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Relativity without the First Postulate

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We are changing Einstein's axiom system for special relativity and propose a new fundamental theory in relativistic physics. We do not assume that all inertial coordinate systems are equivalent (Einstein's first postulate), but we keep the second axiom, that the speed of light c is the same in all inertial frames. Some key results are [7]:
- The limiting energy and momentum of any particle as its speed approaches the speed of light, are *finite* and proportional to its rest mass.
- These upper bounds give the minimum lengths and time intervals of a particle in the sense of uncertainty and the ultraviolet cutoffs in the renormalization in quantum field theories.
- Photons, and all other particles moving at the speed of light have *nonzero* rest mass. They, however, obey the corresponding (modified) equations with *vanishing mass terms*.
We prove results of the new theory which can give answers and solutions to the following problems and difficulties in modern physics: Divergence difficulties in quantum field theories, "zero over zero" operations in momenta-energy calculations, failure in finding Higgs particles in gauge theories, singularities in general relativity.

1 Introduction

In modern physics, there exist some problems and difficulties:

(1) Singularities in general relativity. "*...it is my opinion that singularities must be excluded*" (Einstein [4, p. 164]). And because of this, he underlines that "*One may not therefore assume the validity of the equations for very high density of field and of matter ...*" (Einstein [4, p. 129]).

(2) Divergence difficulties in quantum field theories, which, "*... are symptomatic of a chronic disorder in the small-distance behavior of the theory*" (Bjorken and Drell [2, p. 4]), and "*In any case the existence of divergent quantities leads one to suspect trouble in the theory at large momenta or, equivalently, small distances*" (Bjorken and Drell [1, p. 154]). Because of the irrational calculation, $\infty - \infty$, in renormalizations, Dirac, a founder of renormalizations, repeatedly asserted [3] that fundamental physics, relativity and quantum theory, must be reformed.

(3) The finite momenta-energies of particles moving at speed of light are given by the operation "zero over zero". For example, the finite energies of photons are given by

$$E = h\nu = \frac{0}{\sqrt{1 - c^2/c^2}}\, c^2 = \frac{0}{0}\, c^2$$

which can be of any value for ν can be of any value.

(4) The inconsistency of gauge invariances for short range interactions with nonzero rest masses of the corresponding gauge particles. The Higgs mechanism seems to be helpful in trying to reach consistency, but there is no experimental evidence which indicates the existence of Higgs particles. Moreover, too many parameters caused by the mechanism make the theory look like a phenomenological rather than a basic theory, as T.D. Lee [5] pointed out. S. Weinberg [8] as the first person who used the Higgs mechanism to establish a unified gauge theory for electromagnetic and weak interactions, proposed a model without the Higgs mechanism in 1981, several years after he won the Nobel prize for that work. Some physicists believe that they will be able to

find Higgs particles in the superconducting super collider. But Weinberg [9], speaking of the proposed SSC accelerator, says "*I refuse to believe that fundamental physics will stop at that point ... We do not know these underlying laws ... We may never know the ultimate laws of nature*".

In this paper, we will change Einstein's axiom system and propose a new fundamental physics based on a new axiom structure. Einstein's theory of relativity is based in its entirety on two postulates [4]:

P1: The laws of physics take the same form in all inertial frames.

P2: The speed of light c is the same in all inertial frames.

From these two postulates Einstein derived that the laws of motion are invariant under Lorentz transformations, in particular

$$dS'^2 = dS^2 \quad \text{for any inertial frames } S,\ S', \qquad \text{and} \qquad dx'^{\mu} = \alpha^{\mu}_{\nu} dx^{\nu},$$

where α^{μ}_{ν} are the matrix elements of the Lorentz transformation from S to S'. Einstein pointed out [4, p. 35] that assuming only P2 one can allow more general transformations than Lorentz transformations of the form $dS'^2 = \lambda(v) dS^2$, where $\lambda(v)$ is a function of the relative velocity v of the inertial frames. If in addition one assumes P1 he showed that $\lambda(v) = \text{const} = 1$. In other words the Lorentz transformations are a necessary result of Einstein's axioms P1 and P2.

In this paper we study the consequences if we abandon the first postulate P1 but keep the second postulate P2. We do not assume that two inertial coordinate systems are still "equivalent" when their relative velocity is high enough, and we allow the limiting deviations (as $V \to c$) of the new theory from the current one to be large enough. This will lead us to more general linear transformations which leave the speed of light invariant and the new equations of laws of physics will be invariant under these transformations, called *c-invariant transformations*. We will derive the corresponding c-invariant equations of particle mechanics, the c-invariant Klein–Gordon, Proca and Maxwell equations and their interactions. No matter whether the deviations from the classical Einstein theory can be verified experimentally or not, we prove theoretical results of the new theory which can give us answers and solutions to the problems and difficulties mentioned above. For details we refer to [7].

2 c-invariant groups

We introduce a new type of general linear transformations leaving the speed of light invariant. Their algebraic structure is *not* the one of a group but of a groupoid (see Section 3); we call it a *c-invariant group* . In Section 3 we will discuss this algebraic structure. These transformations will generalize the Lorenz transformations from the classical theory.

Let Σ be the set of all inertial coordinate system and set for $S \in \Sigma$

$$dS^2 = dx^{\mu} dx^{\mu} = \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 - c^2 dt^2. \tag{1}$$

We are not considering dS as a distance element as in general geometrical models of flat space-time but rather as a formal definition because the generalized transformations we will consider will not be a symmetry of this dS^2 but for a different quantity which will define our geometry.

