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# Realizations of Indecomposable Solvable 4-Dimensional Real Lie Algebras

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Inequivalent classes of realizations for indecomposable four-dimensional solvable real Lie algebras in the space of four variables are obtained.

Inequivalent two- and three-dimensional Lie algebras were classified in XIX century by Lie [1]. In 1963 Mubaraksyanov classified three- and four-dimensional real Lie algebras [2] (see also those results in Patera and Winternitz [3]). In 1989 Mahomed and Leach obtained realizations of three-dimensional Lie algebras in terms of vector fields defined on the plane [4]. Mahomed and Soh tried to obtain realizations of three- and four-dimensional Lie algebras in the space of variables $(t, x, y)$ [5], but their attempt can not be considered successful. Their article contains some misprints and a number of realizations are omitted. The results of [5] are used in [6] to solve the problem of linearization of systems of second-order ordinary differential equations, so some results from [6] are incorrect too. Realizations of solvable third-dimensional Lie algebras in the space of any number of variables are considered in [7].

In the present paper we give a complete set of inequivalent realizations of real indecomposable four-dimensional Lie algebras in the space of four variables, realizations of real decomposable four-dimensional Lie algebras are considered in [8]. Obtained results can be applied to integration of ordinary differential equations (or systems of ordinary differential equations) (see, for example, [9]), and to the problems of group classification (see, for example, [10]). We look for the realizations in the class of vector fields:

$$Q = \sum_{i=1}^{4} \xi^i \left(x_1, x_2, x_3, x_4\right) \partial_{x_i}.$$

After using Mubaraksyanov classification [2] we consider ten indecomposable solvable four-dimensional Lie algebras:

$A_{4.1}$ $[Q_2, Q_4] = Q_1,$ $[Q_3, Q_4] = Q_2;$

$A_{4.2}$ $[Q_2, Q_4] = Q_2,$ $[Q_1, Q_4] = qQ_1,$ $[Q_3, Q_4] = Q_2 + Q_3,$ $q \neq 0;$

$A_{4.3}$ $[Q_1, Q_4] = Q_1,$ $[Q_3, Q_4] = Q_2;$

$A_{4.4}$ $[Q_1, Q_4] = Q_1,$ $[Q_2, Q_4] = Q_1 + Q_2,$ $[Q_3, Q_4] = Q_2 + Q_3;$

$A_{4.5}$ $[Q_1, Q_4] = Q_1,$ $[Q_2, Q_4] = qQ_2,$ $[Q_3, Q_4] = pQ_3,$ $-1 \leq p \leq q \leq 1,$ $pq \neq 0;$

$A_{4.6}$ $[Q_1, Q_4] = qQ_1,$ $[Q_2, Q_4] = pQ_2 - Q_3,$ $[Q_3, Q_4] = Q_2 + pQ_3,$ $q \neq 0,\ p \geq 0;$

$A_{4.7}$ $[Q_2, Q_3] = Q_1,$ $[Q_1, Q_4] = 2Q_1,$ $[Q_2, Q_4] = Q_2,$ $[Q_3, Q_4] = Q_2 + Q_3;$

$A_{4.8}$ $[Q_2,Q_3]=Q_1,\quad [Q_1,Q_4]=(1+q)\,Q_1,\quad [Q_2,Q_4]=Q_2,$
$[Q_3,Q_4]=qQ_3,\quad |q|\le 1;$

$A_{4.9}$ $[Q_2,Q_3]=Q_1,\quad [Q_1,Q_4]=2qQ_1,\quad [Q_2,Q_4]=qQ_2-Q_3,$
$[Q_3,Q_4]=Q_2+qQ_3,\quad q\ge 0;$

$A_{4.10}$ $[Q_1,Q_3]=Q_1,\quad [Q_1,Q_4]=-Q_2,\quad [Q_2,Q_4]=Q_1,$
$[Q_2,Q_3]=Q_2.$

For each algebra we write down only the non-zero commutation relations. We start from a given Lie algebra with a set of structure constants and look which vector fields in at most four variables satisfy the given set of commutator relations with none of the operators vanishing. We thus look for possible realizations or representations of our Lie algebra. Two realizations of the same Lie algebra will be considered equivalent or similar if there exists an invertible transformation mapping one of the realizations to the other.

