Mykhailo Vasyl'ovych Ostrohrads'kyi

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The paper describes life and research of the great Ukrainian mathematician Mykhailo Ostrohrads'kyi, whose 200th anniversary was marked in 2001. His development as a mathematician and his joint work with other most prominent scientists of his time are presented. Mykhailo Ostrohrads'kyi published more than 50 research papers, and laid foundation to many areas in calculus, differential equations and mathematical physics. The paper gives a review of some important results and formulae by Ostrohrads'kyi, shows their importance to further development of science.

On September 12, 2001 (it is September 24 according to the new calendar) there is 200th anniversary of Mykhailo Vasyl'ovych Ostrohrads'kyi, an outstanding Ukrainian mathematician, whose papers rightfully belong to the treasury of science and profoundly influenced development of mathematical analysis, the theory of differential equations, mathematical physics, and mechanics and rightfully belong to the treasury of science. His attention was always concentrated on extremely important problems of his time of both theoretical and practical nature. Similarly to Lagrange, he sought general approaches to the investigation of problems of different nature, discovering, as a result, original ways of reaching his goal.

Investigations of Ostrohrads'kyi embraced the entire spectrum of problems studied by prominent European mathematicians of that time, such as N. Abel, W. Hamilton, C. Gauss,



A. Cauchy, J. Lagrange, P. Laplace, J. Liouville, S. Poisson, J. Fourier, C. Jacobi, etc. For this reason, his results had certain intersections with results of these scientists, but they were never inferior to them in the generality of problems considered the rigorousness and originality of exposition, and efficiency of applications. He was a star of the first magnitude in the constellation of these outstanding personalities.

For better understanding of the significance of the scientific heritage of Ostrohrads'kyi, one must characterize, at least in general terms, the epoch in which his views were formed and his scientific activity developed. It fell mainly to the first half of the 19th century, one of the most remarkable and productive periods in the history of exact natural sciences. At the beginning of the 19th century, many fundamental works were published, such as the five-volume *Treatise on Celestial Mechanics* (1799–1825) by Laplace and the two-volume *Analytical Mechanics* (1811–1815) by Lagrange. In these works, deep in content and masterful in exposition, the results of predecessors in mechanics and astronomy were outlined and systematized, and foundations for future investigations in these branches of science were laid. It is quite possible that the general methods presented in the indicated work of Lagrange most strongly affected the formation of scientific views of Ostrohrads'kyi. He substantially developed and generalized them in numerous papers and lectures.

As V.A. Steklov noted about the first half of the 19th century in his speech at the celebration of the centenary of Ostrohrads'kyi, "this was an exciting period of time, when almost every day brought new ideas and new discoveries in various areas of mathematical physics and, with it, in mathematical analysis. Without no overstatement one can say that little has been added to the scientific ideas of that era, and today's efforts are mainly at streamlining and developing the theories of the great thinkers of that time, of extending their applications, and of perfecting the proofs". Mykhailo Ostrohrads'kyi participated in almost all mentioned areas of mathematical physics. His works in the theories of heat, elasticity, and attraction, as well as hydrodynamics, are not inferior in their significance to the works of the luminaries of science of that time.

Mykhailo Vasyl'ovych Ostrohrads'kyi, a son of a landowner Vasyl Ivanovych and of Iryna Andriyivna (born Ustymovych) was born in his father's estate in the village Pashenna of Kobelyaky povit (district), Poltava region¹.

At 9 Myshko entered the pension at the Poltava Gymnasium named *House for Education* of *Poor Gentry*, where one of the tutors was another famous person from Poltava, a poet I.P. Kotlyarevs'kyi. From here Mykhailo Ostrohrads'kyi was transferred to Romny Postal Office where he soon was awarded with a civil title of Collegial Registrar. In 1815 14-year old civil servant was dismissed from his position and entered the same pension once more. Ostrohrads'kyi was not distinguished by a particular diligence, but was noticed as a lively, capable and smart boy.

The following data with respect to first years of Ostrohrads'kyi's studies at the Poltava Gymnasium were discovered in its archive. During the first month of his studies at the Gymnasium he was marked as "average" by capability, "diligent" by diligence and "fair" by manners, and at the end of the first year of studies – "smart" by capability and "courteous" by manners.

During 1813 Ostrohrads'kyi, with the 9-grade system, had the following grades: 6 for psychology, 7 for moral philosophy, 2 for history and geography, 0 for Latin, French and German. We can see that Ostrohrads'kyi did not like languages, especially Latin.

