Proceedings of Institute of Mathematics of NAS of Ukraine 2002, Vol. 43, Part 1, 339–343

High-Frequency Absorption by a Soliton Gas in One-Dimensional Magnet

Galyna OKSYUK

Institute for Low Temperature Physics & Engineering, 47 Lenin Ave., Kharkov 61103, Ukraine
E-mail: *oksyuk@ilt.kharkov.ua*

The additional mechanism of the super-high-frequency power absorption in the quasy-one-dimensional antiferromagnet was considered. This absorption works due to the generation of the stationary nonlinear excitations of kinks type. The estimation of the effect is obtained for the real physical system. The shape of the signal of absorption is analyzed for some values of the external magnetic field. The quantity of the effect is detectable.

1 Introduction

We investigate theoretical problem of the super-high-frequency (SHF) field absorption by a gas of kink type solitons in the model of one-dimensional easy axis antiferromagnet (AFM) to show that the effect of linear response can be visible. The frequencies of solitons are comparable to the frequencies of magnons, and can be in intersection with the second ones. The external stationary homogeneous magnetic field, applied along the easy axis, causes the phase of system state, the eigenvalue frequencies spectrum of this phase and the value of the gap, particularly. The external microwave magnetic field is applied at the same direction. The shape of the expected absorption signal has the marked intense and is analyzed for some values of the external constant magnetic field.

The paper has the following structure. In the Section 2, the known results about the magnetization created by one kink [1, 2, 3], we obtain using the method of adiabatic approximation. This way turns out very convenient for the following calculation of the contribution of the weak uniform magnetic field into the energy of interaction between the kink and magnetic field, and to the magnetization also (see (1), (6) and (7)). The Section 3 is devoted to the calculation of the average energy absorbed from the external field over one period of our system. The theoretical investigations are illustrated by the numerical calculated curves of the dependence of the absorbed capacity on the frequency for some values of external magnetic field. The calculation of the quantity of the effect is based on the data of computer simulation, and was done with papameters of well-investigated quasi-one-dimensional AFM system, which admits the existence of the soliton excitations [4, 5]. We discuss obtained results in the Conclusion.

2 "Mechanical" aspects of solitons

Familiar model of one-dimensional two-sublattice AFM in an external magnetic field was considered in the paper of Bar'yakhtar and Ivanov [2]. This system was described in terms of weak FM $\vec{m}$ and AFM $\vec{l}$ vectors, such that $(\vec{m}, \vec{l}) = 0$, $\vec{m}^2 + \vec{l}^2 = 1$ (here $\vec{m} = \frac{\vec{M}_1 + \vec{M}_2}{2M_0}$, $\vec{l} = \frac{\vec{M}_1 - \vec{M}_2}{2M_0}$, and $\vec{M}_1$, $\vec{M}_2$ are the sublattices magnetizations, $|\vec{M}_1| = |\vec{M}_2| = M_0$). This formulation of the effective equations for magnetizations of the two subblattices was obtained for the natural assuming for AFM that the energy of relativistic interaction is small comparing to the exchange energy. The magnetization $\vec{m}$, created by one kink, was expressed in terms of $\vec{l}$ and $\partial\vec{l}/\partial t$

$$\vec{m}(x,t) = \frac{2}{\omega_0\delta}\left[\frac{\partial\vec{l}}{\partial t},\vec{l}\right] + \frac{2}{\delta}\left[\vec{h} - (\vec{h},\vec{l})\vec{l}\right], \tag{1}$$

where $\omega_0 = 2\mu_0 M_0/\hbar$, μ_0 is the Bohr magneton, and M_0 is an equilibrium magnetization. In angular variables θ and φ for vector $\vec{l}$, $|\vec{l}| = 1$, $l_z = \cos\theta$, $l_x + l_y = \sin\theta\exp(\imath\varphi)$,

The well-known [2] nonperturbed kink type solutions were obtained

$$\begin{aligned} &\cos\theta = \sigma\tanh B(x - vt - x_0), \\ &\varphi = \omega t - \varphi_0 + \Delta(x - vt - x_0), \end{aligned} \tag{2}$$

where $\sigma = \pm 1$, $B = \frac{\kappa(v;\omega)}{1-v^2}$, $\kappa^2(v;\omega) = \gamma^2(1-v^2) - (\omega - h_3)^2$, $\gamma = (c/\omega_0)\sqrt{(\beta/\alpha)}$, $\Delta = \frac{v}{1-v^2}(h_3 - \omega)$.

The method of adiabatic approximation proposed in our paper allows to justify the results obtained earlier [1, 2, 3] as well as to study the further applications.