We arrange all results in the next Table 1 (below $\partial_i=\partial_{x_i}$, $i=1,\dots,4$; $A^k_{n.m}$ denotes the $k$-th realizations of algebra $A_{n.m}$).

**Table 1.** Realizations of real indecomposable solvable four-dimensional Lie algebras.

| | |
|---|---|
| $A^1_{4.1}$ | $\partial_1,\ \partial_2,\ \partial_3,\ x_2\partial_1+x_3\partial_2+\partial_4;$ |
| $A^2_{4.1}$ | $\partial_1,\ \partial_2,\ \partial_3,\ x_2\partial_1+x_3\partial_2+x_4\partial_3;$ |
| $A^3_{4.1}$ | $\partial_1,\ \partial_2,\ \partial_3,\ x_2\partial_1+x_3\partial_2;$ |
| $A^4_{4.1}$ | $\partial_1,\ \partial_2,\ x_3\partial_1+x_4\partial_2,\ x_2\partial_1+x_4\partial_3-\partial_4;$ |
| $A^5_{4.1}$ | $\partial_1,\ \partial_2,\ -\frac{x_3^2}{2}\partial_1+x_3\partial_2,\ x_2\partial_1-\partial_3;$ |
| $A^6_{4.1}$ | $\partial_1,\ x_3\partial_1,\ \partial_2,\ x_2x_3\partial_1-\partial_3$ |
| $A^7_{4.1}$ | $\partial_1,\ x_2\partial_1,\ x_3\partial_1,\ -\partial_2-x_2\partial_3;$ |
| $A^8_{4.1}$ | $\partial_1,\ x_2\partial_1,\ \frac{x_2^2}{2}\partial_1,\ -\partial_2$ |
| $A^1_{4.2}$ | $\partial_1,\ \partial_2,\ \partial_3,\ qx_1\partial_1+(x_2+x_3)\,\partial_2+x_3\partial_3+\partial_4;$ |
| $A^2_{4.2}$ | $\partial_1,\ \partial_2,\ \partial_3,\ qx_1\partial_1+(x_2+x_3)\,\partial_2+x_3\partial_3;$ |
| $A^3_{4.2}$ | $\partial_1,\ \partial_2,\ x_3\partial_1+x_4\partial_2,\ qx_1\partial_1+x_2\partial_2+(q-1)\,x_3\partial_3-\partial_4;$ |
| $A^4_{4.2}$ | $\partial_1,\ \partial_2,\ e^{(1-q)x_3}\partial_1+x_3\partial_2,\ qx_1\partial_1+x_2\partial_2-\partial_3,\ q\ne 1$ |
| $A^5_{4.2}$ | $\partial_1,\ x_3\partial_1,\ \partial_2,\ (qx_1+x_2x_3)\,\partial_1+x_2\partial_2+(q-1)\,x_3\partial_3;$ |
| $A^6_{4.2}$ | $\partial_1,\ x_3\partial_1,\ \partial_2,\ (x_1+x_2x_3)\,\partial_1+x_2\partial_2+\partial_4,\ q=1;$ |
| $A^7_{4.2}$ | $\partial_1,\ x_2\partial_1,\ x_3\partial_1,\ qx_1\partial_1+(q-1)\,x_2\partial_2+((q-1)\,x_3-x_2)\,\partial_3;$ |
| $A^8_{4.2}$ | $\partial_1,\ x_2\partial_1,\ \frac{x_2}{1-q}\ln\lvert x_2\rvert\partial_1,\ qx_1\partial_1+(q-1)\,x_2\partial_2,\ q\ne 1$ |
| $A^1_{4.3}$ | $\partial_1,\ \partial_2,\ \partial_3,\ x_1\partial_1+x_3\partial_2+\partial_4;$ |
| $A^2_{4.3}$ | $\partial_1,\ \partial_2,\ \partial_3,\ x_1\partial_1+x_3\partial_2+x_4\partial_3;$ |
| $A^3_{4.3}$ | $\partial_1,\ \partial_2,\ \partial_3,\ x_1\partial_1+x_3\partial_2;$ |
| $A^4_{4.3}$ | $\partial_1,\partial_2,\ x_3\partial_1+x_4\partial_2,\ x_1\partial_1+x_3\partial_3-\partial_4;$ |
| $A^5_{4.3}$ | $\partial_1,\ \partial_2,\ x_3\partial_2+ce^{-x_3}\partial_1,\ x_1\partial_1-\partial_3,\ c\in\{0;1\};$ |
| $A^6_{4.3}$ | $\partial_1,\ x_3\partial_1,\ \partial_2,\ (x_1+x_2x_3)\,\partial_1+x_3\partial_3$ |
| $A^7_{4.3}$ | $\partial_1,\ x_2\partial_1,\ x_3\partial_1,\ x_1\partial_1+x_2\partial_2+(x_3-x_2)\,\partial_3;$ |
| $A^8_{4.3}$ | $\partial_1,\ x_2\partial_1,\ -x_2\ln\lvert x_2\rvert\partial_1,\ x_1\partial_1+x_2\partial_2$ |

