Proceedings of Institute of Mathematics of NAS of Ukraine 2000, Vol. 30, Part 1, 210–217.

Periodic Soliton Solutions to the Davey–Stewartson Equation

Masayoshi TAJIRI † *and Takahito ARAI* ‡

Department of Mathematical Sciences, College of Engineering,
Osaka Prefecture University, Sakai 599-8531, Japan
† *E-mail: tajiri@nm.ms.osakafu-u.ac.jp*
‡ *E-mail: arai@nm.ms.osakafu-u.ac.jp*

The periodic soliton resonances and recurrent wave solutions to the Davey–Stewartson equation are presented. The solutions that described the interaction between a y-periodic soliton and a line soliton are analyzed to show the existence of the soliton resonances. The various recurrent solutions (The growing-and-decaying mode, breather and rational growing-and-decaying mode solutions) are presented. The y-periodic soliton and breather solutions can be constructed as the imbricate series of algebraic soliton solutions and rational growing-and-decaying mode solutions, respectively.

1 Introduction

It is well known that spin and statistics in quantum mechanics come from symmetries of transformation. The soliton solutions to some soliton equations show Fermion-like behavior. We could not obtain the solutions from the initial value problem which forgive the coexistence of completely same solitons in the wave field to some soliton equations. It is very interesting to know that what symmetries are hidden in soliton equations related to this problem. Before studying the symmetries to the soliton equations from the point of view, we will show some propagation properties of solitons to the Davey–Stewartson (DS) equation which is the two-dimensional generalization of the nonlinear Schrödinger equation [1].

The higher-dimensional nonlinear wave fields have richer phenomena than one-dimensional ones since various localized solitons may be considered in higher-dimensional space. The DS equation has four kinds of soliton solutions: the conventional line, algebraic, periodic and lattice solitons. The conventional line soliton has an essentially one-dimensional structure. On the other hand, the algebraic, periodic and lattice solitons have a two-dimensional localized structure.

The solutions to the DS equation have been studied previously in various aspects [2–9]. The existence of solitons having the structures peculiar to a higher-dimensionality may contribute to the variety of the dynamics of nonlinear waves. To clarify the dynamics, we must investigate various interactions between two different kinds of solitons. In the previous papers [10, 11], the various interactions between two y-periodic solitons, line and periodic and periodic and algebraic solitons were investigated. And we found the periodic resonant interactions which are qualitatively different from the interaction between two line solitons. We expect that the periodic soliton resonances play fundamental role in the nonlinear development of higher-dimensional wave field as the existence of the periodic soliton resonances may be related to the instability of the solitons, accompanied by their decay and merging.

The governing equations for the description of the long time evolution of unstable wave train have been studied by many authors. The extension to the two-dimensional case was examined by Zakharov [12], Benny and Roskes [13] and Davey and Stewartson [1]. The time evolution of the solution of the 1D-NLS equation with periodic boundary condition and with a Benjamin–Feir unstable initial condition was studied numerically by Lake *et al.* [15]. They found that a modulated unstable wave train achieves a state of maximum modulation and returns to an unmodulated initial state, which is well known as the Fermi–Pasta–Ulam (FPU) recurrence. One of the important feature of the solutions of the NLS equations in one- and two-dimensions is the recurrence of the unstable wavetrain to its initial state.

The purposes of this study are (i) to review periodic soliton solutions and recurrent solutions and (ii) to show that these solutions can be constructed by imbricate series of rational soliton solutions or rational growing-and-decaying mode solutions.