The dynamics of free kink in 4-dimensional phase-space (X, Φ, I_1, I_2) is defined by the Hamiltonian equations

$$\frac{dI_1}{dt} = -\frac{\partial\mathcal{H}_0}{\partial X}, \qquad \frac{dI_2}{dt} = -\frac{\partial\mathcal{H}_0}{\partial\Phi}, \qquad v = \frac{dX}{dt} = \frac{\partial\mathcal{H}_0}{\partial I_1}, \qquad \omega = \frac{d\Phi}{dt} = \frac{\partial\mathcal{H}_0}{\partial I_2}. \tag{3}$$

Here X and Φ are the parameters defining the kink spatial arrangement and form, correspondingly, I_2 is the adiabatic invariant of two-parameter solution of the Landau–Lifshitz equations, I_1 is the field impact, $\mathcal{H}_0$ is an unperturbed Hamiltonian. For the first two variables we can write the definitions $\Phi(t) = \int dt\omega(t)$, $X = \int dtv(t)$. The variables $I_1 = I_1(v,\omega)$, $I_2 = I_2(v,\omega)$ will be defined later. The solution (2) can be rewritten in the form

$$\varphi = \Phi + \Delta(v,\omega)(x - X), \qquad \cos\theta = \sigma\tanh B(x - X).$$

The Lagrange function L_0 in new variables can be obtained as follows:

$$L_0 = I_1\frac{dX}{dt} + I_2\frac{d\Phi}{dt} - \mathcal{E}_0(I_1, I_2),$$

where $I_1 = 2\gamma^2 v/\kappa(v,\omega)$, $I_2 = 2(\omega - h_3)/\kappa(v,\omega)$.

In [11] it was shown that with a week uniform magnetic field $h(\vec{t}) = \vec{h_0}\cos 2\Omega t$, $(\vec{h_0}, \vec{M}) \ll \mathcal{H}_0$, which is polarized along the easy magnetization axis, the breather dynamics equations (the adiabatic approximation equations) will have the form (3) as before, where $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm int}$. Moreover, it was investigated the FM linear response to the SHF field with the frequency the same with the initial soliton one. It was expected that under the weak uniform magnetic field the resonance interaction between the breathers having frequency $\omega = \Omega$ and the external magnetic field can exist, but the numerical calculations showed that the effect was small sufficiently.

The relation for $\mathcal{H}_{\rm int}$ for our AFM system (the energy of interaction between the kink and magnetic field) obtained by us is the following:

$$\mathcal{H}_{\rm int} = \mathcal{H}_1\exp(\imath\Phi) + \bar{\mathcal{H}}_1\exp(\imath\Phi), \tag{4}$$

where

$$\mathcal{H}_1 = h_0\frac{i\pi\sigma}{2B^2}\frac{v}{(1-v^2)}\cosh^{-1}\left(\frac{\pi\Delta}{2B}\right)\left\{\gamma^2 + h_3(\omega - h_3)\right\}. \tag{5}$$

The whole magnetization created by one kink can be written as

$$M_x = \int dx\, m_x = M_1 \exp(\imath\Phi) + \bar{M}_1 \exp(-\imath\Phi). \tag{6}$$

The calculating of M_1 leads to the following relation

$$M_1 = -\gamma^2 \left(\frac{\imath\pi\sigma}{\delta B^2}\right) \frac{v}{(1-v^2)} \cosh^{-1}\left(\frac{\pi\Delta}{2B}\right). \tag{7}$$

3 High-frequency properties of solitons

Investigations of the contribution of the solitons of different types to the specific heat, magnetization and the dynamical structure factor, defining non-elastic neutron dissipation and so on, are actual problems of the solid state physics during last years [6–10].

The magnet state involving a great number of kinks, the average distance between which is much larger than the average size of the kinks (a "gas" of kinks), can be described by the distribution function $\rho(X, \Phi, I_1, I_2)$, determining the number of quasiparticles per an element of phase volume $\triangle\Gamma$. It would appear reasonable that in the thermodynamic equilibrium state $\rho = \rho_0 = \exp(-\tilde{\beta}\mathcal{E}_0)$, where the inverse energetic temperature $\tilde{\beta} = (M_0 a)^2 \alpha\omega_0 / c\mathcal{T}$, $\mathcal{T}$ is the temperature.

