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Towards Uniform T-Duality Rules

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In this contribution based on the talk given at the SUSY'01, Dubna, Russia, I discuss the reasons of appearing the different sets of fields entering into the T-duality transformations and a way to construct the uniform T-duality rules.

1 Introduction

Discovery in the past decade of the new class of non-compact symmetries allowed to revise one of the main problems of Superstrings: The puzzle of having too much "fundamental" theories. The resolution was in the conjecture on the M-theory in framework of which it turned out possible to unify all the five different superstring theories. The useful tool for establishing the M-theory evidence are dualities connected to the new symmetries of perturbative or non-perturbative sectors of Superstrings.

In this Contribution I would like to discuss some questions related to the so-called T-duality (see [1] for review). Namely, I will focus my attention on the following:

Q1: What are the different ways of T-duality rules derivation?

Q2: Do the T-duality rules coincide?

Q3: How uniform T-duality rules could be derived?

For the sake of simplicity I restrict myself in what follows to the pure classical frames, that means neglecting the dilaton field, which receives its corrections from quantum effects, and will consider backgrounds with single isometry direction. However, these restrictions are not so crucial, and more general case can be considered more or less straightforwardly in the same manner.

2 How one can derive the T-duality rules?

There are at least three different ways for getting the T-duality rules. The first one is based on the consideration of fundamental string dynamics in special kind of backgrounds allowing for existence of isometry direction [2]. Such consideration gives a possibility to derive the T-duality rules for Neveu–Schwarz (NS) sectors of the original and dual theories. The second way appeals to the unification of D=10 type IIA and D=10 type IIB supergravity (SUGRA) theories, which are the low energy effective actions for superstrings, after dimensional reduction down to nine dimensions. Requirement of having the same spectra in nine dimensions leads to the T-duality rules relating both the NS and Ramond (RR) sectors of the original (say, type IIA) and dual (type IIB) SUGRAs [3, 4]. Finally, one can derive the T-duality rules from the consideration of Dirichlet (D)-branes [5, 6, 7]. Again, the requirement of having the same physics after direct dimensional reduction of action for the Dp-brane, propagating in the background of type IIA theory, and double dimensional reduction of action for the D(p+1)-brane in the type

IIB background gives a chance to arrive at the T-duality transformations which relate the NS and the RR fields of the type IIA and type IIB theories to each other.

3 Do the T-duality rules coincide?

Let us consider the picture we have dealt with again. From the point of view of the fundamental string dynamics in the background with isometry no any *additional* redefinitions of the original fields which enter into the T-duality transformations are required. It should be pointed out since derivation of the T-duality rules is accompanied by dimensional reduction, target-space gauge fields can always be redefined to pick up the term which is proportional to the Kaluza–Klein vector field. This is due to the so-called transgression of the field strengths which is well-known in Kaluza–Klein literature. In such a situation *additional* means other possible redefinitions which do not relate to the transgression.

However, remind that in the case of fundamental string we are dealing with the NS sector only. If we extend our consideration to involve the RR sector into the game and to consider the T-duality rules from the point of view of the SUGRAs, we observe that the RR fields entering into the T-duality transformations are not always the same as the original ones [3, 4]. In other words some of the RR fields *additional* redefinition is required. The reason of this phenomenon is easy to understand. One of the features of the D=10 type IIB SUGRA is S-duality symmetry under the global $SL(2,\mathbb{R})$ transformations. This symmetry is absent in the case of the D=10 type IIA SUGRA. Since we can establish the connection between these theories only after dimensional reduction, we expect the trace of the higher dimensional $SL(2,\mathbb{R})$ transformations to be in lower dimensions. To make this symmetry manifest it is necessary to redefine the fields and to recollect them into the $SL(2,\mathbb{R})$ multiplets [3, 4]. On the other hand there always exists a freedom in the field (re)definition due to Bianchi identities (BIs) for the RR fields. But nevertheless, having the same BIs for two different sets of fields one describes the same physics [8]. Important point, which is worth to note, is that the additional redefinitions of the RR fields just fall into the class of admissible from the point of view of the BIs ones.