**Continuation of Table 1.**

| | |
|---|---|
| $A^1_{4.4}$ | $\partial_1,\, \partial_2,\, \partial_3,\, (x_1+x_2)\,\partial_1 + (x_2+x_3)\,\partial_2 + x_3\partial_3 + \partial_4;$ |
| $A^2_{4.4}$ | $\partial_1,\, \partial_2,\, \partial_3,\, (x_1+x_2)\,\partial_1 + (x_2+x_3)\,\partial_2 + x_3\partial_3;$ |
| $A^3_{4.4}$ | $\partial_1,\, \partial_2,\, x_3\partial_1 + x_4\partial_2,\, (x_1+x_2)\,\partial_1 + x_2\partial_2 + x_4\partial_3 - \partial_4;$ |
| $A^4_{4.4}$ | $\partial_1,\, \partial_2,\, -\frac{{x_3}^2}{2}\partial_1 + x_3\partial_2,\, (x_1+x_2)\,\partial_1 + x_2\partial_2 - \partial_3;$ |
| $A^5_{4.4}$ | $\partial_1,\, x_3\partial_1,\, \partial_2,\, (x_1 + x_2x_3)\,\partial_1 + x_2\partial_2 - \partial_3$ |
| $A^6_{4.4}$ | $\partial_1,\, x_2\partial_1,\, x_3\partial_1,\, x_1\partial_1 - \partial_2 - x_2\partial_3;$ |
| $A^7_{4.4}$ | $\partial_1,\, x_2\partial_1,\, \frac{x_2^2}{2}\partial_1,\, x_1\partial_1 - \partial_2$ |
| $A^1_{4.5}$ | $\partial_1,\, \partial_2,\, \partial_3,\, x_1\partial_1 + qx_2\partial_2 + px_3\partial_3 + \partial_4$ |
| $A^2_{4.5}$ | $\partial_1,\, \partial_2,\, \partial_3,\, x_1\partial_1 + qx_2\partial_2 + px_3\partial_3;$ |
| $A^3_{4.5}$ | $\partial_1,\, \partial_2,\, x_3\partial_1 + x_4\partial_2,\, x_1\partial_1 + qx_2\partial_2 + (1-p)\,x_3\partial_3 + (q-p)\,x_4\partial_4;$ |
| $A^4_{4.5}$ | $\partial_1,\, \partial_2,\, x_3\partial_1,\, x_1\partial_1 + qx_2\partial_2 + \partial_4;$ |
| $A^5_{4.5}$ | $\partial_1,\, \partial_2,\, x_3\partial_1,\, x_1\partial_1 + qx_2\partial_2;$ |
| $A^6_{4.5}$ | $\partial_1,\, \partial_2,\, x_3\partial_2,\, x_1\partial_1 + qx_2\partial_2 + \partial_4;$ |
| $A^7_{4.5}$ | $\partial_1,\, \partial_2,\, x_3\partial_2,\, x_1\partial_1 + qx_2\partial_2;$ |
| $A^8_{4.5}$ | $\partial_1,\, \partial_2,\, x_3\partial_1 + f\,(x_3)\,\partial_2,\, x_1\partial_1 + qx_2\partial_2 + \partial_4;$ |
