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Relation between Periodic Soliton Resonance and Instability

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There are periodic soliton resonances in interactions between two periodic solitons and between periodic soliton and another kind of soliton. A close relation exists between the periodic soliton resonance and soliton instability to transverse disturbances. It is shown that the instability occurred in the line soliton is relaxed by emission of the periodic soliton and the periodic soliton resonance is inverse process of the instability.

1 Introduction

The dynamics of nonlinear waves in higher-dimensional space have richer phenomena than one-dimensional case, since various localized structures may be considered as candidates for solitons. The two-dimensional generalization of the Korteweg–de Vries (KdV) equation was given by Kadomtsev and Petviashvili to discuss the stability of one-dimensional KdV soliton (line soliton in two dimensions) against transverse long-wave disturbances, which is known as the Kadomtsev–Petviashvili (KP) equation and is expressed as follows [1],

$$(u_t + 6uu_x + u_{xxx})_x + 3su_{yy} = 0, \qquad s = \pm 1. \tag{1}$$

The propagation property of solitons depends essentially on the sign of s in equation (1). In the media with negative dispersion ($s = +1$), line solitons are stable to long transverse perturbations. On the other hand, line solitons are unstable for positive dispersion ($s = -1$) [1]. This leads to the conjecture that solutions having two-dimensional localized structures should be found in the positive dispersion case. Such solitons were found by Manakov *et al.* [2] and Ablowitz and Satsuma [3], which have no longer exponential tails, but instead take the form of rational functions in both space and time variables.

The N line soliton solution was obtained by Zakharov and Shabat [4] and Satsuma [5]. Miles [6] found the existence of the singular parameter regime for multi-soliton solutions in the negative dispersion case, where the solutions become singular. He showed that the two-soliton solution reveal a resonant interaction when it is just on the borderline between regular and singular regimes in the parameter space. In this case, when the relative inclination between wave normals is at a certain small critical angle, two solitons interact strongly to make a resonant soliton from a point at which two incident solitons meet together. Such a resonant interaction is called “soliton resonance”. On the other hand, in the positive dispersion media, the solution which describes the interaction between two line solitons never satisfies the resonant condition.

Another kind of two-dimensional localized soliton in the positive dispersion media is a periodic soliton, which was found by Zaitsev at first by superposition of rational solitons (algebraic solitons) [7]. The periodic soliton solutions which describe the multi-soliton interactions have been obtained by Tajiri and Murakami [8]. The stability for the y-periodic and x-periodic solitons has been discussed by Zhdanov [9], using the dressing method. He also obtained the solution which describes the nonlinear stage of instability of the line soliton propagating in the x-direction and pointed out the decay of a one-dimensional soliton into a soliton of lower amplitude and a periodic soliton.

As there are three kinds of solitons, line soliton, algebraic soliton and periodic soliton, various interactions between two different kinds of solitons may happen in the positive dispersion media. The interactions between two periodic solitons, periodic soliton and line soliton, line soliton and algebraic soliton and periodic soliton and algebraic soliton have been studied by several authors [10–16]. Thing to be most emphasized among them is the existence of resonant interactions between two periodic solitons and between periodic soliton and another kind of soliton. Here we call them "periodic soliton resonances", which are qualitatively different from the resonant interaction between two line solitons (soliton resonances) of the KP equation with negative dispersion. Periodic soliton resonances are irrelevant to the divergence of soliton solutions. This is crucially different from the case of soliton resonances.

Another equation having two-dimensional localized soliton solutions is the Davey–Stewartson (DS) equation [17]:

$$\begin{aligned} &iu_t + pu_{xx} + u_{yy} + r|u|^2u - 2uv = 0, \\ &v_{xx} - pv_{yy} - r\left(|u|^2\right)_{xx} = 0, \end{aligned} \tag{2}$$

where $p = \pm 1$, r is constant. This is the two-dimensional generalization of the nonlinear Schrödinger (NLS) equation. Equation (2) with $p = +1$ and $p = -1$ are called the DSI and DSII equations, respectively. Anker and Freeman [18] studied the interactions between two dark line solitons which are skewed with respect to each other and found the existence of soliton resonances. The DS equation has algebraic soliton and periodic soliton solutions in addition to the dark line soliton solution [19]. In the previous paper, the interactions between two periodic solitons, between dark line soliton and periodic soliton, and between algebraic soliton and periodic soliton were investigated [20, 21]. We found the existence of periodic soliton resonances in each case.

