

Numerical Simulation of Replacing Oil by Water in a Scale-Invariant Porous Medium

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The objective of the paper is to study the interface between the fluids having different viscosities in their joint movement through the nonhomogeneous random scale-invariant porous media. The surface tension and the force of gravity are not taken into account. Porosity and permeability have pulsations from an extreme wide range of scales and logarithmic-normal statistics. Statistical parameters of the interface are found. The problem is numerically solved in a unit size cube with 256^3 grid in spatial variables.

1 Introduction

The interface of oil displaced by water is unstable. This instability has the origin of different viscosities and explains the viscous fingering between fluids while the displacement takes place [1]. Another origin of the complex interface is the fluctuations of permeability of the porous media. Geological porous media are characterized by multiple-scale heterogeneities that have a significant influence on the transport of water and oil. The permeability heterogeneity is often identified as the main factor determining transport and dispersion of impurities in the subsurface [2]. In this paper, both factors are studied and compared in a direct computer experiment. We concentrate on them and do not discuss other important factors such as the surface tension and the gravitational force. The physical properties of such media are simulated by means of random fields. The probability distribution is simplified by the refined scaling hypothesis by Kolmogorov [3]. This hypothesis is associated with the fractal properties of the porous media that have a certain experimental support [4]. In particular, correlation functions of the parameters are asymptotically described by power laws. This fact enables us to use the scaling theory for modeling permeability and porosity.

2 Statement of problem

A displacement of a more viscous fluid (oil) by a less viscous one (water) in the 3D-scale-invariant porous medium is considered. At a low Reynolds number, Darcy's law gives the velocity $\mathbf{v}(\mathbf{x}) = -\varepsilon(\mathbf{x})\nabla p$, where p is pressure, $\varepsilon(\mathbf{x})$ is permeability. The incompressibility condition brings about the equation for p :

$$\nabla [\varepsilon(\mathbf{x}) \nabla p(\mathbf{x})] = 0, \tag{1}$$

where the permeability $\varepsilon(\mathbf{x})$ is equal to $\varepsilon_1(\mathbf{x})$ if the first phase (water), is located at point \mathbf{x} and equal to $\varepsilon_2(\mathbf{x})$, if the second phase (oil) is located at this point \mathbf{x} . The displacement of one fluid by another one is displayed with passive particles which label the interface between fluids. The

trajectories of particles that label the interface between fluids are calculated from the equations:

$$m(\mathbf{x}) \frac{d\mathbf{x}_i(t)}{dt} = \mathbf{v}(\mathbf{x}), \quad \mathbf{x}_i(0) = \mathbf{x}_{i0}, \quad (2)$$

where i stands for the number of particle $i = 1, \dots, N$, $m(\mathbf{x})$ is porosity. We solve equation (2) using the pressure from equation (1) and find the new coordinates of our phases. For the next time step we solve equation (1) with the new permeability coefficient $\varepsilon(\mathbf{x})$. A more viscous fluid (oil, solute) is displaced by a less viscous one (water). The ratio of permeabilities is determined by the ratio between viscosities of phases. The medium does not change while the flow moves. How does heterogeneity of a porous medium affect the average front coordinate, the variance of width of the front, etc.?

For describing functions of permeability and porosity we use the scaling theory proposed by Kolmogorov [3]. The full details of the scaling approach for the porous medium are considered in [5]. Here, we only briefly outline such an approach. The fluctuations of $\varepsilon(\mathbf{x})$, $m(\mathbf{x})$ of various sizes may be identified by a spatial smoothing at $l \rightarrow 0$, $\varepsilon(\mathbf{x}, l) \rightarrow \varepsilon(\mathbf{x})$, $m(\mathbf{x}, l) \rightarrow m(\mathbf{x})$. We assume that there exists such a minimum scale l_0 , that $\varepsilon(\mathbf{x}, l_0) \approx \varepsilon(\mathbf{x})$, $m(\mathbf{x}, l_0) \approx m(\mathbf{x})$.