Consider the following coordinate transformations connecting two inertial coordinate systems $S, S' \in \Sigma$:

$$dx'^{\mu} = f_S^{1/2}(\vec{V}_{S'S}) \alpha_{(\mu\nu)}(\vec{V}_{S'S}) dx^{\nu} \equiv T^{\mu}_{\nu}(S'S) dx^{\nu}, \tag{2}$$

where $\vec{V}_{S'S}$ is the velocity of S' relative to S (measured in S) and $f_S(\vec{V}_{S'S})$ is a positive function, called the *transformation factor* from S to S', and $\alpha_{(\mu\nu)}(\vec{V}_{S'S})$ are the matrix elements of the

Lorentz transformation from S to S' (we use the index notation $\alpha_{(\mu\nu)}$ to indicate that $\alpha_{(\mu\nu)}$ is not covariant under the new transformations) and $T^{\mu}_{\nu}(S'S) \equiv f_S(\vec{V}_{S'S})\alpha_{(\mu\nu)}(\vec{V}_{S'S})$ are the matrix elements of the corresponding more general linear transformation. In general the map f_S might depend on S, especially when the relative velocity of the coordinate systems is high enough. Thus we have the transformation rule

$$dS'^2 = f_S(\vec{V}_{S'S})dS^2, \qquad \text{for all} \quad S, S' \in \Sigma. \tag{3}$$

These transformations leave the speed of light invariant; indeed let $u^i = dx^i/dt$, $i = 1, 2, 3$ be the coordinate velocity in S then for $u = c$ in S and $u' = c'$ in S' we have $dS'^2 = dx'^{\mu}dx'^{\mu} = (c'_1 dt')^2 + (c'_2 dt')^2 + (c'_3 dt')^2 - c^2(dt')^2 = \left(c'^2 - c^2\right)(dt')^2 = f_S(\vec{V}_{S'S})dS^2 = f_S(\vec{V}_{S'S})dx^{\mu}dx^{\mu} = 0$, hence $c' = c$.

Remark 1. These transformations are *not* conformal transformations, because dS^2 and dS'^2 are not two metrics; see Definition 1.

We now study the important properties of these new transformations $T^{\mu}_{\nu}(S'S)$. For any $S, S', S'' \in \Sigma$ we have $dS''^2 = f_{S'}(\vec{V}_{S''S'})dS'^2 = f_{S'}(\vec{V}_{S''S'})f_S(\vec{V}_{S'S})dS^2$ and $dS''^2 = f_S(\vec{V}_{S''S})dS^2$, hence

$$f_{S'}(\vec{V}_{S''S'})f_S(\vec{V}_{S'S}) = f_S(\vec{V}_{S''S}). \tag{4}$$

In particular, we get $f_S(\vec{V}_{S'S})f_{S'}(\vec{V}_{SS'}) = 1$, $(dS'^2 = f_S(\vec{V}_{S'S})dS^2 = f_S(\vec{V}_{S'S})f_{S'}(\vec{V}_{SS'})dS'^2)$ which implies $f_S^{-1}(V_{S'S}) = f_{S'}(V_{SS'})$. For the matrix representation we find that they satisfy the *consistency condition*

$$T^{\mu}_{\sigma}(S''S')T^{\sigma}_{\nu}(S'S) = T^{\mu}_{\nu}(S''S), \qquad \text{for all} \quad S, S', S'' \in \Sigma, \tag{5}$$

that means $f_{S'}^{1/2}(\vec{V}_{S''S'})\alpha_{(\mu\sigma)}(\vec{V}_{S''S'})f_S^{1/2}(\vec{V}_{S'S})\alpha_{(\sigma\nu)}(\vec{V}_{S'S}) = f_S^{1/2}(\vec{V}_{S''S})\alpha_{(\mu\nu)}(\vec{V}_{S''S})$ for all S, S', $S'' \in \Sigma$.

More abstractly we write the consistency condition (5) as

$$T(S''S')T(S'S) = T(S''S), \qquad \text{for all} \quad S, S', S'' \in \Sigma, \tag{6}$$

whose algebraic meaning we will explain in Section 3.

Let $S_0 \in \Sigma$ be a fixed but arbitrary inertial frame, then for any $S \in \Sigma$,

$$dS_0^2 = f_S(\vec{V}_{S_0S})\delta_{(\mu\nu)}dx^{\mu}dx^{\nu}$$

(the Kronecker symbol $\delta_{(\mu\nu)}$ is not covariant under the general transformations). Define

$$\sigma_{\mu\nu} \equiv f_S(\vec{V}_{S_0S})\delta_{(\mu\nu)} \tag{7}$$

then from straightforward calculations we have

Proposition 1. *$\sigma_{\mu\nu}dx^{\mu}dx^{\nu}$ is invariant under the transformations $T(SS_0)$ for all $S \in \Sigma$, i.e.*

$$\sigma_{\mu\nu}dx^{\mu}dx^{\nu} = dS_0^2, \qquad \textit{for all} \quad S \in \Sigma. \tag{8}$$

Definition 1. Let S_o be a fixed absolutely isotropic inertial frame, i.e. there exists a function g such that $f_{S_o}(\vec{X}) \equiv g(|\vec{X}|)$. We define the *distance element* by

$$dS_o^2 = f_S(\vec{V}_{S_oS})dS^2 = f_S(\vec{V}_{S_oS})\delta_{(\mu\nu)}dx^{\mu}dx^{v} \equiv \sigma_{\mu\nu}dx^{\mu}dx^{\nu}, \tag{9}$$

where $\sigma_{\mu\nu} \equiv f_S(\vec{V}_{S_oS})\delta_{(\mu v)}$ is the metric tensor for general flat space-time. Note that Minkowski's space time is a special case with $f_S(\vec{V}_{S_oS}) \equiv 1$ for all $S \in \Sigma$.

The Lorentz transformations (both homogeneous and inhomogeneous) are special cases of our more general transformations, namely those with transformation factors equal to 1.

The transformation factor in (3) depends on the velocity $\vec{V}_{S'S}$ between S and S', so we regard f_S as a function on $\mathbb{R}^3$ for any given inertial frame $S \in \Sigma$. We call f_S the *factor function* of S. More precisely, for $S \in \Sigma$ let $f_S : \mathbb{R}^3 \to \mathbb{R}_+$, $f_S(\vec{V}) = f_S(v^1, v^2, v^3)$, $\vec{V} = v^i \vec{e}_i$, be a function on $\mathbb{R}^3$, where $\{\vec{e}_i, i = 1, 2, 3\}$ is the orthonormal basis of S. If for any $S' \in \Sigma$, $f_S(\vec{V}_{S'S}) = f_S(\vec{V})_{|\vec{V}=\vec{V}_{S'S}}$, then f_S is called the factor function of S, which gives the transformation factors from S to all other inertial coordinate systems $S' \in \Sigma$.