To complete the picture we need to know what happens in the D-brane case. The action for any D-brane (modulo instanton in type IIB) has the following structure (see, e.g., [9] for details)

$$S^{Dp} = -\int d\xi^{p+1}\sqrt{-\det(g_{mn}+\mathcal{F}_{mn})} + \int_{M^{p+1}} C\wedge e^{\mathcal{F}_2}, \qquad C=\sum_{n=0}^{d/2(-1)} C^{IIB(IIA)}_{2n(+1)}. \tag{1}$$

The first term is the kinetic term represented by generalization of Dirac action for relativistic membrane and the action for nonlinear electrodynamics proposed by Born and Infeld and called therefore by Dirac–Born–Infeld (DBI) term, and the second term is the Wess–Zumino (WZ) term. The DBI term is constructed out the NS target-space fields and worldvolume gauge field a_1 entering by means of $\mathcal{F}_2 = da_1 - B_2$, while both the NS 2-form B_2 and the RR target-space fields enter in the WZ term. In T-duality business two parts of the action play their roles independently giving the T-duality rules for the NS and for the RR sectors respectively [5, 7]. One can demonstrate that in such a case no original fields redefinition is required. The reason for this lies, in particular, in the natural restriction on the RR fields when they are fixed by the canonical form of the WZ term (cf. (1)), where C is treated as formal sum over the RR potentials in d-spacetime dimensions.

Therefore we are able to conclude the T-duality rules which we can read off from the Fundamental String, the SUGRA and the D-brane considerations coincide in the NS sector. However, even if the RR T-duality rules which follow from the SUGRA and the D-brane considerations coincide, the fields entering into the rules are not always the same.

4 Can we derive the uniform T-duality rules?

The key point here is that the analysis based mainly on the two-dimensional CFT and the vertex operators technique shows that *T-duality is the exact symmetry of string theory* [10, 11] (and Refs. therein). Therefore, in the language of low-energy classical effective dynamics of quantum string theory, it should be the symmetry of the following action

$$S = S_{\text{SUGRA (NS+RR)}} + S_{\text{NS SOURCES}} + S_{\text{RR SOURCES}}, \tag{2}$$

where the first term denotes schematically the action for the SUGRA with the NS and the RR fields (strictly speaking with their field strengths) and the last terms denote dynamical sources for the NS and the RR fields which are the actions for fundamental string and D-branes. This approach leads to the complete set of classical dynamical equations of motion describing dynamics of the SUGRA in presence of matter-type sources as well as dynamics of sources in the dynamical-type background, and gives therefore enclosed interaction picture.

In such a consideration it is naturally to expect that one can derive the uniform T-duality rules for both the NS and RR sectors since all the possible additional redefinitions coming from the SUGRA consideration will be under control and shall be in accordance with T-duality rules for the sources. One can observe some details in favour of this claim in [12]. There the model with action [13, 14]

$$S = S_{\text{SUGRA (NS)}} + S_{\text{Fundamental String SOURCE}} \tag{3}$$

was investigated from the point of view of invariance under the T-duality transformations.

The result on the T-duality invariance of such a system is predictable in view of the statement in the beginning of this Section. However, as a by-product, it became clear that as it can be expected from the discussion in the Section before the NS T-duality rules which come from equation (3) are in accordance with the rules derived from the fundamental string and the SUGRA considerations and no additional fields redefinition is required.

This observation simplifies explicit derivation of T-duality transformations, because, roughly speaking, having fundamental string as a source in effective action one can "forget" about the DBI part of D-brane sources and consider the contributions coming only from the WZ part of the D-branes action.

5 Discussion and conclusions

To summarize, I have discussed the reasons of appearing different sets of fields in the T-duality transformations and have sketched a way of the T-duality rules derivation in uniform basis of fields. Actually, the main problem is not even in derivation of T-duality rules in the uniform basis of fields, but rather in verification at the dynamical level the statement on T-duality as the exact symmetry of (perturbative and non-perturbative) string theory.

Vice versa, as we believe in the result achieved in the framework of quantum approach of the CFT and vertex operators, this gives confidence that it should be correct for effective theory describing the low-energy classical dynamics of quantum string theory. Hence, as a by-product, we can derive the T-duality transformations in the uniform basis of fields.

Beside the questions discussed above, consideration of T-duality in the SUGRA+SOURCES type interacting systems plays important role in String Cosmology with the pre-big-band scenario [15] where it is supposed that one deals with the string theory effective action in the background with isometries (see, e.g., the discussion in [16]). But this model should include actions for matter fields (strings and branes) and they have to be invariant under duality transformations.

Finally, in view of the recent paper [17] where an example of supersymmetric interacting system of the SUGRA+SOURCES type has been proposed, it looks very attractive to derive the supersymmetric T-duality rules and to compare the result with that of [18].

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