| $A^9_{4.5}$ | $\partial_1,\, \partial_2,\, x_3\partial_1 + f\,(x_3)\,\partial_2,\, x_1\partial_1 + qx_2\partial_2;$ |
| $A^{10}_{4.5}$ | $\partial_1,\, \partial_2,\, c_1e^{(1-p)x_3}\partial_1 + c_2e^{(q-p)x_3}\partial_2,\, x_1\partial_1 + qx_2\partial_2 + \partial_3,$<br>$c_i \in \{0;1\}$, $c_1 = 0$ when $p = 1$, $c_2 = 0$ when $q = p$ |
| $A^{11}_{4.5}$ | $\partial_1,\, x_3\partial_1,\, \partial_2,\, x_1\partial_1 + px_2\partial_2 + (1-q)\,x_3\partial_3;$ |
| $A^{12}_{4.5}$ | $\partial_1,\, x_2\partial_1,\, x_3\partial_1,\, x_1\partial_1 + (1-q)\,x_2\partial_2 + (1-p)\,x_3\partial_3;$ |
| $A^{13}_{4.5}$ | $\partial_1,\, x_2\partial_1,\, x_3\partial_1, x_1\partial_1 + \partial_4,\, p = q = 1;$ |
| $A^{14}_{4.5}$ | $\partial_1,\, x_2\partial_2,\, f\,(x_2)\,\partial_1,\, x_1\partial_1 + \partial_3,\, f \neq c_1x_2 + c_2,\, p = q = 1;$ |
| $A^{15}_{4.5}$ | $\partial_1,\, x_2\partial_2,\, f\,(x_2)\,\partial_1,\, x_1\partial_1,\, f \neq c_1x_2 + c_2,\, p = q = 1;$ |
| $A^{16}_{4.5}$ | $\partial_1,\, e^{(1-q)x_2}\partial_1,\, e^{(1-p)x_2}\partial_1,\, x_1\partial_1 + \partial_2,\, q \neq 1,\, p \neq 1,\, q \neq p$ |
| $A^1_{4.6}$ | $\partial_1,\, \partial_2,\, \partial_3,\, qx_1\partial_1 + (px_2 + x_3)\,\partial_2 + (-x_2 + px_3)\,\partial_3 + \partial_4;$ |
| $A^2_{4.6}$ | $\partial_1,\, \partial_2,\, \partial_3,\, qx_1\partial_1 + (px_2 + x_3)\,\partial_2 + (-x_2 + px_3)\,\partial_3;$ |
| $A^3_{4.6}$ | $\partial_1,\, \partial_2,\, x_3\partial_1 + x_4\partial_2,\, (qx_1 - x_2x_3)\,\partial_1 + (p - x_4)\,x_2\partial_2 + (q - p - x_4)\,x_3\partial_3 - \left(1 + x_4^2\right)\partial_4;$ |
| $A^4_{4.6}$ | $\partial_1,\, \partial_2,\, c\left(\sqrt{x_3^2+1}e^{(p-q)\arctan x_3}\right)\partial_1 + x_3\partial_2,$<br>$\left(qx_1 - cx_2\left(\sqrt{x_3^2+1}e^{(p-q)\arctan x_3}\right)\right)\partial_1 + (p - x_3)\,x_2\partial_2 - \left(x_3^2+1\right)\partial_3,\, c \in \{0;1\}$ |