Theorem 1. *If the factor function of one $S \in \Sigma$ is given then the factor functions of all other inertial coordinate systems $S' \in \Sigma$ are determined.*

Proof. Let $\oplus$ denote the addition of velocity vectors. For any $S, S', S'' \in \Sigma$ we have $\vec{V}_{S''S} = \vec{V}_{S'S} \oplus \vec{V}_{S''S'}$ and the consistency condition becomes $f_{S'}(\vec{V}_{S''S'}) = \frac{f_S(\vec{V}_{S''S'} \oplus \vec{V}_{S'S})}{f_S(\vec{V}_{S'S})}$. With $\vec{V}' \equiv \vec{V}_{S''S'}$, we find $f_{S'}(\vec{V}') = f_S(\vec{V}' \oplus \vec{V}_{S'S})/f_S(\vec{V}_{S'S})$, for all $\vec{V}'$, $0 \le V' < c$, and all $S, S' \in \Sigma$. This expresses the factor function $f_{S'}$ of any $S' \in \Sigma$ in terms of the factor function f_S of S. ■

Let $\mathrm{Vec} = \{\vec{V} \in \mathbb{R}^3 | 0 \le V < c\}$ and $S \in \Sigma$. We denote the set of all factor functions generated by f_S by

$$\mathbf{F}_S = \left\{ f_{S'} \in C(\mathrm{Vec}, \mathbb{R}_+) | S' \in \Sigma, f_{S'}(\vec{V}') = \frac{f_S(\vec{V}' \oplus \vec{V}_{S'S})}{f_S(\vec{V}_{S'S})}, \quad \text{for all } \vec{V}' \in \mathrm{Vec} \right\}.$$

Proposition 2. *For any $S, S' \in \Sigma$ with $\vec{V}_{S'S} \neq 0$ we have $f_S = f_{S'}$ if and only if $f_S(\vec{V}) \equiv 1$.*

3 Algebraic structure of the transformations $T(S'S)$

Let $\mathbb{R}^4 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times (ic\mathbb{R})$ and let $\varepsilon = \{\varepsilon_\alpha | \alpha \in J\}$ be the collection of all events, where J is some index set. The event ε_α has coordinates X_α in the S frame: $X_\alpha \in \mathbb{R}^4$, $X_\alpha = (x^1_\alpha, x^2_\alpha, x^3_\alpha, x^4_\alpha)$, $x^4_\alpha \equiv ict_\alpha$. Denote $\mathbf{X} = \{X_\alpha \in \mathbb{R}^4 | \alpha \in J\}$ and let $T(S'S) : \mathbf{X} \to \mathbf{X}' = \{X'_\alpha \in \mathbb{R}^4 | \alpha \in J\}$ be a mapping such that $X'_\alpha = T(S'S)(X_\alpha)$ for all $\alpha \in J$. Then the consistency condition for the set $\{T(S'S) | S, S' \in \Sigma\}$ is $T(S''S')T(S'S) = T(S''S)$, for all $S, S', S'' \in \Sigma$. With this (2) becomes $dX' = T(S'S)dX = f_S^{1/2}(\vec{V}_{S'S})\alpha(\vec{V}_{S'S})dX$ where $\alpha(\vec{V}_{S'S})$ is the matrix for the Lorentz transformation from S to S'. We call the matrix $[T^\mu_\nu(S'S)] = \left[f_S^{1/2}(\vec{V}_{S'S})\alpha_{(\mu\nu)}(r\vec{V}_{S'S})\right] = \left[f_S^{1/2}(\vec{V}_{S'S})\alpha(\vec{V}_{S'S})\right]$ the *matrix representation* for the mapping $T(S'S)$.

More abstractly we have the following algebraic situation: Let $\mathbf{A}$ be a collection of sets and for any $A_\alpha, A_\beta \in \mathbf{A}$ let $T(A_\alpha A_\beta)$ be a transformation from A_β to A_α. Denote by $\mathbf{T_A}$ the set of all such transformations defined in $\mathbf{A}$. The product of two such transformations $T(A_\alpha A_\beta)T(A_\sigma A_\gamma)$ is only defined if $A_\beta = A_\sigma$, in which case $T(A_\alpha A_\beta)T(A_\beta A_\gamma)$ is called the *physical product* where the two transformations are successive from A_γ to A_β, then from A_β to A_α.

Definition 2. A set of transformations $\mathbf{T_A}$ is called a *physical group* if it is closed under the physical product; in other words if the transformations satisfy the *consistency condition*

$$T(A_\alpha A_\beta)T(A_\beta A_\gamma) = T(A_\alpha A_\gamma), \qquad \text{for all} \quad A_\alpha, A_\beta, A_\gamma \in \mathbf{A}. \tag{10}$$

Proposition 3. *A) For every α the set $\mathbf{T}_\alpha \equiv \{T(A_\alpha A_\beta) | A_\beta \in \mathbf{A}\} \subset \mathbf{T}_A$, has a left unit element and for every β the set $\mathbf{T}_\beta \equiv \{T(A_\alpha A_\beta) | A_\alpha \in \mathbf{A}\}$ has a right unit element.*

B) For every element of a physical group there exist a right and a left inverse, which are identical to each other.

C) The physical product is associative.

Theorem 2. *Let $S \in \Sigma$ and f_S be given. The set $\mathbf{T}_S = \{T(S'S'') = f_{S''}^{1/2}(\vec{V}_{S'S''})\alpha(\vec{V}_{S'S''}) \mid S', S'' \in \Sigma, f_{S''} \in \mathbf{F}_S\}$ (where $\alpha(\vec{V}_{S'S''})$ is the matrix of the Lorentz transformation from S'' to S') is a physical group, called a c-invariant group.*

4 Model case of factor functions

We give an example of factor functions which shows how these ideas can be realised and which can serve as a model. Let $S_0 \in \Sigma$ be a fixed inertial frame and fix a parameter $N > 0$. Define $f_{S_0}(\vec{V}) = f_{S_0}(V) \equiv \left(1 - (V/c)^N\right)^{-1}$, $V = |\vec{V}| < c$. This factor function of S_0 generates a set $\mathbf{F}_{S_0} = \left\{ f_S(\vec{V}) = \frac{f_{S_0}(\vec{V}\oplus\vec{V}_{SS_0})}{f_{S_0}(\vec{V}_{SS_0})} \;\middle|\; S \in \Sigma \right\}$, where $f_{S_0}\left(\vec{V} \oplus \vec{V}_{SS_0}\right) = \left(1 - B^N\right)^{-1}$ with $B = \left(1 + \vec{V}\cdot\vec{V}_{SS_0}/c^2\right)^{-1}\left[\left(1 + \vec{V}\cdot\vec{V}_{SS_0}/c^2\right)^2 - \left(1 - V_{SS_0}^2/c^2\right)\left(1 - V^2/c^2\right)\right]^{1/2}$.