In 1816 father took him to St. Petersburg to enlist him into one of the guard 's regiment, but, at the advice of P.A. Ustymovych, M.V. Ostrohrads'kyi's uncle, changed his decision and decided to enlist him to Kharkiv University.

Ostrohrads'kyi attended the university at first as a free listener, but later, in 1817, entered as a student of the department of physics and mathematics. During the first year of the university course and the first half of the second year he studied badly and continued to dream about the military service, and every moment he was ready to exchange the university for any regiment.

At the age of 17 on October 3, 1818 Ostrohrads'kyi completed his studies at the university, and received his student's diploma noting that he studied algebra, trigonometry, curve-line geometry, civil architecture, practical geometry, history, statistics of Russia and world history with very good success, and military studies, function theory, integral and variation calculus and Russian language and literature with excellent success.

In 1820 he had exams together with other students, and at the general meeting of the university his name was distinguished. At the time, seeing Ostrohrads'kyi's success, the professor of mathematics T.F. Osipovs'kyi wanted to award Ostrohrads'kyi with the candidate's degree. To get this degree Ostrohrads'kyi had to take the exam on philosophy, but the philosophy professor refused to take an exam for the reason he did not attend lectures on philosophy.

The Ostrohrads'kyi went to the university management, produced his diploma and gave it to professors who had a meeting, with the request to "remove his name from the student list".

Mykhailo Vasyl'ovych went to the village to his father, stating his firm intent to go abroad and to study with famous French mathematicians. Father listened to his son and quite favored his intent.

In May 1822 Ostrohrads'kyi started his journey, but he was robbed in Chernihiv. Father gave him money once more, and in August of the same year he was in Paris already. Having reached his goal with great difficulties, Ostrohrads'kyi attended lectures in Sorbonna and Collége de

¹Biographical data were taken from the sketch by P.I. Trypols'kyi [1].

France, and his bright talent attracted attention of famous French mathematicians: Laplace, Fourier, Ampere, Poisson, Cauchy etc. He was very friendly with the two latter researchers, and later exchanged letters with them, and he was accepted in Laplace's home as a family member.

The scientific talent of Ostrohrads'kyi was powerful, versatile, and, at the same time, original. In only six years of his stay in Paris, which was the center of mathematical research at that time, he got well informed about diverse new ideas and theories, concentrated on the most important problems that were the object of the work of the constellation of French genii (Laplace, Poisson, Cauchy, Fourier, etc.), and succeeded in their solution, getting ahead of these scientists in many issues. In the memoir *On Definite Integrals Taken between Imaginary Limits* submitted to the Académie Française in 1825, Cauchy expressed this by the following words: "Monsieur Ostrohrads'kyi, a young Russian man gifted by an extraordinary insight and very skillful in the analysis of infinitesimals, also applied these integrals and, transforming them to ordinary ones, gave a new proof of the formulae mentioned above, and communicated other formulae, which I now present ..." In his works, as we already noted, Cauchy repeatedly referred to Ostrohrads'kyi. He reviewed his scientific works and was one of the numerous French mathematicians who enthusiastically supported the candidacy of Ostrohrads'kyi for the election as an *Immortal* of the Académie Française. In 1856, he was elected the corresponding member of this academy.

The real mathematical debut of Ostrohrads'kyi took place in 1826, when he submitted his *Mémoire sur la Propagation des Ondes dans un Bassin Cylindrique* to the Académie Française. Under various additional physical assumptions, the problem of wave propagation on the surface of water was studied by Newton, Laplace, Lagrange, Cauchy, and Poisson. The main input of M.V.Ostrohrads'kyi to this issue was that he was the first who considered this problem in a closed cylinder of finite depth. Poisson and Cauchy, who were present at the talk given by Ostrohrads'kyi, highly evaluated the results presented, after which it was decided to publish them in *Mémoires Présentées par Divers Savants*. This was a great honor for Ostrohrads'kyi, who was only 25 years of age. This success strengthened the reputation of the scientist. Warm relationships between him and French mathematicians, such as Cauchy, Poisson, J. Sturm, G. Lamé, etc., were established and lasted for many years.

