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Double Symmetry and Infinite-Component Field Theory

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Qualitative characteristics and the rigorous definition of the concept of the double symmetry is given. We use double symmetry for constructing a theory of fields not investigated before which transform as the proper Lorentz group representations decomposable into infinite direct sums of finite-dimensional irreducible representations. All variants of the double-symmetric free Lagrangian of such fields are described in brief. The solution of the problem of changing Lagrangian mass terms due to a spontaneous breaking of the secondary symmetry is stated. The general properties of the mass spectra of fermions in the considered theory are given. A region of free parameters is pointed out where the theoretical mass spectra qualitatively correspond to a picture typical for the parton model of hadrons.

1 Qualitative characteristics and the rigorous definition of the double symmetry

The main objective of our work presented here is a study of the possibility of an alternative description of hadrons and their interactions.

It is supposed that the composite particles can have an effective description with the help of monolocal fields which transform as representations of the proper Lorentz group $L_+^\uparrow$ decomposable into infinite direct sums of finite-dimensional irreducible representations. We call such infinite-component fields the ISFIR-class fields. Until recently, there was no study of them. It is necessary to note that the physical quantities, including the amplitudes of processes with participation of the particles described by ISFIR-class fields, are expressed through infinite functional series whose terms are proportional to the matrix elements of finite transformations of the proper Lorentz group. Due to this circumstance we can expect any behavior of a given physical quantity as the function of its variables, including such behavior which holds in hadron physics.

The elimination of an infinite number of arbitrary parameters in the relativistically invariant Lagrangians of the ISFIR-class fields is achieved due to an additional invariance of the theory. Initial (primary) and additional (secondary) symmetries of the theory form together the double symmetry. This concept can be considered as generalization for the long-known symmetry of the Gell-Mann–Levy σ-model [1] and supersymmetry.

In the σ-model, particles with different values of the spatial parity, namely, π- and σ-mesons, being a pseudoscalar and a scalar, accordingly, with respect to orthochronous Lorentz group $L^\uparrow$ are united in one multiplet. In order to ensure the invariance of the theory under space reflection P, Gell-Mann and Levy have evidently declared that the parameters of transformations, connecting between themselves the π- and σ-mesons, were the space pseudoscalars. The σ-model symmetry group $SU(2)_L \otimes SU(2)_R$ has found broad applications in the field theory and particle physics. However, except for Gell-Mann and Levy nobody, spoke in explicit form about the transformation properties of these group parameters under the P-reflection.

If one reformulates the left-right symmetric model of electroweak interactions [2–4] in the manner of Gell-Mann–Levy, identifying some parameters of the local gauge group with space scalars and the others with space pseudoscalars, it is possible to provide the initial P-invariance of such model and to reproduce all experimentally observed consequences of the Weinberg–Salam model. In this case the nature of the P-symmetry violation appears logically simple and clear. If the Higgs field has both scalar and pseudoscalar components, and they both have nonzero vacuum expectation values whose relative phase is not equal to $\pm\pi/2$, then either left or right weak charged current dominates depending on the relative signs of the quantities which are present in gauge transformations. It means that, as a result of spontaneous breaking of the local symmetry, the physical vacuum does not possess a certain P-parity. The fields of all intermediate bosons constitute a superposition of polar and axial 4-vectors, these vectors having equal weights in the fields of W-bosons. Transformation properties of the gauge bosons with respect to the orthochronous Lorentz group have the same powerful significance as the form of weak currents. However, they yield no experimental detection in contrast to the interaction vertices.

In Ref. [5], the grain of new group theoretical approach sown by Gell-Mann and Levy has received its generalization in the concept of double symmetry consisting of the primary and secondary symmetries.

The secondary symmetry has three essential features. First, the parameters of the transformations generating a global or local secondary-symmetry group H_T belong to the given T-representation space of the global group of the primary symmetry G. Second, the H_T-group transformations operate on the vectors of some S-representation space of the group G and do not take them out from this space, i.e. the secondary symmetry does not violate the primary one. Third, the primary- and secondary-symmetry groups G and H_T have no common elements except for the unity, and even in that case when they are isomorphic.

We restrict ourselves with the consideration of the global symmetries. The above-listed features find their realization in the following rigorous definition.