2 Periodic soliton resonances

The Davey–Stewartson equation may be written as

$$\begin{cases} \mathrm{i}u_t + pu_{xx} + u_{yy} + r|u|^2u - 2uv = 0, \\ pv_{xx} - v_{yy} - pr(|u|^2)_{xx} = 0, \end{cases} \tag{1}$$

where $p = \pm 1$, r is constant, eq. (1) with $p = 1$ and $p = -1$ are called the DS I and DS II equations, respectively. In this section, we study the resonant interactions between y-periodic soliton and line soliton mutually parallel propagating to the x-direction of the DS I equation with $r > 0$. The solution that describes the interaction between a y-periodic soliton and a line soliton is written as [11]

$$u = u_0 e^{\mathrm{i}(kx+ly-\omega t)} \frac{g}{f}, \qquad v = -2p(\log f)_{xx} \qquad (p = 1) \tag{2}$$

with

$$\begin{aligned} f = {} & 1 - \frac{1}{\alpha^2}\exp(\xi_1)\cos\eta + \frac{M}{4\alpha^4}\exp(2\xi_1) \\ & + \exp(\xi_2)\left\{1 - \frac{N}{\alpha^2}\exp(\xi_1)\cos\eta + \frac{MN^2}{4\alpha^4}\exp(2\xi_1)\right\}, \end{aligned} \tag{3}$$

$$\begin{aligned} g = {} & 1 - \frac{1}{\alpha^2}\exp(\xi_1 + \mathrm{i}\phi)\cos\eta + \frac{M}{4\alpha^4}\exp 2(\xi_1 + \mathrm{i}\phi) \\ & + \exp(\xi_2 + \mathrm{i}\psi)\left\{1 - \frac{N}{\alpha^2}\exp(\xi_1 + \mathrm{i}\phi)\cos\eta + \frac{MN^2}{4\alpha^4}\exp 2(\xi_1 + \mathrm{i}\phi)\right\}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} & \xi_1 = \alpha x - \Omega_P t + \sigma_1, \qquad \xi_2 = \beta x - \Omega_L t + \sigma_2, \qquad \eta = \delta y - \gamma t + \kappa, \\ & \sin^2\frac{\phi}{2} = \frac{\alpha^2 + \delta^2}{2ru_0^2}, \qquad \sin^2\frac{\psi}{2} = \frac{\beta^2}{2ru_0^2}, \\ & \Omega_P = 2k\alpha - (\alpha^2 - \delta^2)\cot\frac{\phi}{2}, \qquad \Omega_L = \beta\left(2k - \beta\cot\frac{\psi}{2}\right), \qquad \gamma = 2l\delta, \\ & M = 1 \Big/ \left[1 - \frac{(\alpha^2 + \delta^2)^2}{2\delta^2 ru_0^2}\right], \qquad N = \frac{2ru_0^2 \sin\frac{\phi}{2}\sin\frac{\psi}{2}\cos\frac{\phi-\psi}{2} - \alpha\beta}{2ru_0^2 \sin\frac{\phi}{2}\sin\frac{\psi}{2}\cos\frac{\phi+\psi}{2} - \alpha\beta}. \end{aligned} \tag{5}$$

We can investigate the phase shifts after the collision between y-periodic soliton and line soliton by using the solution. The condition $|N| = \infty$, corresponds to the phase shift in the propagation direction becomes infinite for the case $\alpha\beta > 0$. This means that the period of the intermediate state, where the periodic soliton propagates together with the line soliton, persist infinitely. This is thought as a resonance between the y-periodic soliton and the line soliton. By equating the denominator of N to zero, this condition is given by

$$2ru_0^2 \sin\frac{\phi}{2}\sin\frac{\psi}{2}\cos\frac{\phi+\psi}{2} - \alpha\beta = 0. \tag{6}$$

The condition $N = 0$ corresponds to the phase shift in the propagating direction becomes negative infinity for $\alpha\beta > 0$. This means that two solitons can interact infinitely apart each other. This is thought as extremely repulsive or long range interaction between the y-periodic soliton and the line soliton. The explicit expression of the condition is obtained by equating the numerator of N with zero as

$$2ru_0^2 \sin\frac{\phi}{2}\sin\frac{\psi}{2}\cos\frac{\phi-\psi}{2} - \alpha\beta = 0. \tag{7}$$

We can also show the existence of periodic soliton resonances in the interactions of periodic soliton-periodic soliton and periodic soliton-algebraic soliton [10, 11].