The Lorentz model is nothing but the limiting case of this model as $N \to \infty$.

5 Dynamics

Now let us derive the equations of fundamental laws of nongravitational physics which are invariant under the c-invariant groups. We call these equations *c-invariant*. We will see that the transformation factors will appear in these equations. When we let all the transformation factors be 1, then all the equations will go back to their counter-parts in the Lorentz invariant theory. When we take some c-invariant groups with transformation factors having the same limiting behavior, some important theoretical results will be obtained.

5.1 *c*-invariant classical mechanics

Let S^* be the instantaneous rest frame of a particle and let $\vec{u} = \vec{V}_{S^*S}$ be the instantaneous velocity of the particle measured in the S-frame. The interval of proper time is

$$d\tau = \sqrt{-dS^{*2}/c^2} = \left[-f_S(\vec{V}_{S\Sigma})dS^2/c^2\right]^{1/2} = f_S^{1/2}(\vec{u})\gamma^{-1}dt, \qquad \text{where} \quad \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}}$$

is called the *Lorentz factor*. Define the 4-velocity $\mathbf{U}^\mu \equiv dx^\mu / d\tau = f^{-1/2}(\vec{u})\gamma dx^\mu / dt$ and the 4-momentum $P^\mu \equiv m_o\mathbf{U}^\mu$.

The particle mechanics invariant under the c-invariant groups is

$$\mathbf{F}^\mu = m_o\frac{d\mathbf{U}^\mu}{d\tau} = \frac{dP^\mu}{d\tau}, \tag{11}$$

where m_o is the rest mass of the particle and $\mathbf{F}^\mu$ is the 4-force determined by electromagnetical and gravitational fields through the corresponding formula. Denote $\vec{P} = (P^1,\ P^2, P^3)$ and $P^4 = iE/c$, then $\vec{P} = m_o f_S^{-1/2}(\vec{u})\gamma\vec{u}$ and with $\lambda \equiv f_S^{-1/2}(\vec{u})$

$$E = m_o f_S^{-1/2}(\vec{u})\gamma c^2 = m_o\lambda\gamma c^2. \tag{12}$$

Thus, $P^\mu P^\mu = P^2 - E^2/c^2 = -m_o^2c^2 f_S^{-1}(\vec{u})$, and hence $E^2 = P^2c^2 + m_o^2c^4 f_S^{-1}(\vec{u})$.

5.2 *c*-invariant quantum mechanics

The de Broglie wave of a free particle is

$$\psi = A\exp(i\hbar^{-1}\Lambda^{-1}P^\mu x^\mu) \tag{13}$$

where the transformation property of A is determined by the spin of the particle and Λ is defined as $\Lambda \equiv f_{S_0}(\vec{V}_{SS_0})$ with S_0 being a fixed absolutely isotropic inertial frame, (i.e. there exists a function g such that $f_{S_o}(\vec{X}) = g(|X|)$). The phase is invariant under c-invariant groups; indeed $\Lambda^{-1}P^\mu x^\mu = f_S(\vec{V}_{S_0S})P^\mu x^\mu = f_S(\vec{V}_{S_0S})\delta_{(\mu\nu)}P^\mu x^\nu = \sigma_{\mu\nu}P^\mu x^\nu$, where $\Lambda^{-1} \equiv f_{S_0}^{-1}(\vec{V}_{SS_0}) = f_S(\vec{V}_{S_0S})$.

Generally, we have

Theorem 3. *Let A^μ and B_μ be a contravariant and a covariant 4-vector under c-invariant groups respectively. Then $\Lambda^{-1}A^\mu$ and ΛB_μ are covariant and contravariant 4-vectors under c-invariant groups respectively.*

Proposition 4. *The c-invariant Klein–Gordon equation for free spin zero particles in the observer-frame S is*

$$\left(\Box - c^2\hbar^{-2}\underline{m}^2\right)\psi = 0, \qquad \textit{where} \quad \Box = \partial^\mu\partial_\mu, \quad \textit{with} \quad \partial^\mu \equiv \Lambda\partial_\mu \tag{14}$$

and

$$\underline{m} \equiv m_o\lambda\Lambda^{-1/2}, \qquad \textit{with} \quad \lambda \equiv f_S^{-1/2}(\vec{u}), \quad \textit{and} \quad \Lambda \equiv f_{S_0}(\vec{V}_{S_0S}). \tag{15}$$

We call $\underline{m}$ the *apparent mass* of the particle in S.

Proof. For any free particle with spin zero, we have $\psi = A\exp\left(i\hbar^{-1}\Lambda^{-1}P^\mu x^\mu\right)$, where A is a scalar. It is clear that ψ obeys (14) for one can easily check

$$\Box\psi = \Lambda\left(i\hbar^{-1}\Lambda^{-1}\right)^2 P^\mu P^\mu\psi = \hbar^{-2}m_0^2c^2\lambda^2\Lambda^{-1}\psi = \underline{m}^2c^2\hbar^{-2}\psi \tag{16}$$

which holds for any $S \in \Sigma$ since $\Lambda^{-1}P^\mu x^\mu$ is an invariant and A is a scalar. ■

Moreover, we have

Theorem 4. *The apparent mass $\underline{m}$ is an invariant under c-invariant groups, i.e. $\underline{m'} = \underline{m}$.*

Proof. This is true simply because

$$\underline{m} = m_o\lambda\Lambda^{-1/2}m_of_S^{-1/2}(\vec{u})f_{S_0}^{-1/2}(\vec{V}_{SS_0}) = m_of_{S_0}^{-1/2}(\vec{V}_{S^*S_o}),$$

which is independent of the choices of the observer-frame S. ■

For the Lorentz group we have $\lambda = 1$ and $\Lambda = 1$ for all $S \in \Sigma$, hence $\underline{m} = m_o$ and the c-invariant Klein–Gordon equation goes back to the Lorentz invariant Klein–Gordon equation.