| $A^5_{4.6}$ | $\partial_1,\, x_2\partial_1,\, x_3\partial_1,\, qx_1\partial_1 + ((q-p)\,x_2 + x_3)\,\partial_2 + ((q-p)\,x_3 - x_2)\,\partial_3$ |
| $A^6_{4.6}$ | $\partial_1,\, e^{(q-p)x_2}\cos x_2\partial_1,\, -e^{(q-p)x_2}\sin x_2\partial_1,\, qx_1\partial_1 + \partial_2$ |
| $A^1_{4.7}$ | $\partial_1,\, \partial_2,\, x_2\partial_1 + \partial_3,\, \left(2x_1 + \frac{{x_3}^2}{2}\right)\partial_1 + (x_2 + x_3)\,\partial_2 + x_3\partial_3 + \partial_4$ |
| $A^2_{4.7}$ | $\partial_1,\, \partial_2,\, x_2\partial_1 + \partial_3,\, \left(2x_1 + \frac{{x_3}^2}{2}\right)\partial_1 + (x_2 + x_3)\,\partial_2 + x_3\partial_3;$ |
| $A^3_{4.7}$ | $\partial_1,\, \partial_2,\, x_2\partial_1 + x_3\partial_2,\, 2x_1\partial_1 + x_2\partial_2 - \partial_3;$ |
| $A^4_{4.7}$ | $\partial_1,\, x_2\partial_1,\, -\partial_2,\, \left(2x_1 - \frac{{x_2}^2}{2}\right)\partial_1 + x_2\partial_2 + \partial_3;$ |
| $A^5_{4.7}$ | $\partial_1,\, x_2\partial_1,\, -\partial_2,\, \left(2x_1 - \frac{{x_2}^2}{2}\right)\partial_1 + x_2\partial_2$ |
| $A^1_{4.8}$ | $\partial_1,\, \partial_2,\, x_2\partial_1 + \partial_3,\, (1+q)\,x_1\partial_1 + x_2\partial_2 + qx_3\partial_3 + \partial x_4;$ |
| $A^2_{4.8}$ | $\partial_1,\, \partial_2,\, x_2\partial_1 + \partial_3,\, (1+q)\,x_1\partial_1 + x_2\partial_2 + qx_3\partial_3,\, q \neq 0;$ |
| $A^3_{4.8}$ | $\partial_1,\, \partial_2,\, x_2\partial_1 + \partial_3,\, x_1\partial_1 + x_2\partial_2 + x_4\partial_3,\, q = 0;$ |
| $A^4_{4.8}$ | $\partial_1,\, \partial_2,\, x_2\partial_1 + \partial_3,\, x_1\partial_1 + x_2\partial_2 + c\partial_3,\, q = 0,\, c \in \mathbb{R};$ |
| $A^5_{4.8}$ | $\partial_1,\, \partial_2,\, x_2\partial_1 + x_3\partial_2,\, 2x_1\partial_1 + x_2\partial_2 + \partial_4,\, q = 1;$ |