In 1828, Ostrohrads'kyi moved to St. Petersburg. Only at that moment, after coming back to Russia, Ostrohrads'kyi was appreciated by his compatriots, and a circle of people who loved mathematics was established around him at once, who wanted to find out about new views and methods in calculus. In the same year, in 1928 (on December 17) the Imperial Academy of Science elected him as Adjunct of Applied Mathematics, in 1830 he received the title of Extraordinary Academician, and in a year – the title of Ordinary Academician. In July 1830 he was sent to Paris with a research purpose and at that time presented to the Paris institute his course of celestial mechanics, where he showed great independence, mainly in simplification of explanation of general methods. Arago and Poisson, having considered this work at the request of the Paris Academy, awarded Ostrohrads'kyi with a praising reference that was finished by the following words: "We believe that the paper by Ostrohrads'kyi deserves the Academy's praise and approval"; in this Arago puts Ostrohrads'kyi's name along with that of immortal Laplace.

Inspired by the first successes, Ostrohrads'kyi set the grand problem of presentation of various sections of mathematical physics by means of mathematical methods. In one of his reports submitted to the St. Petersburg Academy in 1830, he wrote: "The followers of Newton developed the great law of universal gravitation in detail and applied mathematical analysis to numerous important problems in general physics and physics of weightless substances. The collection of their works about the system of universe forms the immortal folios of *Celestial Mechanics*, from which astronomers will take the elements for their tables for a long time. However, physical and mathematical theories are still not unified; they are distributed over numerous collections of academic memoirs and are investigated by different methods, often very doubtful and imperfect; moreover, there are theories developed but never presented. I set it as my aim to combine these theories, present them by using a uniform method, and indicate their most important applications. I already collected the necessary materials on the motion and equilibrium of elastic bodies, propagation of waves on the surface of incompressible liquids and propagation of heat inside solid bodies and, in particular, inside the globe. However, these theories will constitute only the necessary part of the entire work, which will also embrace the distribution of electricity and magnetism in bodies capable of being electrified or magnetized through electrodynamical influence, motion of electric fluids, motion and equilibrium of liquids, action of capillarity, distribution of heat in liquids, and probability theory; in this last part, I will dwell upon several issues in which the famous author of Celestial Mechanics was apparently wrong."

In this respect it is interesting to recall that D. Hilbert, who, as a true mathematician, was concerned with the absence of order in the triumphal progress of physics at the beginning of the 20th century and decided to give a mathematical presentation of physics by using the axiomatic approach (the sixth Hilbert problem). However, despite his deep faith in the omnipotence of the axiomatic method and its capability to bring an order into chaos, Hilbert realized that mathematics alone is insufficient for the solution of all physical problems. Although Hilbert spent a lot of effort and time to be well informed about new physical investigations, he failed to implement his plan concerning physics.

In the most general statement, the problem indicated was formulated by Ostrohrads'kyi in his report made at a session of the St. Petersburg Academy of Sciences on November 5, 1828, and published as an academic edition in 1831 in French under the title *Note sur la Theorie de la Chaleur*. Steklov wrote the following words with respect to this paper: "After Fourier constructed the differential equation of heat propagation in solids, the need for formulation of techniques to determine the temperature of a body that is sought for according to conditions of the problem.

Fourier himself and also Poisson considered the cases of cooling of a solid ball, cylinder, cube and rectangular parallelepiped.

In all these cases Fourier employed the same technique knows now as the Fourier method, but he was unlikely to see the property of its generality in its total. At least we cannot see that from Fourier's research papers, and I am hardly wrong to say that the Fourier method in all its generality was first formulated by Ostrohrads'kyi, and then (in 1829) by Lamé and Duhamel."

In this research Ostrohrads'kyi in part went ahead of Cauchy who in 15 years in his memoir *Recherches sur les intégrales des équations linéares aux differences partielles* obtained the same results once more, and in the note to this memoir Cauchy said: "I would like to compare the theorems I found with those obtained by Ostrohrads'kyi in one of his memoirs, but having bad memory and even not knowing whether this memoir by Ostrohrads'kyi was published anywhere, I am unable to do that". Evidently that the memoir by Ostrohrads'kyi being considered contains just the same conclusions Cauchy was interested in, or at least, part of them.

Ideas of Ostrohrads'kyi's report of 1828 were continued in his two Notes on the Theory of Heat, submitted to the St. Petersburg Academy of Sciences on September 5, 1828, and July 8, 1829. Maybe, this title does not adequately reflect the content of these notes, but it indicates that, in the 1820s, the analytical theory of heat was the leading topic in mathematical physics (for the most part, this is true for Paris, where Ostrohrads'kyi worked in 1822—1828).