The kinetic equation in general case has the form $\frac{\partial\rho}{\partial t} + \mathrm{div}(\rho\vec{u}) = -\frac{\rho-\rho_0}{\tau}$, where τ is the relaxation time. The kinetic equation determining the small nonequilibrium correction ρ_1, owing to the presence of a weak external magnetic field which varies in time, has the following form in the linear approximation:

$$\frac{\partial\rho_1}{\partial t} + \frac{\partial\mathcal{E}_0}{\partial I_2}\frac{\partial\rho_1}{\partial\Phi} + \frac{\rho_1}{\tau} = \frac{\partial\rho_0}{\partial I_2}\cos(\Omega\tau)\frac{\partial\mathcal{H}_{\mathrm{int}}}{\partial\Phi}. \tag{8}$$

The average energy $\bar{Q}$ absorbed from the external field over one period is given by the expression

$$\bar{Q} = -\frac{1}{TLa^2}\int dt \int d^3x \frac{\partial\vec{h}}{\partial t}\vec{\mathcal{M}}, \qquad \mathcal{M}_x = \int \frac{d\Gamma}{(2\pi\hbar)^2}\rho(\Gamma)\, M_x,$$

where L is the length, $\vec{\mathcal{M}}$ is the total magnetization of a sample, $T = 2\pi/\Omega$ which is determined by the kink distribution function ρ, and M_x is the whole magnetization, created by one kink.

The equation (8) can be solved by means of relations (4)–(7). The term with ρ_0 will vanish by averaging t over.

It is essentially to point out that the range of the kinks existence defined by the inequalities $v^2 + \frac{(\omega-h_3)^2}{\gamma^2} < 1$ $(0 < h_3 < \gamma)$ is the ellipse. It is important that there exists the region of negative frequencies. We can accept that the external SHF field can have negative frequencies also. So, the appropriate frequencies of field are defined by the inequality $|\Omega| < \gamma + h_3$. By this means we have three domains of frequencies:

$$\begin{aligned} &\mathrm{I.}\ \Omega_2 < \Omega < \Omega_1, && q_{\mathrm{I}} = J(\Omega), \\ &\mathrm{II.}\ \Omega_4 < \Omega < \Omega_2, && q_{\mathrm{II}} = J(\Omega) + J(-\Omega), \\ &\mathrm{III.}\ \Omega_5 < \Omega < \Omega_4, && q_{\mathrm{III}} = J(-\Omega), \end{aligned} \tag{9}$$

where $\Omega_1 = \gamma + h_3$, $\Omega_2 = \gamma - h_3$, $\Omega_4 = -(\gamma - h_3)$, $\Omega_5 = -(\gamma + h_3)$.

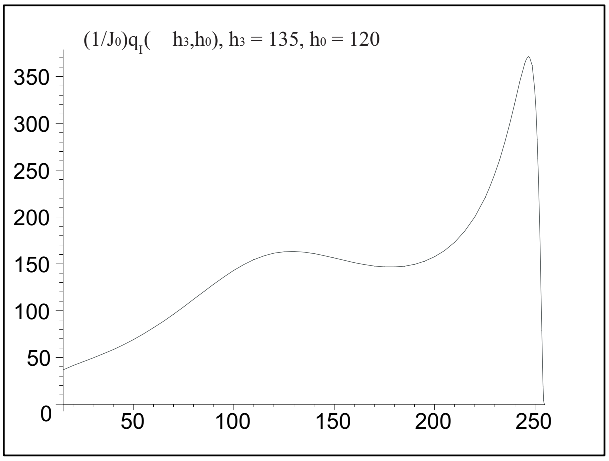

Figure 1.

Now we introduce the expression for the absorbed energy in the linear-response approximation. Here for convenience we use the substitution $x = \gamma v/\kappa(\tilde{v})$, $dx = (\gamma^3\tilde{v}v)dv/\kappa^3$, in which $J(\Omega)$ has the form:

$$J = J(\Omega; h_3, h_0) = J_0(h_0)\,|\Omega|\,\frac{F(\Omega; h_3)}{\tilde{v}} \int_1^\infty \sqrt{x^2-1}\left[1 + \frac{(\Omega - h_3)^2}{\gamma^2\,\tilde{v}^2}x^2\right]^2 \times \cosh^{-2}\left[\frac{\pi}{2}\left(\frac{\Omega - h_3}{\gamma}\right)\sqrt{x^2-1}\right] \exp\left[-2\gamma\tilde{\beta}\frac{F(\Omega; h_3)}{\tilde{v}}x\right] dx, \tag{10}$$

where

$$J_0 = 2h_0^2\pi^3\varepsilon^2\tilde{\beta}/\delta, \qquad F(\Omega; h_3) = 1 + \frac{h_3}{\gamma}\left(\frac{\Omega - h_3}{\gamma}\right), \qquad \tilde{v} = \sqrt{1 - \frac{(\Omega - h_3)^2}{\gamma^2}},$$

Note that J depends on one variable Ω, but thereafter we will include the parameters h_0 and h_3 as arguments of J to emphasize their important role.