Definition 1. Let the group $\mathcal{G}_T$ contain the subgroup G and the invariant subgroup H_T, with $G \neq \mathcal{G}_T$, $H_T \neq \mathcal{G}_T$, and be their semidirect production $\mathcal{G}_T = H_T \circ G$. If any element $h_T \in H_T$ can be written in the form

$$h_T = h(\theta_1)h(\theta_2)\cdots h(\theta_n), \qquad n \in \{1, 2, \ldots\}, \tag{1}$$

where θ_i $(i = 1, 2, \ldots, n)$ is some vector of the representation space of T of the group G, and if for any element $g \in G$

$$gh(\theta)g^{-1} = h(T(g)\theta), \tag{2}$$

then the group G will be called the primary symmetry group, and the groups H_T and $\mathcal{G}_T$ will be respectively called the secondary and double symmetry groups generated by the representation T of the group G.

Constructing the double-symmetric field theories is based on the following consequence of the secondary-symmetry definition.

Corollary 1. *Let $\Psi(x)$ be any field vector in the S-representation space, and $\theta = \{\theta_a\}$ be a vector in the T-representation space of the group G. Then the transformation*

$$\Psi'(x) = \exp(-iD^a\theta_a)\Psi(x) \tag{3}$$

will be a secondary-symmetry transformation iff the condition

$$D^a = S^{-1}(g)D^bS(g)[T(g)]_b{}^a \tag{4}$$

is fulfilled, i.e. the operators D^a are the T-operators (transform as the representation T) of the group G.

In our construction [6,7] of the theory of infinite-component fields of the ISFIR-class the secondary symmetry is generated by transformations (3), with the index μ instead of the index a, in which the parameters θ_μ are the components of the polar or axial 4-vectors of the orthochronous Lorentz group, and the operators D^μ have the matrix realization. We postulate no closing of the algebra of the operators D^μ. It arises automatically in each variant of the considered theory due to those restrictions which are imposed by the requirement that the corresponding Lagrangians possess the double invariance.

2 Double-symmetric Lagrangians of the free ISFIR-class fields

The general structure of relativistically invariant free Lagrangians of the form

$$\mathcal{L}_0 = \frac{i}{2}[(\Psi, \Gamma^\mu \partial_\mu \Psi) - (\partial_\mu \Psi, \Gamma^\mu \Psi)] - (\Psi, R\Psi) \tag{5}$$

was found and described by Gelfand and Yaglom [8,9]. Let us remind that, according to Gelfand and Yaglom, the finite-dimensional irreducible representation τ of the proper Lorentz group is defined by such a pair of numbers (l_0, l_1) that $2l_0$ and $2l_1$ are integers of the same parity and $|l_1| > |l_0|$. The canonical basis vectors of this representation space are related to the subgroup $SO(3)$ and are denoted by $\xi_{(l_0,l_1)lm}$ where l is a spin, m is its projection onto the third axis, $m = -l, -l+1, \ldots, l$, and $l = |l_0|, |l_0|+1, \ldots, |l_1|-1$. The pairs (l_0, l_1) and $(-l_0, -l_1)$ define the same representations, $(l_0, l_1) \sim (-l_0, -l_1)$.

The requirement that the Lagrangian (5) is invariant under the group $L_+^\uparrow$ gives for the operator Γ^μ a condition which is equivalent to the relation (4) (at $g \in L_+^\uparrow$) for the operator D^μ from the secondary-symmetry transformation. It has the consequences expressed through the following equalities

$$\Gamma^i = \left[I^{i0}, \Gamma^0\right] \qquad (i = 1,2,3), \tag{6}$$

$$\begin{aligned}\Gamma^0 \xi_{(l_0,l_1)lm} &= c(l_0+1, l_1; l_0, l_1)\sqrt{(l+l_0+1)(l-l_0)}\,\xi_{(l_0+1,l_1)} \\ &+ c(l_0-1, l_1; l_0, l_1)\sqrt{(l+l_0)(l-l_0+1)}\,\xi_{(l_0-1,l_1)lm} \\ &+ c(l_0, l_1+1; l_0, l_1)\sqrt{(l+l_1+1)(l-l_1)}\,\xi_{(l_0,l_1+1)lm} \\ &+ c(l_0, l_1-1; l_0, l_1)\sqrt{(l+l_1)(l-l_1+1)}\,\xi_{(l_0,l_1-1)lm},\end{aligned} \tag{7}$$

where I^{i0} are the infinitesimal operators of the proper Lorentz group, and $c(l_0', l_1'; l_0, l_1) \equiv c_{\tau'\tau}$ are arbitrary parameters. The same equality are valid for the operator D^μ with arbitrary quantities $d_{\tau'\tau}$.