Proposition 5. *The c-invariant Proca equation for free particles with spin 1 is*

$$\left(\Box - c^2\hbar^{-2}\underline{m}^2\right)\psi_\mu = 0, \qquad \textit{where} \quad \psi_\mu = A_\mu\exp\left(i\hbar^{-1}\Lambda^{-1}P^\mu x^\mu\right) \tag{17}$$

with A_μ being a 4-vector.

We see that $m_o\lambda$ instead of m_o appears in c-invariant equations of law of motion, Klein–Gordon and Proca equations (later we will see also in the c-invariant Dirac equation), where $\lambda = f_S^{-1/2}(\vec{u}) = f_S^{-1/2}(\vec{V}_{S^*S})$ and S^* is the instantaneous rest frame of the particle.

5.3 Limit $u \to c$

Now we consider the limit as $u \to c$, i.e. write $\vec{c} = c\vec{n}$ ($|\vec{n}| = 1$) and let

$$N_S(\vec{n}) \equiv \lim_{\vec{u}\to c\vec{n}} \lambda\gamma, \qquad \lambda \equiv f_S^{-1/2}(\vec{u}). \tag{18}$$

Our fundamental assumption is the following: There exists an inertial frame $S \in \Sigma$, such that

$$N_S(\vec{n}) \equiv \lim_{\vec{u}\to c\vec{n}} \lambda\gamma \equiv \lim_{\vec{u}\to c\vec{n}} f_S^{-1/2}(\vec{u})\gamma < \infty. \tag{19}$$

Under this assumption we have the Theorems 5, 6, 7 and the Results 1, 2, 3.

Theorem 5. *If $N_S(\vec{n}) < \infty$ for all $\vec{n}$ with $|\vec{n}| = 1$ for some $S \in \Sigma$, then $N_{S'}(\vec{n}') = \lim_{\vec{u}'\to c\vec{n}'} \lambda'\gamma' < \infty$ for all $\vec{n}'$ with $|\vec{n}'| = 1$, for all $S' \in \Sigma$, where $\lambda' = f_{S'}^{-1/2}(\vec{u}')$ and $\gamma' = \left(1 - u'^2/c^2\right)^{-1/2}$.*

Theorem 6. *Let $S \in \Sigma$ and denote $\Sigma_S = \{S' \in \Sigma | V_{S'S} = 0\}$. Then whenever there exists an $S_o \in \Sigma$ such that f_{S_o} is isotropic, then $f_{S'_o}$ is isotropic for each $S'_o \in \Sigma_{S_o}$ and f_S is not isotropic for each $S \in \Sigma \setminus \Sigma_{S_o}$.*

In case $N_S(\vec{n}) < \infty$ for all $\vec{n}$ with $|\vec{n}| = 1$ and all $S \in \Sigma$, we get the following results:

Result 1. The contravariant ultraviolet cut-offs are

$$\Lambda^\mu(\vec{n}) \equiv \lim_{\vec{u}\to c\vec{n}} P^\mu = m_o c N_S(\vec{n})(\vec{n}, i),$$

where we use the notion $A^\mu = (\vec{a}, b)$ to indicate that $A^i = a^i$, $i = 1, 2, 3$ and $b = A^4$.

We assume that there exists an $S_o \in \Sigma$ such that f_{S_o} is isotropic. Then in case $\lambda\gamma$ is bounded the upper bound of the 4-momentum for a particle observed in any $S \in \Sigma$ exists, and at least in case $\lambda\gamma$ is nondecreasing, it is given by

$$\Lambda^\mu \equiv \max_{\substack{\vec{n} \in \mathbb{R}^3 \\ |\vec{n}| = 1}} \Lambda^\mu(\vec{n}) = f_{S_o}^{1/2}(\vec{V}_{SS_o}) \frac{\sqrt{1 - V_{SS_o}^2/c^2}}{1 - V_{SS_o}/c} N_0$$

which gives the minimum nonzero lengths and time-intervals for particles in the sense of uncertainty, indicating a true meaning of “discrete” or “quantized” space-time and of any model for non-pointlike elementary particles, e.g. strings. Furthermore, photons and all the particles moving at speed c must have nonzero rest masses which are given by a l’Hospital type limit. For example, consider a photon with energy E in $S \in \Sigma$ which moves along the direction $\vec{n}$. Then its nonzero rest mass is

$$m_o = \frac{Ec^2}{\lim_{\vec{u}\to c\vec{n}} \lambda\gamma} = \frac{Ec^2}{N_S(\vec{n})} \neq 0.$$

In Einstein’s relativity, $\lambda \equiv 1$, $N_S(\vec{n}) = \infty$ and $E = h\nu$, hence $m_o = 0$ while $E = h\nu = 0c^2/\sqrt{1 - c^2/c^2} = 0c^2/0$ can be of any value for ν can be of any value. There is no limit process as there is in our theory.

In case $N_S(\vec{n}) < \infty$ for all $\vec{n}$ and all $S \in \Sigma$, for photons, $E = \Lambda h\nu$ (we will show this later), which can be given by (12) through a l’Hospital-type limit process:

$$E = \Lambda h\nu = m_o c^2 \lim_{\vec{u}\to c\vec{n}} \lambda\gamma = m_o c^2 N_S(\vec{n}),$$

where $m_o = c^{-2}(N_S(\vec{n}))^{-1}\Lambda h\nu$ is the nonzero rest mass of the photons with frequency ν and moving along the unit direction $\vec{n}$. It is impossible to make photons and any particles moving with speed c be at rest. Hence, the so-called "rest mass" of a particle moving with speed c is just a coefficient of proportionality between $N_S(\vec{n})$ and the energy of the particle measured in the S frame, and is independent of S for it is a scalar under c-invariant groups but dependent on both its energy and direction of motion.

Result 2. Since $\gamma \to \infty$ as $u \to c$, our assumption $N_S(\vec{n}) < \infty$ leads to

$$\lim_{u\to c} \lambda \equiv \lim_{u\to c} f_S^{-1/2}(\vec{u}) = 0 \qquad \text{for all} \quad S \in \Sigma. \tag{20}$$

Thus, every scalar or vector particle moving at speed c has *zero apparent mass*, i.e., $\underline{m} \equiv m_o\lambda\Lambda^{-1/2} = 0$. Then they obey the corresponding equations with vanishing mass terms by which the gauge invariances are characterized.