3 Recurrent solutions

One of the important feature of the solution to the DS equation is the recurrence of the unstable wavetrain to its initial state. Three kinds of recurrent solutions, growing-and-decaying mode, breather and rational growing-and-decaying mode solutions are shown in this section, which can be constructed from the two-soliton solution [15]. The two-soliton solution may be written as [3]

$$u = u_0 e^{\mathrm{i}(kx+ly-\omega t)}\frac{g}{f}, \qquad v = -2p(\log f)_{xx}, \tag{8}$$

with

$$f = 1 + e^{\eta_1} + e^{\eta_2} + De^{\eta_1+\eta_2}, \qquad g = 1 + e^{\eta_1+\mathrm{i}\phi_1} + e^{\eta_2+\mathrm{i}\phi_2} + De^{\eta_1+\eta_2+\mathrm{i}(\phi_1+\phi_2)},$$

where

$$\begin{aligned} &\eta_j = K_j x + L_j y - \Omega_j t + \eta_j^0, \qquad \sin^2\frac{\phi_j}{2} = \frac{pK_j^2 - L_j^2}{2ru_0^2}, \\ &\Omega_j = 2pkK_j + 2lL_j - (pK_j^2 + L_j^2)\cot\frac{\phi_j}{2} \qquad (j = 1, 2). \end{aligned} \tag{9}$$

(i) growing-and-decaying mode solution. Taking wave numbers and frequencies pure imaginary and complex, respectively,

$$K_1 = K_2^* = \mathrm{i}\beta, \quad L_1 = L_2^* = \mathrm{i}\delta, \qquad \Omega_1 = \Omega_2^* = \Omega + \mathrm{i}\gamma,$$

$$\phi_1 = \phi_2 = \phi \ \text{ : real}, \qquad \eta_1^0 = \eta_2^{0*}, \qquad e^{\eta_1^0} = e^{\eta_2^{0*}} = -(1/2)e^{-\tilde{\sigma}+\mathrm{i}\theta},$$

we have the following dispersion relation and D

$$\sin^2\frac{\phi}{2} = \frac{\delta^2 - p\beta^2}{2ru_0^2}, \qquad \Omega = -\left(\delta^2 + p\beta^2\right)\cot\frac{\phi}{2},$$

$$\gamma = 2pk\beta + 2l\delta, \qquad D = \frac{2}{1+\cos\phi} > 1.$$

Then, the solution is given by

$$
\begin{aligned}
u = u_0 e^{\mathrm{i}(kx+ly-\omega t+\phi)} & \left[\sqrt{D}\cosh(\Omega t+\sigma-\mathrm{i}\phi)-\cos(\beta x+\delta y-\gamma t+\theta)\right] \\
& \times\left[\sqrt{D}\cosh(\Omega t+\sigma)-\cos(\beta x+\delta y-\gamma t+\theta)\right]^{-1},
\end{aligned}
\tag{10}
$$

$$
v = 2p\beta^2 \frac{\sqrt{D}\cosh(\Omega t+\sigma)\cos(\beta x+\delta y-\gamma t+\theta)-1}{\left[\sqrt{D}\cosh(\Omega t+\sigma)-\cos(\beta x+\delta y-\gamma t+\theta)\right]^2},
\tag{11}
$$

where $\sigma=\tilde{\sigma}+\log\frac{2}{\sqrt{D}}$. The existence condition for the non-singular solution is given by $D>1$, which is satisfied for $\delta^2-p\beta^2>0$. This solution grows exponentially at the initial stage and the growth rate is given by Ω, which is in agreement with the growth rate of the Benjamin–Feir instability. Therefore, we can regard as this growing-and-decaying mode solution as one described the nonlinear evolution of the unstable mode.