The dimensionless field $\psi(\mathbf{x}, l, l') = \varepsilon(\mathbf{x}, l)/\varepsilon(\mathbf{x}, l')$ is similar to the dimensionless ratios of fields according to Kolmogorov (1962). Namely, the field $\psi(\mathbf{x}, l, l')$ is assumed to be translatory homogeneous, isotropic and of the scaling symmetry. The latter means that $\psi(\mathbf{x}, l, l')$ has the same probability distribution as the field $\psi(K\mathbf{x}, Kl, Kl')$ where K is a positive number. The field $\psi(\mathbf{x}, l, l')$ has too many arguments. We define a simpler field having the same information. To introduce such a field we use the following identity that holds by definition of $\psi(\mathbf{x}, l, l')$:

$$\psi(\mathbf{x}, l, l'') = \psi(\mathbf{x}, l, l')\psi(\mathbf{x}, l', l'').$$

Making l' to be infinitesimal as opposed to l , we obtain a simpler field $\varphi(\mathbf{x}, l) = \frac{d\psi(\mathbf{x}, l, ly)}{ldy}|_{y=1}$ that has the same information as $\psi(\mathbf{x}, l, l')$. The permeability $\varepsilon(\mathbf{x})$ is expressed via $\varphi(\mathbf{x}, l)$ as

$$\varepsilon(\mathbf{x}) = \varepsilon_0 \exp \left[- \int_{l_0}^L \varphi(\mathbf{x}, l) \frac{dl}{l} \right]. \quad (3)$$

Doing it in the same way for porosity, we obtain

$$m(\mathbf{x}) = m_0 \exp \left[- \int_{l_0}^L \chi(\mathbf{x}, l) \frac{dl}{l} \right], \quad (4)$$

where L is the scale of the largest fluctuations.

We assume the scale invariance, spatial homogeneity and isotropy to be present for the second centered correlation functions. Here we use a simple model, in which the statistical distributions for $\varphi(\mathbf{x}, l)$, $\chi(\mathbf{x}, l)$ are supposed to be Gaussian-distributed and delta-correlated in the scale logarithm for $\mathbf{x} = \mathbf{y}$:

$$\Phi^{\varphi\varphi}(\mathbf{x}, \mathbf{x}, l, l') = \langle \varphi(\mathbf{x}, l)\varphi(\mathbf{x}, l') \rangle_c = \Phi_0^{\varphi\varphi} \delta(\ln l - \ln l'), \quad (5)$$

where $\langle \rangle_c$ denotes the central moment,

$$\Phi^{\chi\chi}(\mathbf{x}, \mathbf{x}, l, l') = \langle \chi(\mathbf{x}, l)\chi(\mathbf{x}, l') \rangle_c = \Phi_0^{\chi\chi} \delta(\ln l - \ln l'). \quad (6)$$

The correlation between porosity and permeability is determined by the correlation between $\varphi(\mathbf{x}, l)$ and $\chi(\mathbf{x}, l)$:

$$\Phi^{\varphi\chi}(\mathbf{x}, \mathbf{x}, l, l') = \langle \varphi(\mathbf{x}, l)\chi(\mathbf{x}, l') \rangle_c = \Phi_0^{\varphi\chi} \delta(\ln l - \ln l'). \quad (7)$$

Thus, we do not violate the scale invariance of porosity and permeability. We consider a simple model, when statistical distributions for $\varphi(\mathbf{x}, l)$, $\chi(\mathbf{x}, l)$ are assumed to be Gaussian, and hence distributions of porosity and of permeability are logarithmic-normal.

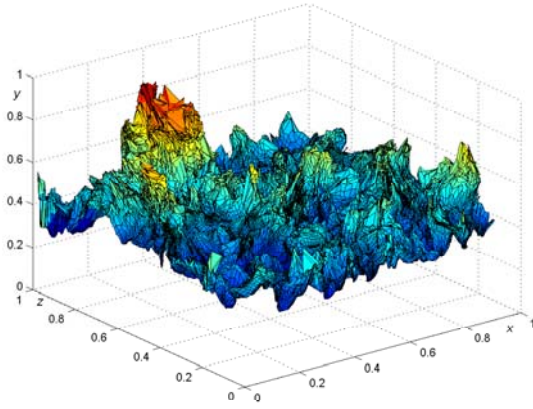


Figure 1. The interface between phases. The ratio of viscosities $\nu = 1$.