The existence of solitons having the structures peculiar to high-dimensionality may contribute to the variety of the dynamics of nonlinear waves. Especially, we expect that the periodic soliton resonances play fundamental role in the nonlinear development of high-dimensional wave fields as the existence of periodic solitons is related to the instability of solitons.

The purpose of this study is to show that we can discuss the problem of the stability of solitons by making use of the solutions of periodic soliton resonance.

2 Soliton stability theory due to periodic soliton resonance solutions

2.1 Stability of the line soliton of the KP equation with positive dispersion

At first, we consider the resonant interaction between line soliton and y-periodic soliton which propagate in the same direction. Although we can obtain the expressions that describe the general interaction between solitons in arbitrary directions, we limit our concern into the solitons that extend in the y-direction and propagate in the x-direction. The solutions that describe the interaction between line soliton and y-periodic soliton are given by [13]

$$u = 2(\log f)_{xx}, \tag{3}$$

with

$$f = 1 - \mathrm{e}^{\xi_P}\cos\eta + \frac{M}{4}\mathrm{e}^{2\xi_P} + \mathrm{e}^{\xi_L}\left(1 - N\mathrm{e}^{\xi_P}\cos\eta + \frac{MN^2}{4}\mathrm{e}^{2\xi_P}\right), \tag{4}$$

where

$$\xi_P = \alpha x - \Omega_P t + \sigma_P, \qquad \xi_L = Px - \Omega_L t + \sigma_L, \qquad \eta = \delta y + \theta,$$

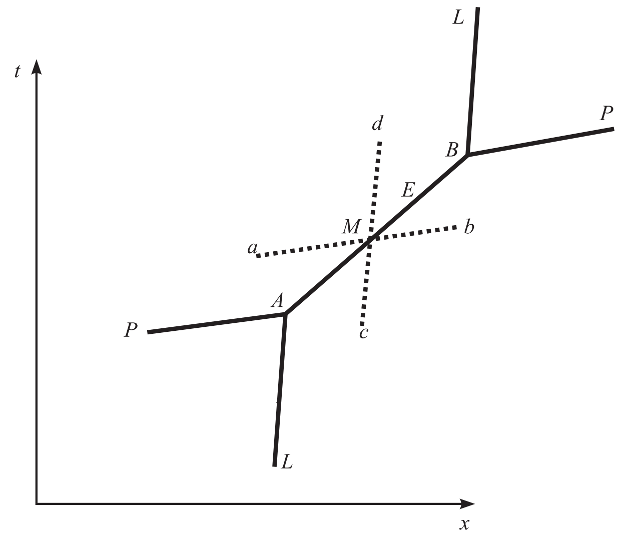

Figure 1. The schematic diagram of the world line of the soliton humps in the $x-t$ plane. AB corresponds to the quasi-resonant state. On line ab; $\xi_P + (1/2)\Gamma \simeq O(1)$ and on line cd; $\xi_L + \Gamma \simeq O(1)$.

$$\Omega_P = \alpha^3 + 3\frac{\delta^2}{\alpha}, \qquad \Omega_L = P^3, \qquad M = \frac{\delta^2}{\delta^2 - \alpha^4}, \qquad N = \frac{(\alpha - P)^2 - (\delta/\alpha)^2}{(\alpha + P)^2 - (\delta/\alpha)^2}, \tag{5}$$

where σ_P, σ_L and θ arbitrary real constants. The phase shift due to the interaction between line soliton and y-periodic soliton is determined only by N. In the case $\alpha P > 0$, the phase shift in the propagating direction is given by $\Gamma = \log|N|$ while that in the transverse direction is determined by the sign of N.