(ii) Breather solution. To obtain analytical expression for the breathing wave solution, we set $K_1=K_2=a$, $L_1=L_2=b$ and $\phi_1=-\phi_2=\mathrm{i}\Phi$ in eq. (9), where a and b are real. Then, frequencies Ω_1 and Ω_2 are complex and are complex conjugate with each other and the solution is given by,

$$
u = u_0 e^{\mathrm{i}(kx+ly-\omega t)} \frac{\sqrt{D}\cosh\xi-\cosh\Phi\cos(\gamma t+\theta)+\mathrm{i}\sinh\Phi\sin(\gamma t+\theta)}{\sqrt{D}\cosh\xi-\cos(\gamma t+\theta)},
\tag{12}
$$

$$
v = -2pa^2 D\frac{1-\frac{1}{\sqrt{D}}\cosh\xi\cos(\gamma t+\theta)}{\left[\sqrt{D}\cosh\xi-\cos(\gamma t+\theta)\right]^2},
\tag{13}
$$

where

$$
\begin{aligned}
&\xi = ax+by-\Omega t+\sigma, \qquad \sinh^2\frac{\Phi}{2}=\frac{b^2-pa^2}{2ru_0^2}>0, \\
&\Omega = 2(pka+lb), \qquad \gamma=\left(b^2+pa^2\right)\sqrt{\frac{2ru_0^2}{b^2-pa^2}+1}, \qquad D=1+\frac{b^2-pa^2}{2ru_0^2},
\end{aligned}
\tag{14}
$$

where σ and θ are arbitrary phase constants.

(iii) Rational growing-and-decaying mode solution. We consider the long wave limit of the growing-and-decaying mode solution. Putting $K_1=K_2^*=\mathrm{i}\varepsilon c$, $L_1=L_2^*=\mathrm{i}\varepsilon d$, $\eta_1^0=\eta_2^{0^*}=\varepsilon(\mathrm{i}\tilde{\theta}'-\tilde{\sigma}')+\mathrm{i}\pi$, and taking the limit as $\varepsilon\to 0$, we have

$$
u = u_0 e^{\mathrm{i}(kx+ly-\omega t)}\left\{1-\frac{4\alpha(\alpha\pm\mathrm{i}\eta)}{\alpha^2+\eta^2+\xi^2}\right\},
\tag{15}
$$

$$
v = -4pc^2\frac{\alpha^2+\eta^2-\xi^2}{(\alpha^2+\eta^2+\xi^2)^2},
\tag{16}
$$

where

$$
\begin{aligned}
&\xi = cx+dy-\gamma t+\theta', \qquad \eta=\Omega t+\sigma', \\
&\Omega = \pm\left(d^2+pc^2\right)\sqrt{\frac{2ru_0^2}{d^2-pc^2}}, \qquad \gamma=2pkc+2ld, \qquad \alpha^2=\frac{d^2-pc^2}{2ru_0^2}.
\end{aligned}
\tag{17}
$$

4 Periodic soliton and recurrent solutions as imbricate series of rational solutions

Zaitsev has succeeded in obtaining a periodic soliton solution by the imbricate series of algebraic soliton solutions for the Kadomtsev–Petviashvili (KP) equation with positive dispersion [16]. It is known that the lattice soliton solution to the KP equation with positive dispersion that have doubly periodic array of the localized structure in the x-y plane was constructed as doubly imbricate series of algebraic soliton solutions, which was expressed by using Weierstrass's $\wp$ function or the Riemann theta functions [17]. In this section, we show that the y-periodic soliton and breather solutions can be constructed as the imbricate series of algebraic soliton solutions and rational growing-and-decaying mode solutions, respectively.

(i) *Y*-periodic soliton solution as imbricate series of algebraic soliton solutions. It is interesting to note that the algebraic soliton solutions is given as the following form:

$$u = u_0 \mathrm{e}^{\mathrm{i}\zeta} \left[1 + \frac{2\mathrm{i}B}{\xi + \mathrm{i}\sqrt{\eta^2 + A^2}}\right] \left[1 + \frac{2\mathrm{i}B}{\xi - \mathrm{i}\sqrt{\eta^2 + A^2}}\right], \tag{18}$$

$$v = 2p \left[\frac{1}{(\xi + \mathrm{i}\sqrt{\eta^2 + A^2})^2} + \frac{1}{(\xi - \mathrm{i}\sqrt{\eta^2 + A^2})^2}\right]. \tag{19}$$

where

$$\begin{gathered} \zeta = kx + ly - \omega t, \qquad \xi = x - \left(2pk - \frac{p - R^2}{B}\right) t + \xi^0, \qquad \eta = R(y - 2lt) + \eta^0, \\ B = \sqrt{\frac{p + R^2}{2ru_0^2}}, \qquad A^2 = 4B^2 \bigg/ \left(2B^2 - \frac{p - R^2}{ru_0^2}\right), \end{gathered} \tag{20}$$