**Continuation of Table 1.**

| | |
|---|---|
| $A^6_{4.8}$ | $\partial_1,\ \partial_2,\ x_2\partial_1 + x_3\partial_2,\ (1+q)\,x_1\partial_1 + x_2\partial_2 + (1-q)\,x_3\partial_3;$ |
| $A^7_{4.8}$ | $\partial_1,\ \partial_2,\ x_2\partial_1,\ (1+q)\,x_1\partial_1 + x_2\partial_2 + \partial_3,\ q \neq 1;$ |
| $A^8_{4.8}$ | $\partial_1,\ \partial_2,\ x_2\partial_1,\ (1+q)\,x_1\partial_1 + x_2\partial_2,\ q \neq 1;$ |
| $A^9_{4.8}$ | $\partial_1,\ \partial_2,\ x_2\partial_1,\ x_3\partial_1 + x_2\partial_2,\ q = -1;$ |
| $A^{10}_{4.8}$ | $\partial_1,\ -x_2\partial_1,\ \partial_2,\ (1+q)\,x_1\partial_1 + qx_2\partial_2 + \partial_3;$ |
| $A^{11}_{4.8}$ | $\partial_1,\ -x_2\partial_1,\ \partial_2,\ (1+q)\,x_1\partial_1 + qx_2\partial_2$ |
| $A^1_{4.9}$ | $\partial_1,\ \partial_2,\ x_2\partial_1 + \partial_3,\ \left(2qx_1 + \frac{{x_3}^2 - {x_2}^2}{2}\right)\partial_1 + (qx_2 + x_3)\,\partial_2 + (qx_3 - x_2)\,\partial_3 + \partial_4;$ |
| $A^2_{4.9}$ | $\partial_1,\ \partial_2,\ x_2\partial_1 + \partial_3,\ \left(2qx_1 + \frac{{x_3}^2 - {x_2}^2}{2}\right)\partial_1 + (qx_2 + x_3)\,\partial_2 + (qx_3 - x_2)\,\partial_3;$ |
| $A^3_{4.9}$ | $\partial_1,\ \partial_2,\ x_2\partial_1 + \partial_3,\ \left(\frac{{x_3}^2 - {x_2}^2}{2} + x_4\right)\partial_1 + x_3\partial_2 - x_2\partial_3,\ q = 0;$ |
| $A^4_{4.9}$ | $\partial_1,\ \partial_2,\ x_2\partial_1 + x_3\partial_2,\ \left(2qx_1 - \frac{{x_2}^2}{2}\right)\partial_1 + (q - x_3)\,x_2\partial_2 - \left(1 + {x_3}^2\right)\partial_3$ |
| $A^1_{4.10}$ | $\partial_1,\ \partial_2,\ x_1\partial_1 + x_2\partial_2 + \partial_3,\ x_2\partial_1 - x_1\partial_2 + \partial_4;$ |
| $A^2_{4.10}$ | $\partial_1,\ \partial_2,\ x_1\partial_1 + x_2\partial_2 + \partial_3,\ x_2\partial_1 - x_1\partial_2 + x_4\partial_3;$ |
| $A^3_{4.10}$ | $\partial_1,\ \partial_2,\ x_1\partial_1 + x_2\partial_2 + \partial_3,\ x_2\partial_1 - x_1\partial_2 + c\partial_3,\ c \in \mathbb{R}$ |
| $A^4_{4.10}$ | $\partial_1,\ x_2\partial_1,\ x_1\partial_1 + \partial_3,\ -x_1x_2\partial_1 - \left(1 + {x_2}^2\right)\partial_2;$ |
| $A^5_{4.10}$ | $\partial_1,\ \partial_2,\ x_1\partial_1 + x_2\partial_2,\ x_2\partial_1 - x_1\partial_2 + \partial_3;$ |
| $A^6_{4.10}$ | $\partial_1,\ \partial_2,\ x_1\partial_1 + x_2\partial_2,\ x_2\partial_1 - x_1\partial_2;$ |
| $A^7_{4.10}$ | $\partial_1,\ x_2\partial_1,\ x_1\partial_1,\ -x_1x_2\partial_1 - \left(1 + {x_2}^2\right)\partial_2$ |

This can be applied to the integrating of fourth-order differential equations of or some system classes of four first-order differential equations and their classification.

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