Infinite number of arbitrary parameters $c_{\tau'\tau}$ in the relativistically invariant Lagrangians of the free ISFIR-class fields was the serious reason for that until recently there were no research of such field theory.

Involved into our consideration are the $L_+^\uparrow$-group representations S, in whose decomposition the multiplicity of each finite-dimensional irreducible representation does not exceed unity.

The Lagrangian (5) (at $R \neq 0$) will be invariant under the secondary-symmetry transformations (3) generated by the polar or axial 4-vector of the group $L^\uparrow$, if the condition

$$[\Gamma^\mu, D^\nu] = 0 \tag{8}$$

is satisfied. This condition is reduced to an algebraic system of the equations with respect to quantities $c_{\tau'\tau}$ and $d_{\tau'\tau}$.

As the analysis shows this system of the equations has nontrivial solutions only for the quite certain countable set of the $L_+^\uparrow$-group representations S. The fermionic field can be described either by any of the representations

$$S^{k_1} = \sum_{n_1=0}^{+\infty} \sum_{n_0=-k_1+1/2}^{k_1-3/2} \oplus \left(\frac{1}{2} + n_0, k_1 + n_1\right), \tag{9}$$

where $k_1 \geq 3/2$, or by the representation S^F which contains all finite-dimensional irreducible half-integer spin representations of the group $L_+^\uparrow$. The bosonic field can correspond either to any of the representations

$$S^{k_1} = \sum_{n_1=0}^{+\infty} \sum_{n_0=-k_1+1}^{k_1-1} \oplus (n_0, k_1 + n_1), \tag{10}$$

where $k_1 \geq 1$, or to the representation S^B which contains all finite-dimensional irreducible integer spin representations of the group $L_+^\uparrow$.

Three variants of the theory are assigned to each of representations (9). In one of them the secondary symmetry is generated by a polar 4-vector of the group $L^\uparrow$, the operators D^μ commute with each other

$$[D^\mu, D^\nu] = 0, \tag{11}$$

i.e. the group H_T is the Abelian one, and the quantities $c_{\tau'\tau}$ are given by equalities

$$\begin{aligned} c(l_0+1, l_1; l_0, l_1) &= c(l_0, l_1; l_0+1, l_1) \\ &= c_0 D(l_1) \sqrt{\frac{(k_1 - l_0 - 1)(k_1 + l_0)}{(l_1 - l_0)(l_1 - l_0 - 1)(l_1 + l_0)(l_1 + l_0 + 1)}}, \end{aligned} \tag{12}$$

$$\begin{aligned} c(l_0, l_1+1; l_0, l_1) &= c(l_0, l_1; l_0, l_1+1) \\ &= c_0 D(l_0) \sqrt{\frac{(k_1 - l_1 - 1)(k_1 + l_1)}{(l_1 - l_0)(l_1 - l_0 + 1)(l_1 + l_0)(l_1 + l_0 + 1)}}, \end{aligned} \tag{13}$$

where $D(j) = 1$, and c_0 is a real constant. In the second and third variants of the theory the secondary symmetry is generated by an axial 4-vector of the group $L^\uparrow$. In the second variant, the formulas (9)–(11), where $D(j) = (-1)^{j-1/2} j$, are valid. In the third variant the following inequality holds

$$[G^{\mu\nu}, G^{\rho\sigma}] \neq 0, \tag{14}$$

in which the antisymmetric operator $G^{\mu\nu}$ is involved. This operator is determined as follows

$$G^{\mu\nu} = [D^\mu, D^\nu]. \tag{15}$$

Four variants of the theory at $k_1 \geq 2$ and two variants at $k_1 = 1$ are assigned to each of representations (10). All variants divides into pairs in which the secondary symmetry is generated either by a polar, or by an axial 4-vector of the group $L^\uparrow$. In one pair of the variants $k_1 \geq 1$ and the equality (11) is valid, and in other pair $k_1 \geq 2$ and the inequality (14) holds.

The representation S^F is assigned by one variant of the theory, and the representation S^B is by two variants. In these cases the operators $G^{\mu\nu}$ are nonzero and commute with each other, and the secondary-symmetry transformations of fermionic (bosonic) fields is generated by the axial (polar or axial) 4-vector of the group $L^\uparrow$.