The y-periodic soliton solution can be obtained from two-soliton solution of Satsuma and Ablowitz as follows:

$$\begin{aligned} u = u_0 \mathrm{e}^{\mathrm{i}\zeta} (1 - \tan^2 \frac{\phi}{2}) \cos^2 \frac{\phi}{2} \Bigg[1 - 2 \frac{\tan \frac{\phi}{2}}{1 - \tan^2 \frac{\phi}{2}} \\ \times \frac{\frac{1}{\sqrt{D}} \tan \frac{\phi}{2} \cos(\delta y - \gamma t + \theta) - \mathrm{i} \sinh(\alpha x - \Omega t + \sigma)}{\cosh(\alpha x - \Omega t + \sigma) - \frac{1}{\sqrt{D}} \cos(\delta y - \gamma t + \theta)} \Bigg], \end{aligned} \tag{21}$$

$$v = -2p\alpha^2 \frac{1 - \frac{1}{\sqrt{D}} \cosh(\alpha x - \Omega t + \sigma) \cos(\delta y - \gamma t + \theta)}{\left[\cosh(\alpha x - \Omega t + \sigma) - \frac{1}{\sqrt{D}} \cos(\delta y - \gamma t + \theta)\right]^2}, \tag{22}$$

where

$$\begin{gathered} D = \left[1 - \frac{(\delta^2 + p\alpha^2)^2}{2ru_0^2 \delta^2}\right]^{-1} > 1, \qquad \sin^2 \frac{\phi}{2} = \frac{\delta^2 + p\alpha^2}{2ru_0^2}, \\ \Omega = 2pk\alpha - (p\alpha^2 - \delta^2) \cot \frac{\phi}{2}, \qquad \gamma = 2l\delta. \end{gathered} \tag{23}$$

On the basis of eqs. (18) and (19), we assume the form of the y-periodic soliton solution as follows:

$$u = \hat{u}_0 e^{i\zeta} \left[1 + \sum_m \frac{ib}{\xi' + i\nu(\eta) + im\pi} \right] \left[1 + \sum_m \frac{ib}{\xi' - i\nu(\eta) + im\pi} \right], \tag{24}$$

$$v = \frac{p\alpha^2}{2} \sum_m \left[\frac{1}{(\xi' + i\nu(\eta) + im\pi)^2} + \frac{1}{(\xi' - i\nu(\eta) + im\pi)^2} \right], \tag{25}$$

where the summation $\sum_m$ means $\lim_{N\to\infty} \sum_{m=-N}^{N}$, $\nu(\eta)$ is a function of η to be determined afterward. The function $\sqrt{\eta^2 + A}$ is deformed to $\nu(\eta)$ by nonlinear effects. Equations (24) and (25) are rewritten as follows

$$u = \hat{u}_0 e^{i\zeta} (1 - b^2) \left[1 - \frac{2b}{1 - b^2} \frac{b \cos 2\nu(\eta) + i \sinh 2\xi'}{\cosh 2\xi' - \cos 2\nu(\eta)} \right], \tag{26}$$

$$v = -2p\alpha^2 \frac{1 - \cos 2\nu(\eta) \cosh 2\xi'}{[\cosh 2\xi' - \cos 2\nu(\eta)]^2}. \tag{27}$$