The resonant conditions are obtained by equating the denominator of N to zero as follows,

$$\frac{\delta}{\alpha^2} = \frac{P}{\alpha} + 1, \tag{6}$$

$$\frac{\delta}{\alpha^2} = -\frac{P}{\alpha} - 1. \tag{7}$$

Fig. 1 is the schematic diagram of the world lines of the soliton humps in the $x-t$ plane for the attractive interaction ($\Gamma > 0$). The line AB in Fig. 1 corresponds to the quasi-resonant state and at the midpoint M of the world line AB, the quasi-resonant state reaches the most resonant state. When we replace $\xi_P + \frac{1}{2}\ln|N|$ and $\xi_L + \ln|N|$ with $\tilde{\xi_P}$ and $\tilde{\xi_L}$, respectively, so as to $\tilde{\xi_P} \sim \tilde{\xi_L} \sim O(1)$ at the most resonant point M, that is,

$$\xi_P + \frac{1}{2}\log|N| = \tilde{\xi_P} \sim O(1), \qquad \sigma_P + \frac{1}{2}\log|N| = \tilde{\sigma}_P,$$

$$\xi_L + \log|N| = \tilde{\xi_L} \sim O(1), \qquad \sigma_L + \log|N| = \tilde{\sigma}_L,$$

(see Fig. 1) equation (4) is rewritten as follows;

$$f = f^{(0)} + \frac{1}{\sqrt{|N|}} f^{(1)} + \frac{1}{|N|} f^{(2)}, \tag{8}$$

with

$$f^{(0)} = 1 + \frac{M}{4} e^{\tilde{\xi_L} + 2\tilde{\xi_P}}, \tag{9}$$

$$f^{(1)} = -e^{\tilde{\xi_P}} \left\{ \cos\eta + e^{\tilde{\xi_L}} \cos(\eta + \epsilon\pi) \right\}, \tag{10}$$

$$f^{(2)} = \mathrm{e}^{\tilde{\xi}_L} + \frac{M}{4}\mathrm{e}^{2\tilde{\xi}_P}, \tag{11}$$

where $\epsilon = 0$ for $N > 0$ and $\epsilon = 1$ for $N < 0$. Taking the limit $|N| \to \infty$, we see that the resonant soliton is the line soliton with wave number $(P/2 + \alpha)$. If we take $|N| \gg 1$, equation (8) shows that the quasi-resonant state consists of the resonant line soliton and small disturbance. The disturbance becomes minimum at midpoint M. The line ME in Fig. 1 corresponds to the period of the linear development of the disturbance on the resonant line soliton. Substituting equation (8) into $u = 2(\log f)_{xx}$, and neglecting the terms of $O(1/|N|)$, we have

$$u = u^{(0)} + \frac{1}{\sqrt{|N|}}\left\{-2u^{(0)}\frac{f^{(1)}}{f^{(0)}} + \frac{2}{f^{(0)2}}\left(f^{(0)}_{xx}f^{(1)} + f^{(0)}f^{(1)}_{xx} - 2f^{(0)}_x f^{(1)}_x\right)\right\}, \tag{12}$$

where

$$u^{(0)} = \frac{1}{2}P_0^2\,\mathrm{sech}^2\frac{1}{2}\left(P_0 x - \Omega_0 t + \sigma_0\right),$$

which is resonant line soliton solution, where $\sigma_0 = 2\tilde{\sigma}_P + \tilde{\sigma}_L + \log\left(\frac{M}{4}\right)$. Substituting equations (9) and (10) into equation (12) and replacing $\tilde{\xi}_L$ with

$$\tilde{\xi}_L = \frac{P}{P_0}\xi_0 + \left(\frac{P}{P_0}\Omega_0 - \Omega_L\right)t - \frac{P}{P_0}\sigma_0 + \tilde{\sigma}_L, \tag{13}$$