5.4 *c*-invariant classical electrodynamics

We can now study the equations of classical electrodynamics. The c-invariant classical electrodynamics is given by

$$\partial_\mu F^{\mu\nu}_{em} = -4\pi c^{-1} J^\nu \qquad \text{and} \qquad f^\mu_{em} = c^{-1} F^{\mu\nu}_{em} J_\nu, \tag{21}$$

where

$$F^{\mu\nu}_{em} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu = \Lambda^2(\partial_\mu A_\nu - \partial_\nu A_\mu) \equiv \Lambda^2 F^{em}_{\mu\nu}$$

and $J_\mu = \Lambda^{-1}J^\mu$, $J^\mu \equiv \rho^* U^\mu$ with $\rho^* \equiv dq^*/dV^*$, where dq^*, dV^* and ρ^* are charge element, volume element and charge density measured in $S^* \in \Sigma$ (the instantaneous rest frame of the charged particle). We call ρ^* the *proper charge density* which is frame-invariant as the proper time interval. We keep the assumption that charges are frame-invariant. Then $dq^* = dq$ and $\rho^* = dq/dV^*$. The kinematic effects of moving rods under c-invariant groups give $dV^* = (f_S^{1/2}(\vec{V}_{S^*S}))^3\gamma dV = \lambda^{-3}\gamma dV$. Hence $\rho^* = \lambda^3\gamma^{-1}\rho$. Now denote $A^\mu \equiv (\vec{A}, i\phi)$ which means $\vec{A} \equiv (A^1, A^2, A^3)$, $i\phi \equiv A^4$ and $\vec{E} \equiv -\nabla\phi - c^{-1}\partial\vec{A}/\partial t$, $\vec{B} \equiv \nabla \times \vec{A}$.

Then it is easy to check that (21) leads to

Proposition 6. *The c-invariant Maxwell equations are:*

$$\begin{aligned} &\nabla \cdot \vec{B} = 0, \\ &\nabla \times \vec{E} = -c^{-1}\partial\vec{B}/\partial t, \\ &\nabla \cdot \vec{E} = 4\pi\underline{\rho}, \\ &\nabla \times \vec{B} = c^{-1}\partial\vec{E}/\partial t + 4\pi c^{-1}\underline{\vec{J}}, \end{aligned} \tag{22}$$

where $\underline{\rho} = \Lambda^{-1}\lambda\gamma\rho^* = \Lambda^{-1}\lambda^4\rho$, *and* $\underline{\vec{J}} \equiv \underline{\rho}\vec{u}$.

Also (21) leads to a Lorentz-type force

$$F^{em}_{(\mu)} = q(\vec{E} + \vec{\beta} \times \vec{B}, i\vec{E}\cdot\vec{\beta}), \qquad \vec{\beta} \equiv \vec{u}/c, \tag{23}$$

where

$$F^{em}_{(\mu)} \equiv \frac{d\tau}{dt}\mathbf{F}^\mu_{em} \equiv \lambda^{-1}\gamma^{-1}\int f^\mu_{em} dV^*.$$

Without difficulty, we obtain $F^{\text{space}}_{(\mu)} = m_o c(1 + \vec{A}_S\cdot\vec{\beta})^{-1}\frac{d\lambda}{dt}\gamma^{-1}(-\vec{A}_S, i)$, $\vec{A}_S \equiv \vec{V}_{SS_o}/c$, with $F^{\text{space}}_{(\mu)} \sim N_o^{-2}$. When N_o is large enough, $F^{\text{space}}_{(\mu)}$ does not cause any practically measurable effect.

5.5 *c*-invariant quantum electrodynamics

We can now combine the previous results.

Proposition 7. *The c-invariant Dirac equation for a free particle with spin $\frac{1}{2}$ is*

$$\left(\gamma_\mu \partial_\mu + c\hbar^{-1}\Lambda^{-1/2}\underline{m}\right)\psi = 0. \tag{24}$$

where γ_μ are the Dirac matrices.

Result 3. Equation (24) tells us that neutrinos moving at speed c must have vanishing apparent mass and nonzero rest mass in every c-invariant theory with finite $N_S(\vec{n})$ and hence satisfy the c-invariant Dirac equation with vanishing mass term and a two-component theory.

We now study the electromagnetic coupling. In the presence of electromagnetic fields we obtain

$$\left[\gamma_\mu\left(\partial_\mu - iqc^{-1}\hbar^{-1}A_\mu\right) + c\hbar^{-1}\Lambda^{-1/2}\underline{\hat{m}}\right]\psi = 0, \tag{25}$$

where $\underline{\hat{m}} \equiv m_o\Lambda^{-1/2}\hat{\lambda} \equiv m_o\Lambda^{-1/2}f_S^{-1/2}(\hat{\vec{u}})$.

The replacement $P^\mu \mapsto -i\hbar\Lambda\partial_\mu - qc^{-1}\Lambda A_\mu = -i\hbar\partial^\mu - qc^{-1}A^\mu$ and the identity $\vec{u} = c^2\vec{P}E^{-1}$ give $\hat{\vec{u}} = -c^2(i\hbar\nabla + qc^{-1}\Lambda^{-1}\vec{A})/(i\hbar\partial_t - \Lambda^{-1}q\phi) = c^2\vec{P}_m E_m^{-1}$, where the two operators $\vec{P}_m$ and E_m^{-1} must be regarded as *commutative*. For an eigenfunction of the energy operator E_m^{-1} with eigenvalue E we have $E_m^{-1}\psi = (E - q\phi)^{-1}\psi$ and

$$\vec{P}_m E_m^{-1}\psi = (E-q\phi)^{-1}\vec{P}_m\psi, \qquad \left(\vec{P}_m E_m^{-1}\right)^2\psi = (E-q\phi)^{-2}\vec{P}_m^2\psi, \qquad \text{ect.}$$