The results of the *Notes* are important not only from the viewpoint of their significance for physics. It is difficult to overestimate their general mathematical significance because, on the one hand, they laid the foundation for important theories, which has been successfully developed up to now, and, on the other hand, the statements obtained therein constitute a part of the foundations of contemporary mathematical analysis.

In this context, the first note is the most important. It consists of two parts. In the first part, the general scheme of the solution of boundary-value problems in mathematical physics is described. The formula for the transformation of the volume integral of the divergence type into a surface integral was derived that now is an integral part of any calculus textbook, and is called the Ostrohrads'kyi–Gauss formula.

The appearance of this formula was stimulated by the needs of potential theory, theory of heat, and variational calculus. The first steps related to volume integrals were made by Lagrange, who found a method for their calculation and gave a formula for a change of variables that generalizes the corresponding Euler formula for double integrals. As for the surface integrals, the *Analytical Mechanics* of Lagrange (1813) contains only certain notes related to specific cases. However, the development of electrostatics and the theory of magnetism expanded the circle of problems of potential theory. Furthermore, the investigation of the distribution of static electricity over the surface of a body led to the necessity of introducing the notion of surface integral. It first appeared in the paper by Gauss published in 1813 and related to potential theory, and some theorems from this work can be regarded as partial cases of Ostrohrads'kyi formula. The formula itself was not shown in the indicated work by Gauss.

Hence, the first great merit of Ostrohrads'kyi lies in the fact that he was the first who realized the mentioned formula (Ostrohrads'kyi–Gauss formula) is of independent interest and indicated its general mathematical importance. In his prominent work of 1834 on variational calculus, he extended this formula to the case of arbitrarily many variables. Its vector interpretation was given in the *Treatise on Electricity and Magnetism* by J. Maxwell, who stressed the priority of Ostrohrads'kyi in the discovery of this formula.

The second substantial result, which is also contained in the first part of the mentioned note, is the introduction of an adjoint operator L^* for a linear differential operator L of arbitrary order with constant coefficients and the derivation of the integral relation for them. For many years, numerous mathematicians worked on the generalization of this formula, and today it is one of the cornerstones of the entire theory of boundary-value problems for differential and difference equations.

The second part of the note was devoted to the application of the general scheme presented in the first part to problems of heat propagation in solid bodies of arbitrary form, namely, to the solution of the mixed heat propagation problem in a bounded domain G with smooth boundary ∂G .

The key point of the work considered was the hypothesis that the spectrum of problem under consideration is discrete and that spectral decomposition of an arbitrary function f(x) inside the domain G. M.V. Ostrohrads'kyi understood that decomposition gives mapping of the function only inside the domain G.

On this occasion, Ostrohrads'kyi wrote: "I think that the series of the decomposition obtained always converges, but it is very difficult to prove this wonderful property in the general case". These words indicate that Ostrohrads'kyi was aware of the complexity of the problem of convergence of such series. Indeed, at those times, numerous fields of mathematical analysis did not have necessary tools not only for solving this problem, but even for getting started with it. The validity of the hypothesis advanced by Ostrohrads'kyi was completely confirmed in the 1960s.

Ostrohrads'kyi's decomposition formula has a universal character because it is also applicable to non-self adjoint boundary value problems and for the case when the domain G is not bounded.

"Finally, note that the eigenvalues of problem under consideration are always real, which is a consequence of the law of propagation of heat, but even this general fact must be established by mathematical analysis", Ostrohrads'kyi wrote in the same note. This means that, unlike many known scientists (Poisson, Laplace, Fourier, Poincaré, etc.) who worked in the field of mathematical physics and mechanics and thought that the rigorousness requirements can be weakened in these fields science, Ostrohrads'kyi had an opposite opinion consonant with the convictions of Gauss, Cauchy, and Abel. In the second of his Notes on the Theory of Heat, Ostrohrads'kyi, for the first time, solved a mixed heat propagation problem with the difference that a function T(t, x) instead of zero enters the right-hand side of the boundary condition, i.e., in the case where this condition is inhomogeneous. This problem was considered earlier by Laplace and Poisson in the case where T(t, x) does not depend on t. Ostrohrads'kyi reduced the problem with an inhomogeneous boundary condition to a problem with a homogeneous boundary condition, but for an inhomogeneous equation whose solution was sought in the form of an infinite series. The Ostrohrads'kyi method of reduction of an inhomogeneous boundary-value problem to a homogeneous one is presented in modern textbooks on mathematical physics as the Duhamel principle. Indeed, J. Duhamel solved this problem simultaneously with Ostrohrads'kyi, but he published his result in 1833, whereas Ostrohrads'kyi published his note in the Mémoires de l'Academie des Sciénces de St.-Pétersbourg in 1831.

