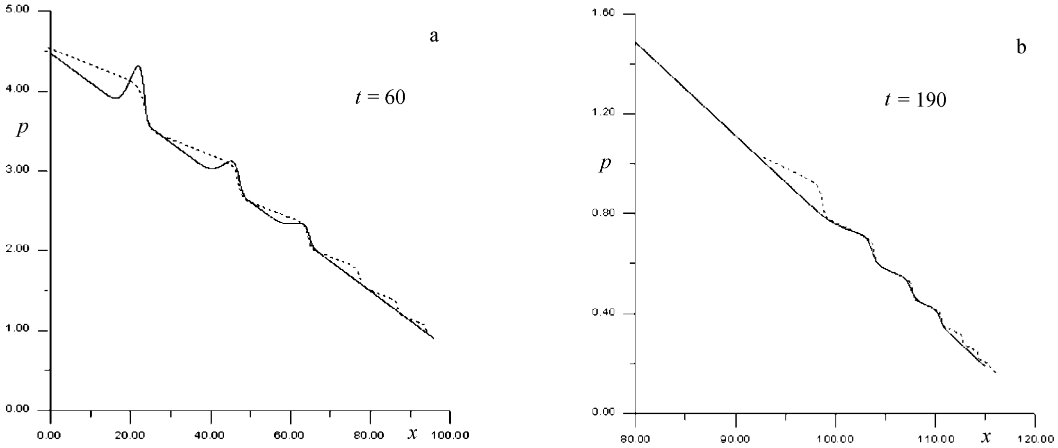

Fig. 3. Wave packs initiated by a piston moving periodically in the pulse regime on the background of a periodic invariant solution, indicated by the dotted line: patterns near the source (a) and at large distance from source (b).

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