For the electron in a hydrogen atom being at rest in any $S \in \Sigma$, we have $\hat{\vec{u}} = cR\nabla$ where $R \equiv -i\hbar\Lambda c/(E + \Lambda^{-1}e^2/r)$. For example, taking our model case

$$\begin{aligned}\lambda &\equiv f_S^{-1/2}(\vec{u}) = f_S^{-1/2}(\vec{V}_{S^*S}) = f_{S_o}^{-1/2}(\vec{V}_{S^*S_o})/f_{S_o}^{-1/2}(\vec{V}_{SS_o})\\ &= \Lambda^{1/2}\sqrt{1-\beta_o^N} = \Lambda^{1/2}\left\{1 - \left[1 - \Gamma^{-2}(1+\vec{A}_S\cdot\vec{u}/c)^{-2}\left(1 - u^2/c^2\right)\right]^{N/2}\right\}^{1/2}\end{aligned}$$

one finds

$$\hat{\lambda} \equiv f_S^{-1/2}(\hat{\vec{u}}) = \Lambda^{1/2}\left\{1 - \left[1 - \Gamma^{-2}(1 + R\vec{A}_S\cdot\nabla)^{-2}\left(1 - R^2\nabla^2\right)\right]^{N/2}\right\}^{1/2},$$

where $\Gamma \equiv (1-A_S^2)^{-1/2}$. The first approximation is $\hat{\lambda} = 1 - \frac{N}{2}\Lambda\Gamma^{-2}A_s^{N-2}R\vec{A}_S\cdot\nabla = 1 - \vec{D}_S\cdot(R\nabla)$, where $\vec{D}_S \equiv \frac{1}{2}N\Lambda A_S^{N-2}\left(1 - A_S^2\right)\vec{A}_S$. Since $f_S(\vec{V})$ is almost 1 within the velocity range of the particles in the accelerators and $u \ll c$ for electrons bound in atoms, we have a very good approximation:

$$\hat{\lambda} \approx 1, \qquad \underline{\hat{m}} \approx m_o\Lambda^{-1/2}.$$

So we obtain an approximate equation for electrons bound in an atom which is at rest in an inertial frame S:

$$\left(-i\hbar c\vec{\alpha}\cdot\nabla + \gamma_4 m_o c\Lambda^{-1}\right)\psi(\vec{r}) = \left(\Lambda^{-1}E + \Lambda^{-2}Ze^2/r\right)\psi(\vec{r}),$$

whose $N \cdot R$ approximation is $E = m_oc^2\left(1 - \frac{1}{2n^2}\Lambda^{-4}Z^2\alpha^2\right)$, $n = 1, 2, 3, \ldots$ and

$$\nu(i \to f) = m_oc^2h^{-1}\Lambda^{-5}\frac{1}{2}Z^2\alpha^2\left(n_f^{-2} - n_i^{-2}\right), \qquad \left(\alpha \equiv e^2\hbar^{-1}c^{-1}\right) \tag{26}$$

which gives the frequency spectrum of the photons emitted from the atom and observed in S; (the notion $(i \to f)$ meaning from initial to final state).

Now let us consider frequency shifts. Label

$$K_\mu \equiv (\vec{K}, i\omega/c) \equiv \hbar^{-1}\Lambda^{-1}P^\mu \equiv \hbar^{-1}\Lambda^{-1}(\vec{P}, iE/c), \qquad \omega = 2\pi\nu,$$

which is the covariant wave vector of a particle. By use of the transformation rule for a covariant 4-vector, we obtain

$$\nu/\nu' = f_S^{1/2}(\vec{V}_{S'S})\left(1 - \vec{n}\cdot\vec{V}_{S'S}/c\right)^{-1}\left(1 - V_{S'S}^2/c^2\right)^{1/2}, \qquad (\vec{n} = \vec{P}/|\vec{P}|). \tag{27}$$

If the particle is a photon, (27) gives the formula of frequency shifts. It is interesting that taking the point of view of an emission theory can also give (27). Let $\vec{u}$ and $\vec{u}'$ be the velocities of the same photon (as a "bullet") observed in the S and S' respectively. From the velocity addition law, we know that $u = u' = c$. However, the same velocity addition law gives

$$\lim_{\vec{u}\to c\vec{n}}\left(1 - u^2/c^2\right)\left(1 - u'^2/c^2\right)^{-1} = \left(1 - \vec{n}\cdot\vec{V}_{S'S}/c\right)^2\left(1 - V_{S'S}^2/c^2\right)^{-1}. \tag{28}$$

Thus, (12), (27), and (28) give

$$\nu/\nu' = f_S^{1/2}(\vec{V}_{S'S})(1 - \vec{n}\cdot\vec{V}_{S'S}/c)^{-1}\sqrt{1 - V_{S'S}^2/c^2},$$

which is identical with (27). Of course, light sources are not Galileo–Newton's "guns".

Let S_1 be the instantaneous rest frame of a moving atom and $\nu_{1(S_1)}(i \to f)$ be the spectrum of photons emitted from the atom and observed in the S_1-system. Using (26) one can write

$$\nu_{1(S_1)}(i \to f) = m_o c^2 h^{-1}\Lambda_1^{-5}\frac{1}{2}Z^2\alpha^2\left(n_f^{-2} - n_i^{-1}\right), \qquad \Lambda_1 \equiv f_{S_o}(\vec{V}_{S_1S_o}).$$

Let $\nu_{(S_1)}(i \to f)$ be the spectrum of the same photons observed in S. Using (27) one can obtain the frequency shifts of the spectrum:

$$\nu_{(S_1)}(i \to f) = f_S(\vec{V}_{S_1S})\left(1 - \vec{n}\cdot\vec{V}_{S_1S}/c\right)^{-1}\left(1 - V_{S_1S}^2/c^2\right)^{1/2}\nu_{1(S_1)}(i \to f).$$

The new formula for *Doppler shifts* is given by

$$\frac{\nu_{(S_1)}(i \to f)}{\nu(i \to f)} = f_S^{-9/2}(\vec{V}_{S_1S})\left(1 - \vec{n}\cdot\vec{V}_{S_1S}/c\right)^{-1}\left(1 - V_{S_1S}^2/c^2\right)^{1/2}. \tag{29}$$

When the source-speed V_{S_1S} is high enough, (29) is most sensitive by comparison to the possible deviation of the values of transformation factors from 1, because the transformation factor $f_S(\vec{V}_{S_1S})$ appears in the formula with the exponent $-9/2$. To test (29), we suggest accelerating lithium ions to sufficiently high speed and then observing their light spectrum. This will be a crucial test if $N_S(\vec{n})$ is not too large. In principles, such experiment will find the function form of the factor function of a laboratory-system S. Then according to Theorem 1 the factor function of S will determine the function forms of the factor functions of all inertial coordinate systems. In particular, letting $S' = S_o$ and $\vec{V}_{S''S_o} = \vec{V}_o$, we find the consistency condition that

$$f_{S_o}(\vec{V}_o) = f_S(\vec{V}_o \oplus \vec{V}_{S_oS})/f_S(\vec{V}_{S_oS}), \tag{30}$$

which must be exactly independent of the direction of $\vec{V}_o$ since f_{S_o} is isotropic. The unique solution $\vec{V}_{S_oS}$ which makes the right side of (30) independent of the direction of $\vec{V}_o$ is the velocity of the special and exactly isotropic inertial frame S_o relative to S and $-\vec{V}_{S_oS}$ is just the special velocity of the laboratory-system S relative to the special and exactly isotropic inertial frame S_o. Furthermore, the following theorem is trivially true.