Comparing these eqs. (26) and (27) with eqs. (21) and (22), respectively, we find

$$\begin{aligned} &\hat{u}_0 = u_0 \cos^2 \frac{\phi}{2}, \qquad b = -\tan\frac{\phi}{2}, \qquad \xi' = \frac{1}{2}(\alpha x - \Omega t + \sigma'), \\ &\nu(\eta) = \tfrac{1}{2} \cos^{-1} \left[\tfrac{1}{\sqrt{D}} \cos(\delta y - \gamma t + \theta) \right]. \end{aligned} \tag{28}$$

The substitution of eq.(28) into eqs.(24) and (25) gives the y-periodic soliton solution as an imbricate series of algebraic solitons. Taking the $\lim \phi \to 0$ ($\alpha \to 0$, $\delta \to 0$ but $\delta/\alpha = R$ is finite), we have

$$\xi' = \frac{\alpha}{2}\xi, \qquad \nu(\eta) = \frac{\alpha}{2}\sqrt{\eta^2 + A^2}, \qquad b = \alpha B. \tag{29}$$

This means that the solutions (21) and (22) are simple summations of algebraic soliton solutions for very small ϕ.

Recently, the lattice soliton solution to the DS equation was constructed as doubly imbricate series of algebraic soliton solutions which was expressed by using Weierstrass's $\wp$ function or the Riemann theta functions [18].

(ii) Breather solution as imbricate series of rational growing-and-decaying mode solutions. At first, we have to note that the rational growing-and-decaying mode solution is rewritten as following form,

$$u = u_0 e^{i\zeta} \left[1 \mp \frac{2i\alpha}{\eta + i\sqrt{\xi^2 + \alpha^2}} \right] \left[1 \mp \frac{2i\alpha}{\eta - i\sqrt{\xi^2 + \alpha^2}} \right], \tag{30}$$

$$\begin{aligned} v = -2pc^2 \Bigg[&\left\{ \frac{1}{(\eta + i\sqrt{\xi^2 + \alpha^2})^2} + \frac{1}{(\eta - i\sqrt{\xi^2 + \alpha^2})^2} \right\} \\ &+ \frac{2\alpha}{(\eta + i\sqrt{\xi^2 + \alpha^2})^2} \frac{2\alpha}{(\eta - i\sqrt{\xi^2 + \alpha^2})^2} \Bigg]. \end{aligned} \tag{31}$$

On the basis of eqs. (30) and (31), we assume the form of the breather solution as follows,

$$u = \bar{u}_0 \mathrm{e}^{\mathrm{i}\zeta} \left\{ 1 + \mathrm{i}b \sum_m \frac{1}{\eta' + \mathrm{i}\nu(\xi) + m\pi} \right\} \left\{ 1 + \mathrm{i}b \sum_m \frac{1}{\eta' - \mathrm{i}\nu(\xi) + m\pi} \right\}, \tag{32}$$

$$\begin{aligned} v = 4A\alpha^2 \Bigg[& \sum_m \frac{1}{(\eta' + \mathrm{i}\nu(\xi) + m\pi)^2} + \sum_m \frac{1}{(\eta' - \mathrm{i}\nu(\xi) + m\pi)^2} \\ & + 4\alpha^2 \left\{ \sum_m \frac{1}{(\eta' + \mathrm{i}\nu(\xi) + m\pi)^2} \right\} \left\{ \sum_m \frac{1}{(\eta' - \mathrm{i}\nu(\xi) + m\pi)^2} \right\} \Bigg], \end{aligned} \tag{33}$$

Equations (32) and (33) are rewritten as follows,

$$u = \bar{u}_0 (1 - b^2) \mathrm{e}^{\mathrm{i}\zeta} \left[1 - \frac{2b}{1 - b^2} \frac{b \cos 2\eta' - \mathrm{i} \sin 2\eta'}{\cosh 2\nu(\xi) - \cos 2\eta'} \right], \tag{34}$$

$$v = 16A\alpha^2 \left[\frac{4\alpha^2 + 1 - \cos 2\nu(\xi) \cos 2\eta'}{(\cosh 2\nu(\xi) - \cos 2\eta')^2} \right]. \tag{35}$$