In all variants of the double-symmetric theory the operator R, specifying the mass term of Lagrangian (5), is a multiple of the identity operator.

3 A spontaneous secondary-symmetry breaking

In complete accordance with the known Coleman–Mandula theorem [10] relating to consequences of the Lorentz-group extension, the mass spectrum corresponding to any of the variants of the infinite-component field theory with the double symmetry is infinitely degenerate with respect to spin. This degeneracy forces us to suppose that the secondary symmetry is spontaneously broken, namely, that the scalar (with respect to the group $L^{\uparrow}$) components of one or several bosonic infinite-component ISFIR-class fields have nonzero vacuum expectation values λ^i. A spontaneous secondary-symmetry breaking introduces only one correction into the free-field theory with the initial double symmetry which consists in changing the operator R in the Lagrangian (5). So, in the fermionic-field Lagrangian, this operator takes the form

$$R = \kappa E + \lambda^i Q_i^{(0,1)00}, \tag{16}$$

where κ is a constant. The P-even operators $Q_i^{(0,1)00}$ come to relation (16) from the interaction Lagrangian

$$\mathcal{L}_{\text{int}} = \big(\psi(x), Q_{i'}^{\tau lm} \varphi^{i'}_{\tau lm}(x)\psi(x)\big), \tag{17}$$

where $\psi(x)$ is the fermionic field, $\varphi^{i'}_{\tau lm}(x)$ is the bosonic-field component, and $Q_i^{\tau lm}$ are the matrix operators.

The Lagrangian (17) containing nonzero operators $Q_i^{(0,1)00}$ possesses the considered double symmetry only in two cases denoted by $\mathcal{A}$ and $\mathcal{B}$. In each of them the secondary-symmetry group is the four-parametrical Abelian group, the fermionic field transforms as any of the representations S^{k_1} (9), and bosonic fields transform as the representation S^1 (10). In the case $\mathcal{A}$, the secondary symmetry is generated by the polar 4-vector of group $L^{\uparrow}$, and $D(j) = 1$. In the case $\mathcal{B}$, the secondary symmetry is generated by the axial 4-vector of group $L^{\uparrow}$ and $D(j) = (-1)^{j-1/2}j$. In both cases the relations

$$R\xi_{(l_0,l_1)lm} = r(l_0,l_1)\xi_{(l_0,l_1)lm} = [\kappa + \lambda^i q_i(l_0,l_1)]\xi_{(l_0,l_1)lm}, \tag{18}$$

$$\begin{aligned}
&(k_1 - l_0 - 1)(k_1 + l_0)q_i(l_0+1,l_1) + (k_1 - l_0)(k_1 + l_0 - 1)q_i(l_0-1,l_1) \\
&\quad - (k_1 - l_1 - 1)(k_1 + l_1)q_i(l_0,l_1+1) - (k_1 - l_1)(k_1 + l_1 - 1)q_i(l_0,l_1-1) \\
&\quad = z_i(l_1 - l_0)(l_1 + l_0)q_i(l_0,l_1)
\end{aligned} \tag{19}$$

are valid for all irreducible representations $(l_0,l_1) \in S^{k_1}$. The parameter z_i is expressed through the ratio of normalization constants of the operators D^{μ} in the secondary-symmetry transformations (3) of the bosonic and fermionic fields. It can have any values. If $z_i = 2$, then the bosonic-field transformation (3) is trivial ($D^{\mu} = 0$), i.e. it is not involved in the secondary symmetry.

In what follows we will deal only with fermionic fields which transform according to the "lowest" of the representations S^{k_1} (9) of the proper Lorentz group, namely, according to the representation $S^{3/2}$. In this case the equation (19) has a unique solution, which can be written in the form

$$q_i\left(-\frac{1}{2},l_1\right) = q_i\left(\frac{1}{2},l_1\right) = 2q_{i0}\frac{u_i^N(u_iN+N+1) - w_i^N(w_iN+N+1)}{N(N+1)(u_i-w_i)(2+u_i+w_i)}, \tag{20}$$

where $N = l_1 - 1/2$, $u_i = \left(z_i + \sqrt{z_i^2-4}\right)/2$ and $w_i = \left(z_i - \sqrt{z_i^2-4}\right)/2$.

4 Properties of the mass spectra in the ISFIR-class field theory

We have all what is necessary for solving of the first physical problem which concerns the general properties of the mass spectra in the double-symmetric theory of the ISFIR-class fermionic fields.