Conceptually, the parameter $2\gamma\tilde{\beta}$ (see (2)) causes the quantity of the effect. For this one to be nonzero, it is essential to $2\gamma\tilde{\beta} \leq 1$.

4 Conclusion

The numerical calculated curves of $(1/J_0)q_{\mathrm{I,II}}(\Omega; h_3, h_0)$ (see (9), (10) for definition) for several values of external field h_3 are obtained. For this calculation we used the parameters of exchange $J = 3K$ and anisotropy $H_A = 500\,\mathrm{Oe}$ of the well investigated quasi-one-dimensional AFM system $CsMnCl_3 \cdot 2H_2O$, which admits the existence of the soliton excitations [5]. This choice of parameters of the crystal leads to the following values of the dimensionless parameters: $\varepsilon = 0.62$, $J_0 = 1.5 \cdot 10^{-13}$, $\tilde{\beta} = 3.46 \cdot 10^{-3}$, $\delta = 3.63 \cdot 10^3$. The result is represented on Fig. 1 for the most interesting case of h_3 close to the spin-flop field h_{sf}, i.e. $h_3 = 120$, $h_{sf} = 135$ (take into account that the frequency, field and other variables and parameters are dimensionless).

The fluent peak observed at the $\Omega_m = 120$ with the capacity value $Q_m = 10^3\,\mathrm{Erg/sec \cdot cm^3}$ is the most important for our investigation. This peak is the respective signal connected with the additional contribution of the kinks into our system SHF absorption [4]. The sharp increasing of capacity observed at the frequency $\Omega = 255$, coincided closely with the upper AFM resonance frequencies, is not taken into account. The kinks in this region of frequencies are out of the physical interest. The value of the imaginary part of the susceptibility is $\chi''_m = 0.22$. Thus, the maximum at the frequency $\Omega_m = 120$ is of interest, since it is renegated out of the uniform resonance line, has the marked intensily and therefore can be analyzed in the experiments aimed at the finding of the additional line form in the absorption spectrum, the frequency-field, angle, temperature and other dependencies, followed from the theory, presented in this paper.

[1] Mikeska H.-J., Non-linear dynamics of classical one-dimensional antiferromagnets, *J. Phys. C*, 1980, V.13, 2913–2923.

[2] Bar'yakhtar I.V. and Ivanov B.A., On non-linear waves of magnetization, *Sov. J. Low Temp. Phys.*, 1979, V.5, 759–770.

[3] Gomonai E.V., Ivanov B.A., L'vov V.A. and Oksyuk G.K., Symmetry and dynamics of the domain walls in weak ferromagnets, *Sov. Phys. JETP*, 1990, V.70, N 1, 307–322.

[4] Anders A.G. and Kobetz M.I., Anomalies in microwave absorption in quasi-one-dimensional $CsMnCl_3 \cdot 2H_2O$ in a pilsed magnetis field, *Low Temp. Phys.*, 1999, V.25, N 6, 436–439.

[5] Ivanov B.A. and Kolezhuk A.K., Soliton thermodynamics of a quasi-one-dimensional antiferromagnet in an external magnetic field, *Sov. J. Low Temp. Phys.*, 1991, V.17, 343–336; Quantum tunneling and quantum coherence in a topological soliton of quasi-one-dimensional antiferromagnet, *Sov. J. Low Temp. Phys.*, 1995, V.21, 986–988.

[6] Mikeska H.-J. and Steiner M., Solitary excitetions in one-dimensional magnets, *Adv. Phys.*, 1991, V.40, N 3, 191–356.

[7] Holyst J. and Benner H., Internal oscillations of solitons in (CH3)(4)$NMNCL_3$ above and below T–N, *Phys. Rev. B*, 1995, V.52, N 9, 6424–6430.

[8] Schwartz U.T, English L.Q. and Sievers A.J., Experimental generation and observation of intrinsic localized spin wave modes in an antiferromagnet, *Phys. Rev. Let.*, 1999, V.83, N 1, 223–226.

[9] Kivshar' Yu.S. and Malomed B.A., Pulsed excitation of solitons in easy-plane ferromagnets, *Sov. Phys. Solid State,* 1989, V.31, N 2, 293–294.

[10] Currie J.F., Krumhansl J.A., Bishop A.R. and Trallinger S.E., Statistical mechanics of one-dimensional solitary-wave-bearing scalar fields: exact results and ideal-gas phenomenology, *Phys. Rev. B*, 1980, V.22, 477–496.

[11] Gorelik L.Yu. and Kulinich S.I., Collisionless microwave energy absorption by a breather gas in a one-dimensional magnet, *Sov. J. Low Temp. Phys.*, 1986, V.12, N 8, 494–495.