we have [22]

$$\begin{aligned} u = u^{(0)} + \frac{u^{(0)}}{\sqrt{MN_L}} &\left[\left\{2\,\mathrm{sech}\frac{\xi_0}{2} - \frac{4\alpha\left(\alpha+P\right)}{P_0^2}\left(A\cosh\frac{1}{2}\xi_0 - B\sinh\frac{1}{2}\xi_0\right)\right\}\right. \\ &\times \exp\left(\frac{P}{2P_0}\xi_0 + \frac{1}{2}\left(-\frac{P}{P_0}\sigma_0 + \tilde{\sigma}_L\right)\right)\mathrm{e}^{\gamma t} \\ &+ \left\{2\,\mathrm{sech}\frac{\xi_0}{2} - \frac{4\alpha\left(\alpha+P\right)}{P_0^2}\left(A\cosh\frac{1}{2}\xi_0 + B\sinh\frac{1}{2}\xi_0\right)\right\} \\ &\left.\times \exp\left(-\left\{\frac{P}{2P_0}\xi_0 + \frac{1}{2}\left(-\frac{P}{P_0}\sigma_0 + \tilde{\sigma}_L\right)\right\}\right)\mathrm{e}^{-\gamma t}\right]\cos\eta, \end{aligned} \tag{14}$$

where

$$\gamma = \frac{1}{2}\left(\frac{P}{P_0}\Omega_0 - \Omega_L\right). \tag{15}$$

Substituting the relation $P{=}P_0{-}2\alpha$ into the resonant condition, $\alpha + P - (\delta/\alpha) = 0$, we have

$$\alpha = \frac{1}{2}\left(P_0 - \sqrt{P_0^2 - 4\delta}\right), \tag{16}$$

$$P = \sqrt{P_0^2 - 4\delta}. \tag{17}$$

When we substitute equations (16) and (17) into the dispersion relations of line soliton and y-periodic soliton, Ω_P and Ω_L can be expressed by P_0 and δ. Then we have the growth rate γ,

$$\gamma = 2\delta\sqrt{P_0^2 - 4\delta}, \tag{18}$$

which is the same as the result obtained by Zhdanov by using the dressing method [9]. Taking the long limit $|\delta| \ll 1$ $(\alpha \ll 1)$, we have

$$\alpha \simeq \frac{\delta}{P_0}, \qquad \frac{4\alpha(\alpha+P)}{P_0^2}A \simeq \frac{4\alpha(\alpha+P)}{P_0^2}B \simeq 2\left(1 - \frac{2\delta}{P_0^2}\right),$$

and then

$$\gamma \simeq 2\delta P_0, \tag{19}$$

$$\begin{aligned} u \simeq u^{(0)} &- \frac{2}{\sqrt{MN_L}}\frac{1}{P_0}\left[C\left\{-\frac{P_0^3}{2}\operatorname{sech}^2\left(\frac{1}{2}\xi_0\right)\tanh\left(\frac{1}{2}\xi_0\right)\right.\right.\\ &-\left.\frac{\gamma}{2}\left(\operatorname{sech}^2\left(\frac{1}{2}\xi_0\right)-\frac{\xi_0}{2}\operatorname{sech}^2\left(\frac{1}{2}\xi_0\right)\tanh\left(\frac{1}{2}\xi_0\right)\right)\right\}\mathrm{e}^{\gamma t}\\ &-\frac{1}{C}\left\{-\frac{P_0^3}{2}\operatorname{sech}^2\left(\frac{1}{2}\xi_0\right)\tanh\left(\frac{1}{2}\xi_0\right)-\left(\frac{-\gamma}{2}\right)\left(\operatorname{sech}^2\left(\frac{1}{2}\xi_0\right)\right.\right.\\ &-\left.\left.\left.\frac{\xi_0}{2}\operatorname{sech}^2\left(\frac{1}{2}\xi_0\right)\tanh\left(\frac{1}{2}\xi_0\right)\right)\right\}\mathrm{e}^{-\gamma t}\right]\cos\eta, \end{aligned} \tag{20}$$

where $C = \exp\left\{-(1/2)\left(\sqrt{P_0^2-4\delta}\sigma_0/P_0-\tilde{\sigma}_L\right)\right\}$. It is very interesting to note that these equations are expressed only by the wave number of resonant line soliton, P_0, and the wave number of the disturbance δ. We can see that the solution (20) has the growing term and decaying term.