The systematic investigation of the problem of expansions of the mentioned type in the eigenfunctions of the operator was continued by Ostrohrads'kyi's followers, in particular, by M.G. Krein, O.Ya. Povzner, I.M. Glazman, Yu.M. Berezans'kyi, V.O. Marchenko and by their students.

Among other works of Ostrohrads'kyi that significantly influenced the subsequent development of the theory of partial differential equations and variational calculus, a special place belongs to his fundamental work Mémoire sur le Calcul des Variations des Integrales Multiples submitted to the St. Petersburg Academy of Sciences on January 24, 1834. This memoir immediately drew the attention of mathematicians. In 1836, it was reprinted by the known Crelle's Journal für die reine und angewandte Mathematik, and its complete English version appeared in 1861 in A History of the Calculus of Variations during the 19th Century by I. Todhunter. It was the paper where fundamental results on the integral calculus of functions of many variables were presented. These results are regarded as classical for a long time already, and, up to now, they serve as the main tool in the theory of partial differential equations. First of all, this concerns Gauss–Ostrohrads'kyi formula in the case of arbitrary multiplicity n, the rule of location of the integration limits with respect to each variable when passing from an n-fold integral to a repeated integral, and the method for finding the derivative with respect to a parameter of a multidimensional volume integral with a variable limit of integration that, together with the integrand, depends on this parameter. In the same work, for the first time, Ostrohrads'kyi introduced (simultaneously with Jacobi) the notion of functional determinants (Jacobians). The developed foundations of integral calculus enabled Ostrohrads'kyi to completely solve the problem of calculation of the variation of an *n*-fold integral with variable limits of integration. Note that, under certain restrictions on the domain of integration, a formula for the first variation was obtained earlier by Euler for n = 2 and by Lagrange for n = 3. Without additional restrictions, in the case n = 2, the corresponding formula was established by Poisson simultaneously with the general case considered by Ostrohrads'kyi.

In the same memoir, Ostrohrads'kyi actually showed that the problem of variational calculus on the extremum of a multiple integral is equivalent to the problem of finding a certain solution of a partial differential equation. Later, this fact, which Riemann called the Dirichlet principle, drew the attention of Gauss, Thomson, and Dirichlet. It was established that this principle plays a key role in numerous variational methods for the solution of boundary-value problems for differential equations. A considerable contribution to the development of these methods for various classes of equations was made by mathematicians from Ukraine such as M.M. Bogolyubov, M.M. Krylov, M.P. Kravchuk, N.I. Pol's'kyi, Yu.D. Sokolov, and their followers.

In connection with the investigations carried out by Ukrainian mathematicians, in particular, at the Institute of Mathematics of the Ukrainian Academy of Sciences, it is reasonable to recall Ostrohrads'kyi's work *Note sur la Méthode des Approximations Successives* (1835) devoted to the integration of the nonlinear Duffing equation using the expansion in the small parameter *a*.

Later, it became clear that this equation plays an important role in the investigation of the process of pitching and rolling of a ship. Much later, the method of a small parameter received wide recognition due to the works of Poincaré, O.M. Lyapunov, M.M. Krylov, M.M. Bogolyubov, Yu.A. Mitropol'skii, and their students. Thus, the Ostrohrads'kyi method was a predecessor of the theory of nonlinear oscillations.

Besides the aforementioned programmatic works, in which Ostrohrads'kyi laid the foundations of the theory of partial differential equations, he also wrote many papers related to the integration of specific equations of mathematical physics and mechanics. Among them, one should mention his large (100 p.) work *Memoir on Differential Equations Related to the Isoperimetric Problem*. Among other important results presented in this memoir, it was shown that all differential equations of variational (Euler–Lagrange) problems with one independent variable can be reduced to canonical systems. Most textbooks on the theory of ordinary differential equations contain the Ostrohrads'kyi–Liouville formula published by M.V. Ostrohrads'kyi in his note On Linear Differential Equation of the n-th Order (1838) (in the case n = 2, it was obtained by Abel in 1827). In the works of Liouville, there is no this very formula.

M.V. Ostrohrads'kyi wrote 54 research memoirs, all in French, 50 of these were read at the meetings of the Russian Academy of Sciences and published in its editions, and others were published in the editions of the Paris Academy of Sciences.

As to the appearance of the manuscripts of the great mathematician we can say the following. He was very unwilling to do any rewriting.