Comparing these eqs. (34) and (35) with eqs. (12) and (13), respectively, we find

$$\begin{aligned} & \eta' = \frac{1}{2}\eta = \frac{1}{2}(\Omega t + \sigma), \qquad b = \left[\frac{2ru_0^2}{b^2 - pa^2} + 1 \right]^{-\frac{1}{2}} = \tanh \frac{\Psi}{2}, \qquad A = -\frac{pa^2 r u_0^2}{b^2 - pa^2}, \\ & \bar{u}_0 = \frac{u_0}{1 - b^2}, \qquad \nu(\xi) = \frac{1}{2} \ln \left(\sqrt{D} \cosh \xi + \sqrt{D \cosh^2 \xi - 1} \right), \end{aligned} \tag{36}$$

where

$$D = 1 + \frac{b^2 - pa^2}{2ru_0^2}. \tag{37}$$

Substituting eq. (36) into eqs. (32) and (33), we have the imbricate series constructing breather solution.

5 Conclusion

The DS equation has four kinds of soliton solutions and three kinds of recurrent wave solutions. We have investigated the interaction between y-periodic soliton and line soliton. There are two-types of singular interactions, namely the resonant interaction and the long range interaction. In the long range interaction, the line soliton receives a small transverse disturbance of the same wave number as approaching y-periodic soliton. The disturbance on the line soliton develops into the same y-periodic soliton as approaching soliton. The line soliton emits the y-periodic soliton forward and changes into the messenger line soliton. Then, we observe that the same y-periodic solitons coexist in the wave field when the messenger line soliton is propagating between them. It was also shown that the periodic soliton solutions and the recurrent wave solutions can be constructed as imbricate series of algebraic soliton solutions and rational growing-and-decaying mode solutions, respectively. If we can regard the y-periodic soliton as a sequence of infinite algebraic solitons, we see that same algebraic soliton can not coexist, but infinite algebraic solitons can coexist in the wave field, which is a kind of condensation. We would like to go on to investigate on the symmetries related to these phenomena.

References

[1] Davey A. and Stewartson K., *Proc. R. Soc. London A*, 1974, V.101, 338.
[2] Anker D. and Freeman N.C. *Proc. R. Soc. London A*, 1978, V.360, 529.
[3] Satsuma J. and Ablowitz M.J., *J. Math. Phys.*, 1979, V.20, 1496.
[4] Boiti M., Leon J., Martina L. and Pempinelli F., *Phys. Lett. A*, 1988, V.132, 432.
[5] Boiti M., Leon J. and Pempinelli F., *J. Math. Phys.*, 1990, V.31, 2612.
[6] Fokas A.S. and Santini P.M., *Phys. Rev. Lett.*, 1989, V.63, 1329.
[7] Hietarinta J. and Hirota R., *Phys. Lett. A*, 1990, V.145, 237.
[8] Gilson C.R., *Phys. Lett. A*, 1992, V.161, 423.
[9] Pelinovsky D., *Phys. Lett. A*, 1994, V.196, 181.
[10] Watanabe Y. and Tajiri M., *J. Phys. Soc. Jpn.*, 1998, V.67, 705.
[11] Tajiri M., Arai T. and Watanabe Y., *J. Phys. Soc. Jpn.*, 1998, V.67, 4051.
[12] Zakharov V.E., *J. App. Mech. Tech. Phys.*, 1968, V.9, 86.
[13] Benny D.J. and Roskes G., *Studies App. Math.*, 1969, V.48, 377.
[14] Lake B.M., Yuen H.C., Rungaldier H. and Ferguson W.E., *J. Fluid. Mech.*, 1977, V.83, 49.
[15] Tajiri M. and Arai T., *Phys. Rev. E*, 1999, V.60.
[16] Zaitsev A.A., *Sov. Phys. Dokl.*, 1983, V.28, 720.
[17] Tajiri M. and Murakami Y., *J. Math. Phys.*, 1993, V.34, 2400.
[18] Takashima R. and Tajiri M., *J. Phys. Soc. Jpn.*, 1999, V.68, 2246.