Theorem 7. *If* $N_S(\vec{n}) \equiv \lim_{\vec{u}\to c\vec{n}} \Lambda\gamma \equiv \lim_{\vec{u}\to c\vec{n}} f_S^{-1/2}(\vec{u})\gamma < \infty$, *then there exists a* $u_b < c$ *such that* $f_S(\vec{V})$ *deviates markedly from* 1 *when* $V > u_b$, *i.e. the deviation of* $f_S(\vec{V})$ *from* 1 *is large enough in the case* $V > u_b$ *and will easily be tested by experiment if only the corresponding energy* E_b *is within the power of accelerators man can or will be able to build.*

Our unique results are mainly described in Result 1, 2 and 3, which are based on the assumption

$$\lim_{\vec{u}\to c\vec{n}} f_S^{-1/2}(\vec{u})\gamma < \infty. \tag{31}$$

We emphasize the following

Theorem 8. *If* u_b *is high enough and* E_b *is large enough, then there will never be direct crucial experimental evidence except for indirect evidence, which would be able to tell us Einstein relativity principle, Einstein symmetry and the relevant results, and our assumption with Result 1, 2 and 3 are true or not.*

6 Conclusion

The equations of laws of physics invariant under c-invariant groups are the analogue of the classical Lorentz invariant equations but with the transformation factors appearing in the equations; especially those from the instantaneous rest frames of particles and the special inertial coordinate system to an arbitrarily given observer – inertial-system. All the equations will go back to their counterparts in the Lorentz invariant theory, if one takes the Lorentz group. The Lorentz invariant theory is that with all the transformation factors equal to 1.

In comparison with Newton's principle, Einstein's theory of relativity is a refinement of the classical Newton theory. It is necessary to know what are the phenomena which are most sensitive to the change of an axiom and those which are not affected at all, in order to avoid doing useless experiments and center attention on those phenomena which are proved to be most sensitive in comparison to the change. Evidently, one wants to verify ones faith in the Einstein symmetry and his first postulate, which claims that when

$$\frac{m_o c^2}{\sqrt{1 - V_{S'S}^2/c^2}} = \tilde{E} = \tilde{M}c^2$$

the S and S'-system are still equivalent, one needs a new theory based on the changed axiom-structure. Only such a new theory can provide the information about sensitivity of various phenomena to the change of the axiom system and a possibility to examine different faiths carefully by indicating the most sensitive phenomena by comparison phenomena and relevant crucial tests. Even if in Theorem 8 u_b is too close to c such that E_b is too large to give any practically measurable deviations of the new theory from the current one within the energy region the objects of experiment will be able to reach, the following theoretical results are still valued if only

$$\lim_{\vec{u}\to c\vec{n}} f_S^{-1/2}(\vec{u})\gamma \neq \infty.$$

(1) The upper limit of the momentum-energy of a particle in any observer-inertial-system is finite. The upper bounds give natural and real ultraviolet cut-offs manifestly contravariant, and the minimum lengths and time intervals of particles in the sense of uncertainty indicating a true meaning of "discrete" or "quantized" space-time and of any model for non-pointlike

elementary particles. The unreasonable operations, "infinities minus infinities", will become "finite quantities minus finite ones" in the renormalizations due to the cut-offs.

(2) All particles with nonzero energies must have nonzero rest masses. The finite 4-momenta of the particles moving with speed c are given by l'Hospital-type limits rather than by the irrational calculation, "zero over zero", without acceptable limit process in Einstein's theory. All the particles moving with speed c obey the corresponding equations with vanishing mass terms. The nonzero rest masses of neutrinos consist with a two-component theory and the nonzero rest masses of photons and other free gauge particles consist with the gauge invariances characterized by vanishing mass terms if these particles move with speed c.

Generally, in any theory invariant under a c-invariant group there are gauge invariances for the gauge particles moving with speed c, which are characterized by the vanishing mass terms, no matter whether "the first postulate" is absolutely valid and the free gauge particles possess zero rest masses. The gauge invariances root in the frame-invariance of the finite transmission rate of interactions. The gauge particles obeying the corresponding gauge invariances possess their nonzero rest masses only in the theories invariant under those c-invariant groups which give the ultraviolet cut-offs.

A new gravitational theory whose zero-field limitation will give the non-Minkowski metric will be established in the future. The difference between it and the general relativity will not be big, but hopeful of success in removing the singularities (which, according to Einstein, must be removed), due to the upper bounds of the densities of particle groups.

Einstein underlines that one should not extend his general relativity to where the gravitational field is very strong and the density of matter is very large [4, p. 129]. In the absence of gravitation, is the Lorentz invariance an absolute truth? We certainly do not assert so. W. Rindler [6] expounds profoundly the nature of physical laws: "*... even the best of physical laws do not assert an absolute truth, but rather an approximation to the truth ... no amount of experimental agreement can ever "prove" a theory, partly because no experiment can ever be infinitely accurate, and partly because we can evidently not test all relevant instances ... Although special relativity is today one of the most firmly established theories in physics ... it is well to keep an open mind even here. ... some law of special relativity may one day be found to fail ... every theory is only a model ... theories should not stagnate in complacency.*" Also, T.D. Lee [5] said that it seems more than likely that our present understanding is transitory and our basic concepts and theories will further undergo major changes. Because of the irrational operations, infinities minus infinities in the renormalizations of the Lorentz invariant quantum field theories, Dirac [3] asserted that the foundations of the current theory must be reformed. In this paper, we actually reform the corner-stone of the current fundamental theory.

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