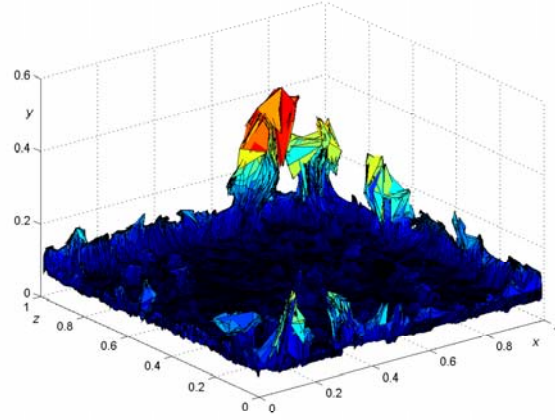


Figure 2. The interface between phases for the ratio of viscosities $\nu = 0.01$.

3 Results of numerical modeling

Equations (1), (2) are numerically solved in the cube with edges L_0 . A constant pressure is set at the faces $y = 0$, $y = L_0$: $p(x, y, z)|_{y=0} = p_1$, $p(x, y, z)|_{y=L_0} = p_2$, $p_1 > p_2$. The pressure of the other faces of the cube is set by a linear dependence in terms of y : $p = p_1 + (p_2 - p_1)y/L_0$. The main filtration flow is directed along the axis y . For the numerical calculation we use dimensionless variables. All lengths are measured in units of L_0 , the permeabilities being measured in units of the less viscous phase ε_{01} . Thus, it is sufficient to solve the problem in a unit size cube with a unit pressure difference and $\varepsilon_{01} = 1$. First we calculate the field of porosity and the field of permeability replacing integrals (3), (4) by the finite difference formula, in which it is convenient to use the logarithm with base 2:

$$\begin{aligned} \varepsilon(\mathbf{x}) &= \varepsilon_{0j} \exp \left[-\ln 2 \sum_{k=-8}^0 \varphi(\mathbf{x}, \tau_k) \delta\tau \right], \\ m(\mathbf{x}) &= m_0 \exp \left[-\ln 2 \sum_{k=-8}^0 \chi(\mathbf{x}, \tau_k) \delta\tau \right], \end{aligned} \quad (8)$$

where $j = 1, 2$, $\varepsilon_{01} = 1$, $\varepsilon_{02} = \mu_1/\mu_2$, μ_1, μ_2 are viscosities of phases, $l_k = 2^{7k}$; $\tau_k = k\delta\tau$, $\delta\tau = 1$ is the discretization step of the scale logarithm, $\tau_k = 0, -1, \dots, \log_2 \frac{1}{256} = -8$. We use a 256^3 grid in spatial variables. The fields $\varphi(\mathbf{x}, l_k)$ and $\chi(\mathbf{x}, l_k)$ are generated independently for every scale l_k because they are delta-correlated in terms of the scale logarithm. The common indices of degree are summarized over the statistically independent layers. For every τ_k , we obtain the two fields:

$$\begin{aligned} \varphi(\mathbf{x}, \tau_k) &= \sqrt{\frac{\Phi_0^{\varphi\varphi}}{\ln 2}} \zeta(\mathbf{x}, \tau_k) + \langle \varphi \rangle \ln 2, \\ \chi(\mathbf{x}, \tau_k) &= \sqrt{\frac{\Phi_0^{\chi\chi}}{\ln 2}} \left(r\zeta(\mathbf{x}, \tau_k) + \sqrt{1-r^2} \varsigma(\mathbf{x}, \tau_k) \right) + \langle \chi \rangle \ln 2, \end{aligned}$$

where $-1 \leq r \leq 1$. The fields $\zeta(\mathbf{x}, \tau_k)$, $\varsigma(\mathbf{x}, \tau_k)$ are independent, Gaussian, having a unit variance, zero average and the same correlation functions:

$$\langle \zeta(\mathbf{x}, \tau_i) \zeta(\mathbf{y}, \tau_j) \rangle_c = \langle \varsigma(\mathbf{x}, \tau_i) \varsigma(\mathbf{y}, \tau_j) \rangle_c = \exp \left[-\frac{(\mathbf{x} - \mathbf{y})^2}{2^{2\tau_i}} \delta_{ij} \right]. \quad (9)$$