In the rest system of a particle with the mass M the Gelfand–Yaglom equation corresponding to the Lagrangian (5) has the form

$$(M\Gamma^0 - R)\psi_{M0} = 0. \tag{21}$$

Its solutions must satisfy the selection condition at which the amplitudes of various processes are finite. We formulate such condition of finiteness of the amplitudes as the relation

$$|(\psi_{M0}, R\psi_{Mp})| < +\infty \tag{22}$$

or

$$(\psi_{M'0}, R\psi_{Mp}) = a(p)\delta(M' - M), \tag{23}$$

where ψ_{Mp} is the particle state vector with the momentum p directed along the third axis, and $a(p)$ is a nonzero number.

In the formula (22) or (23) the bilinear form is expressed through the infinite series whose terms are linear with respect to the components of vector ψ_{M0}, to the components of other vector ψ_{M0} or $\psi_{M'0}$, and to the matrix elements of finite transformations of the proper Lorentz group. These matrix elements for irreducible representation (l_0, l_1) behave as $T_0 \exp(\alpha l_1)/l_1$ at $l_1 \to +\infty$ where $\tanh\alpha = p/\sqrt{M^2c^2 + p^2}$, and the quantity T_0 does not depend on the number l_1 [11].

In the space of the $L_+^\uparrow$-group representation $S^{3/2}$ the equation (21) for the field vector components $(\psi_{M0})_{(\pm 1/2, l_1)lm} \equiv \chi_{lm}(l_1)$ with the spatial parity $(-1)^{l-1/2}$ takes the form

$$D\left(\frac{1}{2}\right)\frac{\sqrt{(l_1 - l)(l_1 + l + 1)}}{2l_1 + 1}\chi_{lm}(l_1 + 1) + D\left(\frac{1}{2}\right)\frac{\sqrt{(l_1 - l - 1)(l_1 + l)}}{2l_1 - 1}\chi_{lm}(l_1 - 1)$$
$$- \left[\frac{D(l_1)(2l + 1)}{4l_1^2 - 1} - \frac{1}{2Mc_0}r(l_1)\right]\chi_{lm}(l_1) = 0, \tag{24}$$

where $l_1 \geq l$, $r(l_1) \equiv r(\pm 1/2, l_1)$. At the replacement of the value M by $-M$ the P-parity of solution of this equation changes to the opposite one.

In a situation when the secondary symmetry of the theory is broken spontaneously we failed either to find solutions of the equation (24) in the form of elementary or special functions, or to find analytical formulas for mass spectra of the theory. However, we are able to make a number of conclusions concerning the mass spectra on the basis of asymptotic behavior of some quantities. On the basis of on these conclusions it is possible to find any number of the lower values of the mass M with the help of numerical methods.

We have analyzed the mass spectrum characteristics in all details in both cases $\mathcal{A}$ and $\mathcal{B}$ of the considered theory provided that the spontaneous breaking of the secondary symmetry is caused by one ISFIR-class bosonic field, and $\kappa = 0$ in formulas (16) and (18).

In the region of the parameter values $z_1 \in (-2, 2)$ the solutions of equation (24) cannot satisfy the condition of finiteness of the amplitudes (22) or (23) at any values of the quantity M, i.e. the mass spectrum is empty.

In the region of the parameter values $z_1 \in (-\infty, -2] \cup (2, +\infty)$ the set of all masses of the theory is restricted below by some positive number. At $l_1 \to +\infty$ the quantities $r(l_1)$ and $\chi_{lm}(l_1)$

have the following asymptotic behavior

$$r(l_1) = r_0 \frac{v^{l_1+\frac{1}{2}}}{l_1+\frac{1}{2}} \left(1+\mathcal{O}\left(l_1^{-1}\right)\right), \tag{25}$$

$$\chi_{lm}(l_1) = A_0 G(l_1) \left(1+\mathcal{O}\left(l_1^{-1}\right)\right) + B_0 G^{-1}(l_1) \left(1+\mathcal{O}\left(l_1^{-1}\right)\right), \tag{26}$$

where

$$G(l_1) = \frac{x^{l_1-\frac{1}{2}} v^{\frac{4l_1^2-1}{8}}}{\left(l_1-\frac{1}{2}\right)!}, \qquad x = -\frac{r_0}{Mc_0 D\left(\frac{1}{2}\right)}, \qquad v = \begin{cases} w_1, & \text{if } z_1 \in (-\infty, -2], \\ u_1, & \text{if } z_1 \in (2, +\infty). \end{cases} \tag{27}$$