2.2 Stability of the line soliton of the DSI equation

In this section, the stability of the line soliton to the DSI equation is studied by making use of the solution of periodic soliton resonance. The solution describing the interaction between line soliton and y-periodic soliton is given by

$$u = u_0 e^{i\zeta}\frac{g}{f}, \qquad v = -2\left(\ln f\right)_{xx} \tag{21}$$

with

$$f(\xi_P,\xi_L) = 1 + e^{\xi_P}\cos\eta + \frac{M}{4}e^{2\xi_P} + e^{\xi_L}\left\{1 + Ne^{\xi_P}\cos\eta + \frac{MN^2}{4}e^{2\xi_P}\right\}, \tag{22}$$

$$g(\xi_P,\xi_L,\phi_P,\phi_L) = f(\xi_P + i\phi_P, \xi_L + i\phi_L), \tag{23}$$

where

$$\begin{aligned} &\zeta = kx + ly - \omega t, \qquad \xi_P = \alpha x - \Omega_P t + \sigma_P,\\ &\xi_L = Px - \Omega_L t + \sigma_L, \qquad \eta = \delta y - \gamma_P t + \theta,\\ &\sin^2(\phi_P/2) = (\alpha^2+\delta^2)/(2ru_0^2), \qquad \sin^2(\phi_L)/2) = P^2/(2ru_0^2),\\ &\Omega_P = 2k\alpha - (\alpha^2-\delta^2)\cot(\phi_P/2), \qquad \gamma_P = 2l\delta,\\ &\Omega_L = P\{2k - P\cot(\phi_L/2)\}, \qquad M = \{1-(\alpha^2+\delta^2)^2/(2\delta^2 ru_0^2)\}^{-1},\\ &N = \frac{2ru_0^2\sin(\phi_P/2)\sin(\phi_L/2)\cos\{(\phi_P-\phi_L)/2\}-\alpha P}{2ru_0^2\sin(\phi_P/2)\sin(\phi_L/2)\cos\{(\phi_P+\phi_L)/2\}-\alpha P}. \end{aligned} \tag{24}$$

When we replace $\xi_P + \frac{1}{2}\ln|N|$ and $\xi_L + \ln|N|$ with $\tilde{\xi}_P$ and $\tilde{\xi}_L$, respectively, so as to $\tilde{\xi}_P \sim \tilde{\xi}_L \sim O(1)$ at the most resonant point M, equations (22) and (23) are rewritten as follows [23]:

$$\begin{aligned} f &= f^{(0)} + \frac{1}{\sqrt{|N|}}f^{(1)}\cos\eta + \frac{1}{|N|}f^{(2)},\\ g &= g^{(0)} + \frac{1}{\sqrt{|N|}}g^{(1)}\cos\eta + \frac{1}{|N|}g^{(2)}, \end{aligned} \tag{25}$$

where

$$f^{(0)} = 1 + \frac{M}{4}\exp(\tilde{\xi}_0), \tag{26}$$

$$g^{(0)} = 1 + \frac{M}{4}\exp(\tilde{\xi}_0 + \mathrm{i}\Phi), \tag{27}$$

$$f^{(1)}(\tilde{\xi}_P, \tilde{\xi}_L) = \exp(\tilde{\xi}_P)\left\{1 + \varepsilon\exp(\tilde{\xi}_L)\right\}, \tag{28}$$