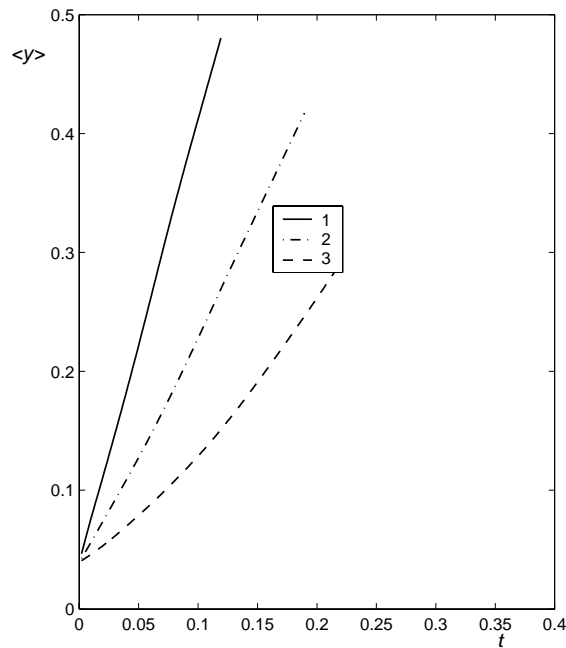


Figure 3. The average position of the front for the different ratios of viscosities: 1 – $\nu = 1$; 2 – $\nu = 0.5$; 3 – $\nu = 0.2$.

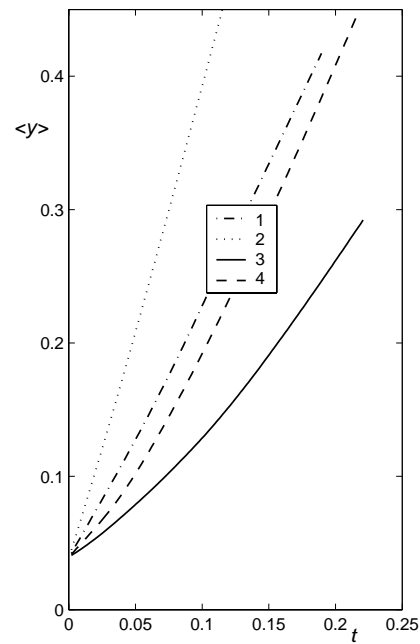


Figure 4. 1 – heterogeneous medium $\nu = 0.5$; 2 – the constant permeability and porosity are equal to mean values, $\nu = 0.5$; 3 – heterogeneous medium, $\nu = 0.2$; 4 – constant porosity and permeability $\nu = 0.2$.

Thus, we consider a simple correlation between the fields. The natural condition for the porosity $0 \leq m(\mathbf{x}) \leq 1$ is satisfied by the choice of parameters $\Phi_0^{\chi\chi}$, $\langle \chi \rangle$.

A particular structure of correlation functions enables us to present a common matrix as product of four correlation matrices. These matrices have a relatively low dimension. This fact allows us to apply the algorithm described in [6]. Equation (1) is solved by the iterative method. In the present calculations, the two upper and the three lower layers are left empty, i.e. the functions φ , χ in these layers are equal to zero. Two empty upper layers indicate to the fact that the largest scale $L = 1/8$. This allows us to replace the probable mean quantities by the space-averaged values. The three lower layers are also left empty to provide a good difference approximation for equation (1).

The parameters of calculation are $\Phi_0^{\varphi\varphi} = 0.3$, $\langle \varphi \rangle = 0.15$, $m_0 = 1$, $\Phi_0^{\chi\chi} = 0.05$, $\langle \chi \rangle = 0.7$, $\Phi^{\varphi\chi} = \sqrt{0.3 * 0.05}$, the coefficient correlation between porosity and permeability $\gamma = 0.92$, the variance of permeability of the first phase is equal to 1.297, the mean porosity being approximately equal to 0.15. Evolution of the front depending on the ratio of viscosities ν is displayed in Figs. 1, 2¹.