If the quantity A_0 is not equal to zero at some value of M, then neither condition (22) nor condition (23) cannot be fulfilled. If $A_0 = 0$ at some value of M, then the condition (22) is fulfilled. Hence, for all z-parameter values from the region $z \in (-\infty, -2)$, the mass spectrum is discrete, if it is not empty. As the calculations show in the whole parameter region $z > 2$, each value of spin and parity corresponds to an accountable set of the masses rising to infinity. The lowest values of mass levels, corresponding to a definite spin, grow with a spin. Thus, in the qualitative sense the theoretical mass spectra correspond to a picture typical for the quark-gluon model of hadrons.

We compare the theoretical mass spectrum with nucleon resonance levels provided that the spontaneous breaking of the secondary symmetry is caused by two ISFIR-class bosonic fields, and $\kappa = 0$ in formulas (16) and (18). In this situation we pay attention to the following property of the particle state vectors in the considered theory.

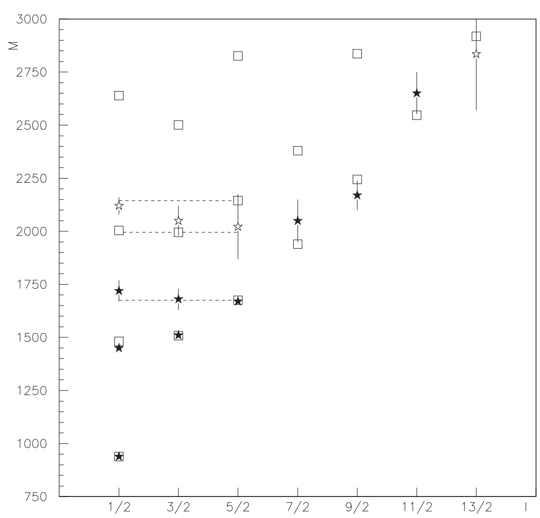

Figure 1.

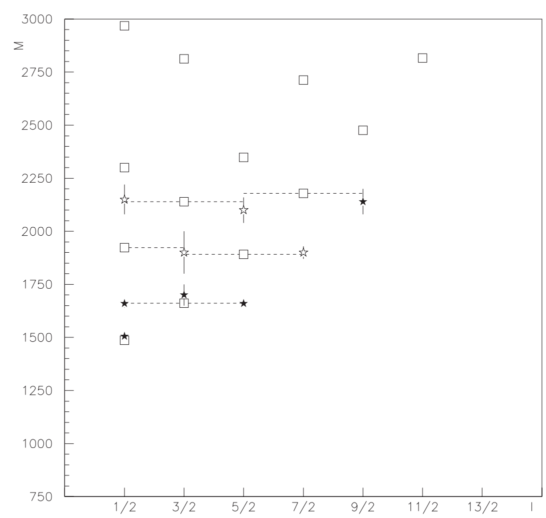

Figure 2.

In the reference system where the ground fermion (boson) of the theory possesses a nonzero velocity, its state vector has nonzero components with all half-integer (integer) spins, irrespective to what spin corresponds to this particle in its rest system. This results in that the amplitude of the resonance decay into ground particles will contain any partial wave in its decomposition. Certainly, only those of them may be observable in experiments which have an appreciable weight. Thus, one resonance can be shown in experiments as a group of several resonances with the same mass and differing only in their spin.

The results given in Figs. 1 and 2 correspond to the case $\mathcal{A}$ of the considered theory at the following choice of parameters $\lambda^1 q_{10}/c_0 = -939/2.4686$ MeV, $z_1 = 2.036$, $z_2 = 0.14$, $\lambda^2 q_{20}/\lambda^1 q_{10} = -0.6724$. The levels with the partial parities $(-1)^{l-1/2}$ and $(-1)^{l+1/2}$ are represented in Figs. 1 and 2 correspondingly. The theoretical masses are shown by the square markers, and the experimental masses (the pole positions) are done by the dark- and light-star markers for, accordingly, well and badly established nucleon resonances [12]. The Breit–Wigner mass is taken for the resonance $N(1440)$ because two poles are found around 1440 MeV, $1370-114i$ and $1360-120i$, whose parameters are much different from the conventional ones $M = 1470$ MeV and $\Gamma = 545$ MeV [13]. The first approximation of the theory to experiments can be estimated as satisfactory.

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