He was a brilliant lecturer, and merits of his lectures were dependent a lot on his mood. Sometimes he gave the whole lecture on mechanics or higher mathematics not using a blackboard, if even complicated formulae were to be introduced.

He lectured with a great passion; wrote huge letters and for this reason made the blackboard full very fast, and then rushed to a large table covered by black impregnated fabric, continued to write at it and then lifted it to show to the listeners what was written. With his passionate lecturing he got tired very soon and sat to rest for a few minutes, drinking a lot of water.

He had a very good memory, remembered many historical and literature works that he read when he was young: knew many poems by heart, his favorite poet was T.H. Shevchenko, almost all poems of which he knew also by heart. His handwriting was so bad that even his close relatives could not read it.

He did not interfere into household issues at all, his wife dealt with that; he preferred walks at hand with his servant Shchak and philosophizing on different issues.

He rarely got ill, and with no problem sustained severe Petersburg climate after the south. He could be often seen at the Neva embankment under a strong rain without an umbrella and galoshes; note that he hated polished boots. In 1830 he had to be treated maybe for the first time. The matter was that during his trip to Paris he injured his eye because of careless using a phosphor match, and he had to go to a doctor. But prompt departure to Russia did not allow him to complete his treatment in Paris, and he got a cold at his eye while going back by sea and after his return to Paris lost his eye at all because of unsuccessful treatment.

In 1831 Ostrohrads'kyi got married, secretly to his father, to Maria Vasylivna Kupfer from Livland that brilliantly wrote verses in German, played and sang, and he encouraged her in all ways to perfect herself in these arts. At the end of his life Ostrohrads'kyi became very religious, and he had an icon-lamp burning even during not so important holidays. His mother's shadow reportedly told him: "Mykhailo, believe and pray!" From that time he became religious.

Many foreign scientific institutions elected Ostrohrads'kyi as their member: he was awarded by one of the most honorary titles for a scientists – a title of a corresponding member of the Paris Academy of Sciences, and titles of a member of Turin, Rome, American Academies, and a title of Honorary Doctor of the Alexander University. Among all that he was especially proud by the title of the member of the American Academy.

Ostrohrads'kyi made history not only as a first-rank scientist. He was a great teacher whose activity had a decisive influence on the increase of the level and role of science, first of all, mathematics, mechanics, and engineering, in the Russian Empire. Any other scientist and pedagogue of the first half of the 19th century can hardly be compared with him in this respect. The time of Euler with his fundamental achievements in the Russian mathematics and mechanics was followed by a certain fall. No systematic investigations were performed in these directions. This fall lasted till the appearance of Ostrohrads'kyi. After he moved to St. Petersburg, this city became the center of the mathematical life of Russia. His scientific works, inimitable lectures, and gifted disciples indicated the rise of the Russian science. Ostrohrads'kyi was an active promoter of new physical and mathematical achievements and the author of many textbooks on mathematics and mechanics, which were used by several generations of scientists and engineers. It is difficult to find a scientific institution of St. Petersburg where he did not give lectures. He devoted much time to pedagogic activity, and, thus, less time left for his scientific work. "A man, without doubt, of brilliant mind", P.L. Chebyshev wrote about Ostrohrads'kyi, "he did not accomplish even a half of what he could have done if he were not "bogged down" with tiresome permanent teaching work". However, this teaching "bog" made its great input into the progress of mathematics and physics in Russia. Under the influence of two Ukrainians, namely, Ostrohrads'kyi and V.Ya. Buniakowski, the first scientific schools were created in these directions, whose branches gave the world such renowned scientists as P.L. Chebyshev, M.E. Zhukovs'kyi, A.M. Lyapunov, V.A. Steklov, G.F. Voronoi, S.A. Chaplygin, etc.

Mykhailo Vasyl'ovych died unexpectedly. In summer of 1861 he came to his own estate in Ukraine and caught cold. Instead of treatment he decided to go to St. Petersburg, but had to stay in Poltava because of his illness. He died on December 20, 1861 (1 January, 1862) at midnight.

The last will of Ostrohrads'kyi was to be buried, as Shevchenko, in Ukraine. In accordance with his will, he was buried in his home village of Pashenna (now Pashenivka), Poltava province. Two hundred years ago, this land gave the world Ostrohrads'kyi, who, in the period of rapid development of science at the beginning of the 19th century, was the only Slavonian who, together with the glorious team of West-European scientists, created the foundations of modern mathematics, physics, and mechanics.

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