Fig. 1 shows the interface between the same phases $\nu = 1$. In Fig. 2 we have the interface between the phases for the ratio of viscosities $\nu = 0.01$ ($\nu = \mu_1/\mu_2$). The geometry of both media and the moment of time are the same. The interface between fluids has an unstable behavior and “viscous fingering” for the $\nu = 0.01$. The numerical simulation shows that within the domain of ratio of viscosities for oil $0.01 \leq \nu \leq 1$, the variances of the fronts strongly depend on heterogeneity of the porous media. In the Hele–Shaw cell permeability heterogeneities are, as a rule, not considered [1]. Fig. 3 shows average coordinates of the front depending on time for $\nu = 0.5, 0.2$. Such ratios of viscosities are typical of Siberian reservoirs. The comparison of the average coordinates of the front in a heterogeneous medium with the coordinates of the

¹Figures in colour will be available only in electronic version.

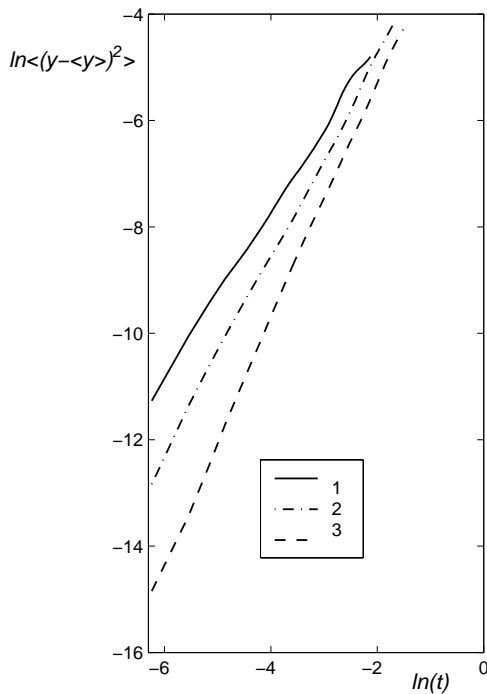


Figure 5. The logarithm of variance of the front along axis y for the different viscosities ratios: 1 – $\nu = 1$; 2 – $\nu = 0.5$; 3 – $\nu = 0.2$.

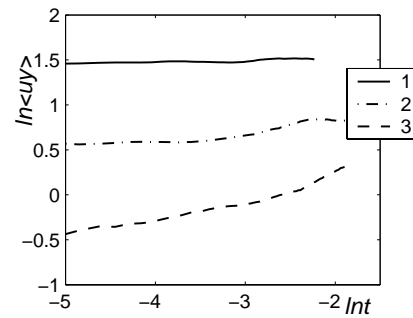


Figure 6. 1 – The logarithm of average velocity of the front for $\nu = 1.0$; 2 – $\nu = 0.5$; 3 – $\nu = 0.2$.

front for a medium with constant permeability and porosity is displayed in Fig. 4.

The variances of width of the front for different ν are displayed in the logarithm coordinates in Fig. 5. The slope angles α increase if the viscosities ratio decreases:

$$\nu = 1, \quad \alpha = 1.56; \quad \nu = 0.5, \quad \alpha = 1.84; \quad \nu = 0.2, \quad \alpha = 2.24.$$

The time dependence of variance of the front is a power function of time with the index of power α . This fact is a consequence of scale-invariance of medium.

The ratio of the longitudinal component and the transverse one of the diffusion tensor is constant and approximately equal to 3.5 for $\nu = 1$. (This value can also be evaluated by the perturbation theory). Non-diagonal components of the tensor are small for our problem. For $\nu = 0.5$, the ratio of the longitudinal component and the transverse one calculated for water at the moment t_{end} is approximately equal to 5.0. The average velocities along the axis y are displayed in Fig. 6.

4 Summary

We have considered 3-D model of displacement of immiscible fluids in fractal porous media. The “viscous fingering” occurs, similar to the flow in a horizontal square Hele–Show cell, although we do not consider the surface tension force. Different spatial moments of the front are calculated, and their statistical characteristics are presented. Dispersion of the water front is described by the second order diffusion tensor D_{ij} . All components of D_{ij} are computed. The value of the transverse diffusion is investigated and compared to the longitudinal one, while statistical parameters of the medium and viscosities of the fluids vary.

Acknowledgements

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