$$f^{(2)}(\tilde{\xi}_P, \tilde{\xi}_L) = \exp(\tilde{\xi}_L) + \frac{M}{4}\exp(2\tilde{\xi}_P), \tag{29}$$

$$g^{(j)} = f^{(j)}(\tilde{\xi}_P + \mathrm{i}\phi_P, \tilde{\xi}_L + \mathrm{i}\phi_L) \qquad (j = 1, 2), \tag{30}$$

where $\Phi = 2\phi_P + \phi_L$, $P_0 = 2\alpha + P$, $\Omega_0 = 2\Omega_P + \Omega_L$, $\tilde{\xi}_0 = 2\tilde{\xi}_P + \tilde{\xi}_L = P_0 x - \Omega_0 t + 2\tilde{\sigma}_P + \tilde{\sigma}_L$ and $\varepsilon = 1$ for $0 < N$, $\varepsilon = -1$ for $N < 0$. Taking the limit $|N| \to \infty$, we have the resonant line soliton solution with wave number P_0 and phase Φ as follows,

$$u_0 = u_0 \mathrm{e}^{\mathrm{i}\zeta}\frac{1 + \mathrm{e}^{\xi_0 + \mathrm{i}\Phi}}{1 + \mathrm{e}^{\xi_0}}, \qquad v_0 = -\frac{P_0^2}{2}\mathrm{sech}^2\frac{1}{2}\xi_0, \tag{31}$$

where $\xi_0 = \tilde{\xi}_0 + (1/2)\log(M/4)$. Using the resonant condition, and dispersion relations of y-periodic soliton and line soliton, we obtain

$$P^2 = P_0^2 - 4\delta^2 \pm 4\left(\sqrt{2ru_0^2 - P_0^2}\right)\delta. \tag{32}$$

Taking account of the relations, $\alpha = (P_0 - P)/2$, $\sin^2(\phi_L/2) = P^2/2ru_0^2$ and $\phi_P = (\Phi - \phi_L)/2$, we see that P, α, ϕ_L, ϕ_P, Ω_P and Ω_L are expressed by P_0, Φ and δ. Therefore, the quasi-resonant solution is expressed by parameters (P_0, Φ, δ), which are the wave number and the phase of resonant line soliton and the wave number of transverse disturbance on the resonant line soliton, respectively. Neglecting the terms of $O(1/N)$, we have [23].

$$u = u_0 \mathrm{e}^{\mathrm{i}\zeta}\left[\frac{g^{(0)}}{f^{(0)}} + \frac{1}{\sqrt{|N|}}\frac{g^{(0)}}{f^{(0)}}\left(\frac{g^{(1)}}{g^{(0)}} - \frac{f^{(1)}}{f^{(0)}}\right)\cos\eta\right], \tag{33}$$

$$v = -2\frac{f_{xx}^{(0)}f^{(0)} - f_x^{(0)2}}{f^{(0)2}} + \frac{1}{\sqrt{|N|}}\left[\left\{4\frac{f_{xx}^{(0)}f^{(0)} - f_x^{(0)2}}{f^{(0)2}}\frac{f^{(1)}}{f^{(0)}}\right.\right.$$
$$\left.\left. - \frac{2}{f^{(0)2}}\left[f_{xx}^{(0)}f^{(1)} - 2f_x^{(0)}f_x^{(1)} + f^{(0)}f_{xx}^{(1)}\right]\right\}\cos\eta\right]. \tag{34}$$

It should be noted that the magnitude of the initial disturbance (the magnitude of the disturbance on the resonant line soliton at the most resonant state) is of the order $(1/\sqrt{|N|})$. The second term of equations (33) and (34) describes the linear development of the disturbance on the resonant line soliton. Regarding the quasi-resonant soliton solution at the most resonant state as the initial state, we can size the full-time evolution up to the nonlinear stage of the soliton with a small disturbance undergoing the instability by using the quasi-resonant solution. From these facts, we can expect that the instability occurred in the line soliton is relaxed by the emission of the periodic soliton and the periodic soliton resonance is inverse process of the instability.

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